XL CILAMCE
IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING
NOVEMBER 11-14, 2019
Praiamar Natal Hotel & Convention
Natal, RN-BRAZIL

OPTIMIZATION OF PRESTRESSED WAFFLE SLABS VIA GENETIC ALGORITHM

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Abstract. In the majority of current buildings, the use of prestressing has enabled the design of projects with larger spans than the conventional ones. Besides, the technique has favoured greater durability of the structures, since it avoids problems of cracking and excessive deformation in service. However, most projects are still heavily guided by the expertise of the structural designer, which does not guarantee optimal projects from an economic point of view. This work aims to present an optimization formulation for waffle slabs executed with current plastic moulds, using the heuristic of genetic algorithms in the program 'BIOS' (Biologically Inspired Optimization System). The analysis of the efforts was integrated into the model using the 'FAST' program (Finite Element Analysis Tool). To evaluate optimum changes in the geometric characteristics of plastic moulds, currently provided in commercial catalogues, design variables related to the cross-section geometry of the slab, the number of tendons per rib and their eccentricity in the middle of the span were selected. Thus, the application of the evolutionary algorithm seeks to minimize the objective function of the model, composed of the costs with the materials and labour used, while attending to the constraints of ultimate and service states in the design, arranged in standards. In all cases, the slabs are simply supported and the prestressing was considered as equivalent load. A comparison is made between the costs of the formulated optimization model and another one that incorporates the geometry of the plastic moulds currently available in the market. The outcomes indicate that reasonable reductions can be achieved in the total execution costs of the waffle slabs by varying the input parameters of the algorithm, such as the spans adopted. Also, is evaluated the effect of adopting slabs with higher spans ratio over the optimization results.

Keywords: Waffle slab, Prestressing, Optimization, Genetic algorithm.

1 Introduction

The use of unbonded post-tensioning in waffle slabs has furthered designs with spans even larger and favoured the adoption of lighter structures that also meet all ultimate strength and service requirements (ABNT [1], ABNT [2] and ABNT [3]). Prestressing seeks to optimize structural elements by compressing the cross-section, limiting or eliminating cracking and excessive displacements, and thus improving strength and durability. Advantages of post-tensioning in bidirectional waffle slab has been studied, cite Moldovan and Mathe [4].

Protensão Impacto® has been one of the pioneer companies in the development of modularized plastic moulds and prestressing systems to be applied in buildings. To uplift cost savings, *PavPlus system*© has been conceived to enable quick construction performance with the employment of fewer materials and the insertion of an industrial-made welded steel grids and post-tension tendons in predefined places on the floor.

Before the new design realities, resources as numerical tools provide a good way to find the optimum geometry of post-tensioned waffle slabs, in which costs are the lowest. Genetic Algorithms (GA's) are widely suggested in the literature for its merit of computational efficiency to handle with probabilistic optimization problems, especially when the design variables are discrete in a finite search space (SAHAB et al. [5]). Thereby, an objective function is computationally drawn, mainly composed of materials costs. The variables concern the cross-section geometry and the tendon's profile, and the problem needs to obey some predetermined constraints.

Quoting Lopes et al. [6], the study led to the choice of bigger plastic moulds even for small spans, as an algorithm's attempt to increase the concrete area and to prescind stirrups in the slab. From the research, most of the cases showed maximum eccentricity as an active constraint, which led to the best performance of tendons to balance loads. The authors ratified that the higher the spans, the greater the costs.

Related to structural analysis, internal forces and displacements are obtained using the Grillage Analogy Method, that transforms the arrangement of waffle slabs in a mesh whose beams have the same inertia moment as the interest geometry section. This method has been commonly added into commercial software for structural design, also academy has recommended it as a non-complex programming method for slab analysis.

Towards the study of optimum arrangements for post-tensioned waffle slabs, this work aims to apply a mono-objective optimum formulation to single simply supported bidirectional slabs, with prestressing modelled as a charge that counterbalances dead and live loads. Based on the adopted methodology from Sobrinho [7], stress-strains are taken from "FAST" and read by "BIOS" to process the optimization. Both are open-code programmes developed in LMCV-UFC (ROCHA et al. [8]). Since the optimization of floors is attached to several and complex variables, the optimization of single slabs has been useful to provide some guidelines for economic, feasible and serviceable solutions. Using current plastic moulds of and also enabling the geometry variables free, optimum designs are obtained in a couple of project cases and finally, some important remarks are made at the end of this paper.

2 Optimization model

2.1 Project variables

The variables chosen were the distance between rib axes - b_f, the height of the moulds - h, the inferior and superior width of ribs - $b_{w,inf}$ and $b_{w,sup}$ respectively, the thickness of the slab table - h_m. Furthermore, the number of post-tension tendons every three ribs - N_p and the position of parabolic tendons, in the middle of spans, from the bottom of the section - y_{cb} were also selected to make up the prestressing problem. Regarding the definition of geometry, all variables were firstly chosen from a current set of available plastic moulds and afterwards, they were set free so that the algorithm could reach for other solutions. All variables representation can be seen in Fig. 1.

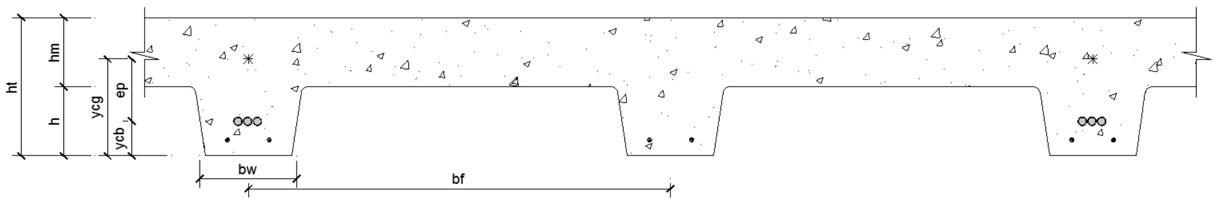

Figure 1. Transversal section of waflle post-tensioned slab

2.2 Objective function

The objective function, observed in Eq. (1), seeks to represent the cost of the prestressed waffle slab, based on the unit consumption of the materials used, such as concrete - c_c (R$/m^3), post-tension steel - c_p (R$/kg) and welded grid of reinforcement steel - c_r (R$/kg). Important parameters for the function are: mass of tendon - w_p (kg/m^2), mass of welded grid of steel - w_p (kg/m^2), and the optimum volume of concrete - V_c (m^3/m^2), all per meter squared.

$$C_t = c_c V_c + c_p w_p + c_r w_r. \tag{1}$$

2.3 Constraints

The main constraints that must be met were divided into three groups. Regarding size constraints, the first group included all variables lower and upper bounds. Then, the second group held service state requirements, such as decompression limit state, cracking limit state, excessive deflections limit states, vibrational limit state, maximum and minimum compression stresses at the application of prestressing force and also under service. Finally, the third group introduced the ultimate strength state by the analysis of shear stress and bending stress. Numerically, all constraints have been normalized to help the algorithms seek for valid solutions in a patterned way. The general constraint formulation can be understood from Eq. (1) as follows.

$$g_i\left(\overrightarrow{\mathbf{x}}\right) = \frac{V_{design}}{V_{limit}} - 1.0; \quad 0 \leq i \leq n. \tag{2}$$

3 Structural analysis

The present work chose ”T” sections to describe the transversal geometry of mesh bars, including its inertia moment and centroid position since it is closer to real waffle slabs layout. Using the Grillage Analogy Method in ”FAST”, the model included distributed loads, due to self-weight, coating, masonry and also live loads. To evaluate tension and compression stresses, post-tension has been considered of level two, in which both frequent and permanent load combinations are admitted. All combination factors followed the item 11.8 from ABNT [1], as well as physical properties of materials were selected following item 8.

4 Genetic algorithm

Belonging to a class of bioinspired algorithms, GA's starts by choosing the initial population, where individuals are randomly combined according to the adopted crossing rate, and giving rise to a new population of solutions every generation. With the elitism rate, the best solutions of the previous generation are preserved, and the mutation probability causes random changes in the individuals to avoid premature convergence. Individuals are selected according to their fitness function, that is, the fittest tend to pass their characteristics to the subsequent generations, while the less fit ones tend to be penalized and discarded by the optimization. The stopping criterion often becomes the maximum number of generations.

In this work, the calibrated input parameters for optimization were: 10 optimizations, 50 number of generations, population size of 20 individuals, the crossover rate at 0.85 and the mutation probability at 0.30. *Adaptive Method* was assumed to evaluate penalized objective function and individuals were selected following the *Ranking* strategy.

5 Application and discussion

For the problem, class II was chosen to the environmental aggressiveness, with coverage of 20 mm to reinforcement steel and 30 mm to tendons. The parameters adopted were: prestressing tendon with a yield stress of 1670 MPa and ultimate stress of 1860 MPa, reinforcement steel with a yield stress of 600 MPa, concrete compressive resistance f_{ck} of 35 MPa, masonry load of 1.0 kN/m^2, coat load of 1.0 kN/m^2, and live load of 2.0 kN/m^2. The materials costs were concrete cost - R$ 326.34, prestressing cost with tendons and labour - R$ 16.00, and reinforcement steel cost - R$ 5.60, obtained from SINAPI /CE 2019 and also from Protensão Impacto®.

To optimize bidirectional slabs of a residential building, the problem variables included the geometry of the plastic moulds and then, were set free in a range from the lower to the upper bounds shown in Table 1. Besides that, the tendons profile hold possible positions using available commercial spacers, and the initial prestressing force adopted was 150 kN, with immediate losses of 6% and final losses of 20%.

Table 1. Variables domain to the optimization problem

Limits	h_m (cm)	h (cm)	b_f (cm)	$b_{w,inf.}$ (cm)	$b_{w,sup.}$ (cm)	N_p	y_p (cm)
Lower bound	10	10	50	12	15	0	4
Upper bound	20	20	90	15	20	9	16

The discrete variables have step equal to 1.0, except the inferior and superior width of ribs, whose step is 0.5. In the aim of show how optimization can modify some parameters of the available plastic moulds to achieve better costs, project cases involved both models and the increasing of λ along the process, also to measure the optimization behaviour of slabs that becomes to the edge of bidirectional criteria.

Table 2. Results of the optimization cases

			Commercial model		Proposed model	
Case	λ	Size (m x m)	b_f, h, N_p, y_{cb}	Cost (R$/m^2)	b_f, h, N_p, y_{cb}	Cost (R$/m^2)
1	1.00	7.0 x 7.0	$\{61, 15, 4, 5\}$	85.31	$\{85, 19, 5, 6\}$	81.61
2	1.25	8.0 x 10.0	$\{61, 20, 5, 6\}$	99.09	$\{80, 20, 6, 6\}$	90.62
3	1.50	9.0 x 13.5	$\{61, 20, 6, 5\}$	106.44	$\{90, 19, 9, 7\}$	99.10
4	1.75	10.0 x 17.5	$\{61, 20, 8, 6\}$	121.31	$\{79, 20, 9, 6\}$	107.52
5	2.00	11.0 x 22.0	$\{61, 20, 9, 6\}$	128.56	$\{61, 20, 8, 4\}$	119.23

The results obtained for the commercial model had an average of 94% for load balancing and a success rate that ranged from 40% to 80%, while the Proposed model showed an average of 97% for load balancing and a level of success rate of 20%, mainly explained for the high number of different individuals throughout optimization. The reinforcement rate declined along with the project cases for both models.

6 Conclusion

A model for optimization of waffle bidirectional slabs with unbonded prestressing tendons was presented, by comparing the outcomes of slabs using current commercial plastic moulds to those with geometric variables of a cross-section set free.

The optimization led to the use of one type of plastic mould in 80% of the commercial model cases, suggesting its good performance to achieve optimum results. As expected, the number of tendons increased as spans were made wide, reaching its maximum number in Case 5. Moreover, eccentricities were around slightly upper the maximum value, owing to constraints of cracking and displacements in the act of prestressing the tendons. Related to the proposed model, although moulds' height was almost the same as the previous model, the distance between rib axes depended on the span adopted, indicating an average of 80 cm. Hence, the algorithm pointed larger distances between rib axes to attain better benefits from post-tensioning in waffle slabs. The number of tendons attached the maximum since Case 3, and the eccentricity became the maximum in Case 5, where loads were the greatest. Also, the costs were lower than previous model, achieving a difference of 13% in Case 4.

Internal forces increased when spans became larger due to the rise of self-weight, and the results tended to include more tendons, exploiting the maximum capacity of the selected plastic mould and obeying constraints as maximum precompression and compressive stresses in the act of prestressing.

Acknowledgements

The authors acknowledge FUNCAP-CE for the financial support given in the form of a scientific initiation grant, to UFC-CE for the help of costs to attend at a scientific event and to Protensão Impacto® for the partnership in the research.

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