AEROACOUSTICS ANALYSIS OF TRAILING EDGE INCLUDING POROSITY EFFECTS

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Abstract. Trailing edge noise on rigid surfaces is a significant source of sound in aircraft. In order to minimize the sound radiated by a process involving acoustic waves, the present article investigates the effects of trailing edge noise generation on rigid semi - infinite and porous plates. For this purpose, the Boundary Element Method is employed to solve the Helmholtz equation, subject to boundary conditions that seek to model the vibration and porosity effects of the plates. The study is developed for a two-dimensional problem, in which the incident source of noise generation was a lateral quadrupole, positioned at the trailing edge. In this paper the lateral quadrupole represents a turbulent vortex in the boundary layer of the plate, according considerations presented in the literature. Initially, the study verified the acoustical directivity for the rigid plate with the wave numbers k = 0.1, 1, 5 and 10. In a second moment, the directives for the porous plate with k = 0.1 and 10 were analyzed. From the results it was found that because the sides of the porous plate have opposite phases, the pressure variation on the surface of the plate opposes pressure in the region above it causing noise reduction. It was also found that the porosity effect causes noise reduction more effectively at low wave numbers.

Keywords: Acoustic scattering, Trailing edge noise, Boundary element method

1 Introduction

The trailing edges noise have been recognized as an important noise source for a long time, the principal mechanism being the scattering of turbulence as it is advected near the edge, as demonstrated by Ffowcs Williams and Hall in the problem of acoustic scattering by a rigid, semi- infinite flat plate [1, 2]. This scattering pressure fluctuations structures produces undesirable noise in a variety of aeronautic applications, as blades, wings and flaps [3].

Due to the imminent importance of the noise generated by leading edges, diverse researches have been studying methods to mitigate this form of acoustic scattering. Elasticity is a solution that has been studied profoundly, and is known to decrease sound radiated by a process involving coupled acoustic and bending waves. In this research field, Manela [4] studied the motion and sound of a thin elastic plate, subject to uniform low-Mach flow and actuated at its leading edge, where the linearized response to arbitrary small-amplitude translation and rotation is analyzed using Fourier decomposition of the forcing signal. Both periodic and non-periodic actuations are investigated. With the study, it was verified that in the case of sinusoidal excitation, the plate elasticity has two opposite effects on sound radiation depending on the forcing frequency. At frequencies close to the near-resonance motion results in the generation of high sound levels. However, at frequencies far from resonance, the plate elasticity reduces the amplitude of plate deflection (compared to that of a rigid plate), leading to noise reduction. In the case of non-periodic actuation, the plate-fluid system amplifies those frequencies that are closest to, which in turn, dominate the acoustic signature. The results identify the trailing edge noise as the main source of sound, dominating the sound generated by direct plate motion.

Nilton et al [3] also carried out studies to verify how the elasticity in plates reduces the acustic noise in leading edge. In the study, plates of composite materials were modeled through a numerical method of boundary elements. The results show that composite materials are advantageous since laminates lead to lower acoustic scattering when compared to structurally equivalent metallic plates. This behavior is due to a lower specific mass, leading to higher coupling between fluid and solid, and thus to more significant elasticity effects, decreasing substantially the radiated sound.

Another approach that has been extremely verified for the reduction of this form of noise, is related to plates with porosity. Thus, several works are being developed to quantify and qualify the parameters of this new noise rusting methodology in leading edges.

Peake and Jaworski [5] studied the effects of elasticity and porosity on the efficiency of aerodynamic noise generation between turbulent eddy with the semi-infinite poroelastic edge. The scattering problem was solved using the Wiener Hopf technique to identify the scaling dependence of the resulting aerodynamic noise on plate and flow properties. Asymptotic analysis of these special cases reveals parametric limits where the far-field acoustic power scales like for a porous edge, and a new finite range of behaviour is found for an elastic edge to be compared with the well-known dependence for a rigid impermeable edge.

Numerical studies are also being developed for the analysis and optimization of this new noise reduction methodology, among which the work published by [6–9] stands out. In these studies, numerical methodologies, such as boundary elements and finite elements were developed to analyze the noise reduction at the trailing edges.

The purpose of the present work is investigates the effects of trailing edge noise generation on rigid semi - infinite and porous plates. For this purpose, the Boundary Element Method (BEM) is employed to solve the Helmholtz equation, subject to boundary conditions that seek to model the vibration and porosity effects of the plates. The study is developed for a two-dimensional problem, in which the incident source of noise generation was a lateral quadrupole, positioned at the trailing edge.

2 Mathematical modeling

The problem in question consists of an incident source of quadrupole noise generation positioned at the point (1.0, 0.004), i.e, at the trailing edge position. However with a distance of $\Delta y = 0.004$ m above the line of the center of the plate. The lateral quadrupole is representative of a turbulent vortex in the boundary layer of the plate, according to Ffowcs Williams and Hall [1]. Rigid and porous plates will be studied, where they have a unit chord, c = 1 and the thickness t = 0.002 m. The elements on the upper and lower surfaces of the plate have constant length Δx and the center of the plate is positioned at $y_2 = 0$. Thus, we wish to determine the scattered sound at a given observer position r, as shown in Fig. 1. The turbulent eddy in the vicinity of the trailing edge generates an incident quadrupolar sound field. A quadrupole in free field has a near-field pressure that is mostly reactive and does not propagate to the far acoustic field. However, owing to the source proximity to the edge, the said near-field pressure is now scattered by the plate and radiates to the far field by this mechanism [10].

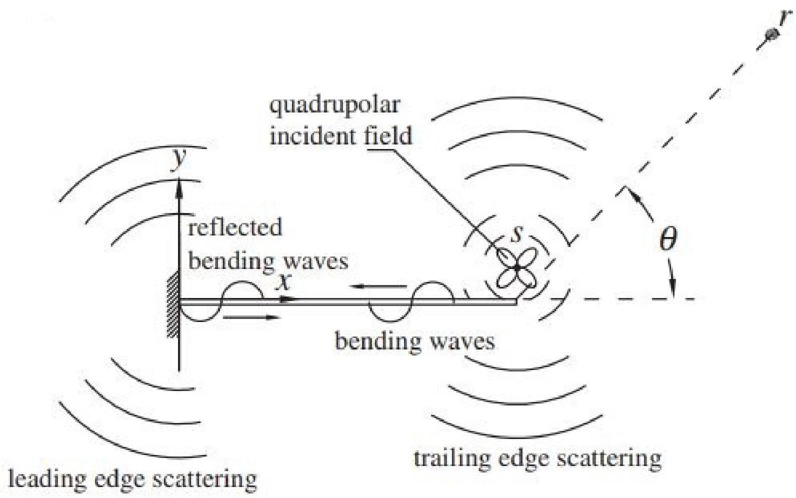


Figure 1: illustration of the problem and its main physical characteristics. Available from: [8]

To obtain the scattered sound, we solve the Helmholtz equation, which describes a reflection of acoustic waves in a quiescent medium, is given according to [11] by:

$$\nabla^2 \hat{p} + k^2 \hat{p} = \hat{S} \tag{1}$$

Where $\hat{p}$ is the acoustic pressure in the frequency domain, $k = \omega / c_0$ is the acoustic wave number and $\hat{S}$ represent the turbulent eddy.

The porosity effects can be used to reduce noise at trailing edges. This mechanism is used in nature in the wings of owls, which are silent hunters. In this work the porosity model based on a conductivity of Rayleigh, K_R was considered. In this model, we can calculate the pressure variation through the pores of the plate as:

$$\left(\frac{\partial p}{\partial y} \right)_{sup} = -\frac{\alpha_H K_R}{2R} \Delta p \tag{2}$$

In the above equation, α_H represents the porous fraction of the plate, K_R is the Rayleigh conductivity which provides a measure of the volume of fluid flowing through the pores as a function of the pressure variation along them, R represents the radius mean of the pores and Δp is a measure of the pressure difference on the plate, Δp=p_{lower}-p_{upper}. This model assumes that the pores can be distributed irregularly along the plate.

3 Numerical analysis

The Boundary Element Method (BEM) is applied to solve numerically the Helmholtz equation, Eq. 1. The BEM approach consists of solving the problem by integrating the contour surface instead of integrating the entire volume of fluid. In the problem, the sound waves are produced by concentrated sources representative of turbulent eddies positioned near the plate trailing edge. The following non-homogeneous Helmholtz equation represents the pressure disturbances induced by the concentrated sources in a quiescent medium.

$$\nabla^2 p(\mathbf{x}) + k^2 p(\mathbf{x}) = -S_i \tag{3}$$

In Eq. 3, S_i represents the ith source strength. A quadrupole source is chosen to provide the incident acoustic field because it represents the noise from a compact turbulent eddy [1, 12]. A fundamental solution for the Helmholtz equation is the free space Greens function, G$(\mathbf{x},\mathbf{y})$, written as:

$$G(\mathbf{x},\mathbf{y}) = \frac{i}{4} H_0^{(1)} \left(k\sqrt{(x_1-y_1)^2 + (x_2-y_2)^2} \right) \tag{4}$$

Here, $H_0^{(1)}$ stands for the Hankel function of the first kind and order zero. The incident quadrupolar field can be computed as the second derivative of the Greens function.

As we can see, the porosity will be added to the equation by the value of $\frac{\partial p}{\partial n} = \frac{\partial p}{\partial y} n_y$. Thus, the equation of contour elements for a porous plate should retain all terms of the integral Helmholtz equation, in terms such as:

$$T(\mathbf{x})p(\mathbf{x}) = \int_\Gamma \left[\frac{\partial p(\mathbf{y})}{\partial n_y} G(\mathbf{x},\mathbf{y}) - \frac{\partial G(\mathbf{x},\mathbf{y})}{\partial n_y} p(\mathbf{y}) \right] d\Gamma - \frac{\partial G(\mathbf{x},z_i)}{\partial z_{im} \partial z_{in}} S(z_i) \tag{5}$$

where T($\mathbf{x}$) = 1/2 when $\mathbf{x}$ is on the analyzed surface Γ, and T($\mathbf{x}$) = 1 when $\mathbf{x}$ is in the acoustic field in the region of the fluid.

In matrix form, the BEM equation is written as:

$$[H]\{p\} - [G]\left\{ \frac{\partial p}{\partial n} \right\} = \{S\} \tag{6}$$

The coefficients of the matrix [H] are given by h_{ij} = 1/2 for i = j e $h_{ij} = \int_\Gamma \frac{\partial G(x_i,x_j)}{\partial n_j} d\Gamma$ for i $\neq$ j. The coefficients of the matrix G are written as $g_{ij} = \int_\Gamma G(x_i, y_j)$. In the latter case, when i = j we have the following analytical solution:

$$g_{ii} = \Gamma_i \left[\frac{1-\gamma - ln(0.25k\Gamma_i)}{2\pi} \right] + 0.25i \tag{7}$$

At where γ = 0.5772156649 is Euler constant, Γ_i is the length of the boundary element i, k is the number wave and i represents the complex number.

Applying the boundary conditions we have the following linear system:

$$([H] - [G][D])\{p\} = \{S\} \tag{8}$$

where the elements of matrix D are calculated as:

$$D_{ik} = -\frac{\alpha_H K_R}{2R} n_{yi}(n_{yk}\delta_{k,i} + n_{yk}\delta_{k,N-i}) \tag{9}$$

In the above equation, N represents the number of boundary elements used in the discretization of the plate.

it was assumed that the boundary elements i and $N-i$ have centroids in the same positions y_1, i.e., if element i is above the plate (below), element $N-i$ is in the same position y_1 below (above) of the plate.

4 Results and discussion

To study an acoustic scattering problem on trailing edge with porosity effects we have calculated the sound radiated near a lateral point quadrupole source of intensity equal to 3. The trailing edge of the studied plate has a unit length (L = 1) and thickness t = 0.002. The quadrupole source is located at a distance 0.002 above the trailing edge $z = [1, 0.004]$. We calculate the relative change in acoustic power level due to effects of porosity and present directivity results to 360 observers at a distance of 50 chords from the plate trailing edge. The polar angle θ is measured according to the schematic in Fig. 1.

To use the present formulation of the BEM code, it is necessary to calculate the radiated sound by a plate of small but finite thickness. In all cases analysed in this work, this thickness is much smaller than the acoustic wavelength and, therefore, the directivities obtained by the BEM are close to the expected results for plates of zero thickness [13]. For the plate discretization, 200 elements were used on the upper surface of the plate and 200 elements on the lower surface. One element was used to "close" the plate on each side.

4.1 Variation of the acoustic wave number

In Fig. 2 the acoustic wave number k of 0.1 to 10 was scanned for an impermeable plate, that is $\alpha_H/R = 0$. For k = 0.1, the wavelength is λ = 10 π which is much larger than the plate length (L = 1), so it can be considered a compact surface, reflecting the waves as a dipole. The amplitude increases considerably with increasing wavelength with k = 1 as there is more reflecting area, however the directivity remains similar to a dipole source because the wave length is still larger than the plate. For higher values of k the wave length is close to or less than the size of the plate, so it can not be considered a compact surface and the waves scattered along the regions of the plate form interference that generate lobes like a cardioid.

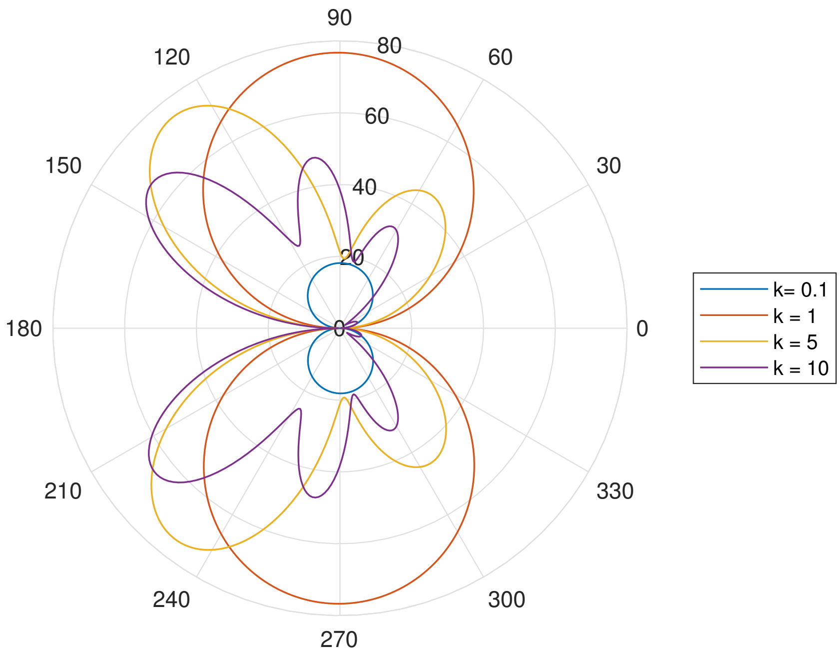
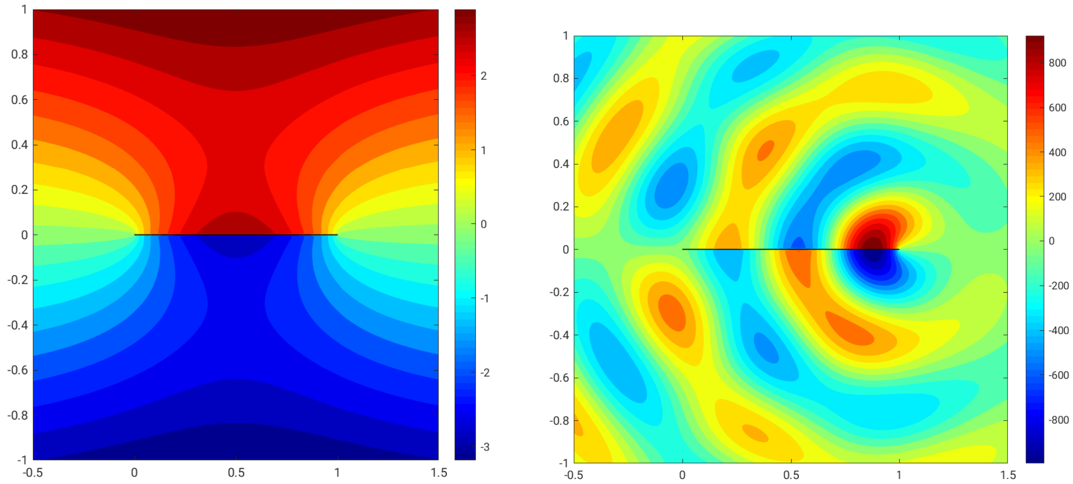


Figure 2: Variation of acoustic wave number in Impermeable plate.

To show the general changes of the radiated sound due to the change of the wave number on the plate, Fig. 6 shows the pressure fields of the plate calculated as $|p|\sin\angle p$ for the most extreme cases analyzed, $k = 0.1$ and 10. It can be seen that for $k = 0.1$, Fig. 6a, a symmetric pressure fluctuation pattern occurs on the plate, this causes the sound to be irradiated as a dipole, as also indicated in Fig. 2. As for the analysis for $k = 10$, we see that in the vicinity of the trailing edge (x, y) = (1, 0) the scattered field presents the cardioid directivity typical of the semi-infinite edge problem [1], with waves with maximal amplitude propagating towards the plate leading edge. As these acoustic waves approach the leading edge, they are backscattered. This process occurs multiple times and leads to a directivity shape close to the original cardioid, modified by lobes [14, 15]. These lobes are visible in Figs. 2 and 6b, especially near the trailing edge.



(a) Pressure field for k =0.1 and $\alpha_H/R = 0$

(b) Pressure field for k = 10 and $\alpha_H/R = 0$

Figure 3: Pressure fields for rigid plate

4.2 Porosity variation

For the evaluation of the plate with porosity, we can verify from the Eq. 2 that $(\partial p/\partial y)_{sup}$ is always in phase opposition with Δp, this phase opposition should diminish the radiated sound. Figures 4 and 5 presents directivity patterns for compact and non-compact perforated plates, where porosity leads to reductions of the radiated sound in all directions. For lower k, illustrated by the $k = 0.1$ results in Fig. 4, the directivity maintains its compact dipole shape, decreasing in amplitude with increasing α_H/R, and for higher k (10 for Fig. 5) the directivity shape changes progressively from a lobed cardioid to the dipole shape for higher α_H/R. Figure. 6 shows the pressure field for the most extreme cases of analyzed wave numbers for porous plates. It is verified that the greater k the smaller the phase opposition, leading to the conclusion that the effect of the porosity in the reduction of the sound decreases when k increases.

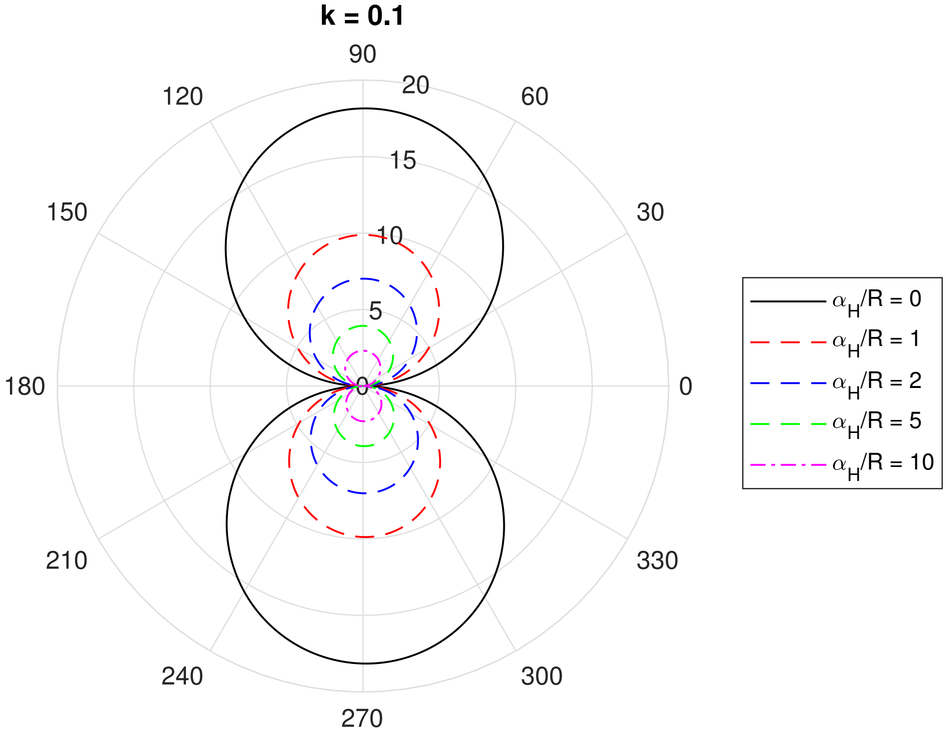


Figure 4: Scanning of porous fraction values for $k = 0.1$

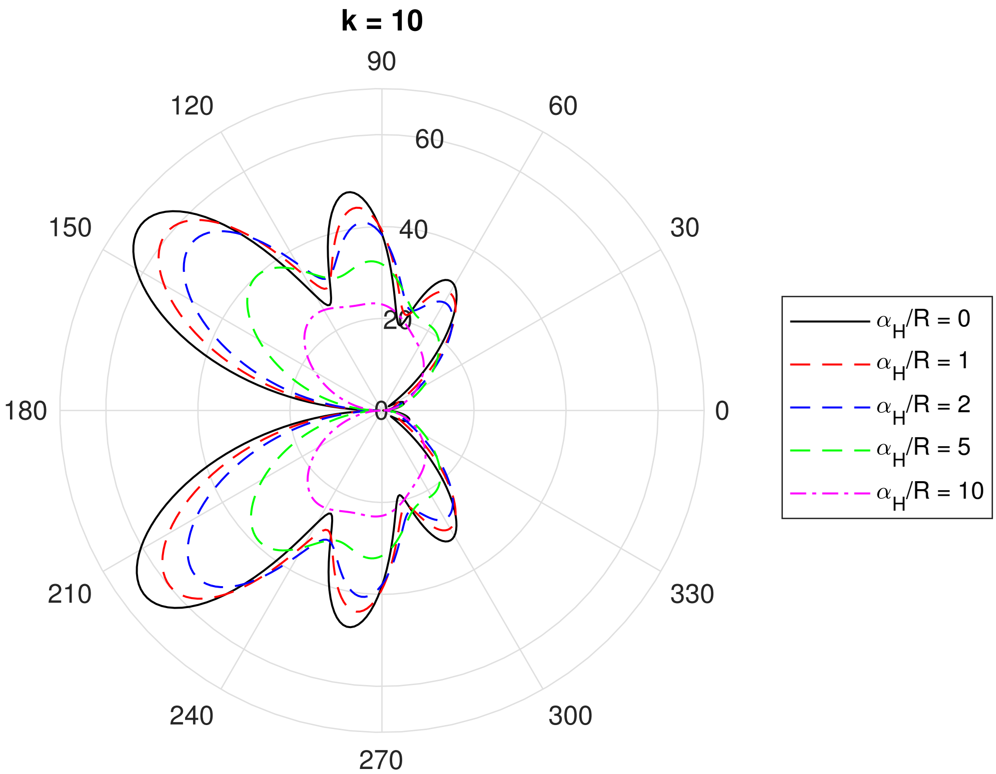
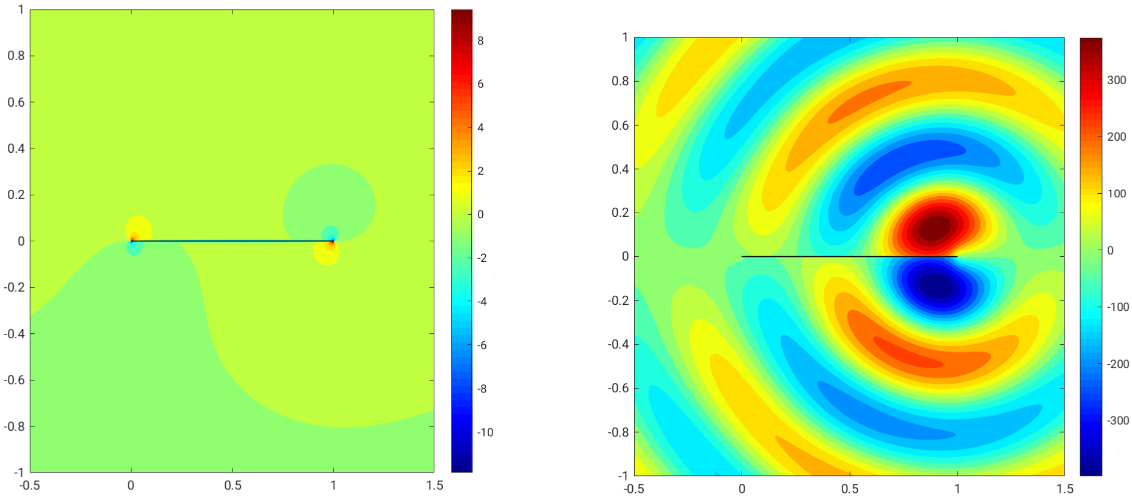


Figure 5: Scanning of porous fraction values for $k = 10$



(a) Pressure field for $k = 0.1$ and $\alpha_H/R = 10$ (b) Pressure field for $k = 10$ and $\alpha_H/R = 10$

Figure 6: Pressure fields for porous plate

5 Conclusion

The work presents a formulation to calculate the acoustic field scattered by rigid and porous plates. The Boundary Element Method (BEM) is used to solve the problem under study. Initially, the adopted numerical method solves the pressure on each plate and then solves the pressure for the 360 observers at a distance of 50 lengths of the plate chord. With the present method, we have observed that the general behaviors observed here emphasize that the effects of porosity lead to significant reductions of the irradiated sound at low values of k, this is due to a phase opposition between $\partial p/\partial y$ and e Δp, which is related to the reduction of scattered sound. However, this reduction decreases as the acoustic parameter increases.

With the results presented for high values of wave number, it is concluded that the porosity in plates decreases the sound radiation only for small ranges of this parameter. Thus, future analyzes will seek by means of simulations to consider the plate as elastic and porous, to verify if this new effect, elasticity, reduces the noise radiated to high wave numbers.

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