XL CILAMCE
IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING
NOVEMBER 11-14, 2019
Praiamar Natal Hotel & Convention
Natal, RN-BRAZIL

DESIGN OF REINFORCED CONCRETE UNDER ECCENTRIC LOADS WITH NEWTON-RAPHSON'S METHOD

Jéssica Yasmin Vieira Lima
Danilo Menezes Santos
Jorge Carvalho Costa
jessicavieiral94@gmail.com
dmsantosse@gmail.com
jorgecostase@gmail.com
Universidade Federal de Sergipe
Av Marechal Rondom S/N, São Cristóvão, Sergipe, Brasil

Abstract. This article aims to use a numerical algorithm in Matlab for the design of rectangular reinforced concrete columns under eccentric loads. Usually, both in academic and professional practice, the design of such columns makes use of dimensionless charts or tables. These charts are subject to specific restrictions as section shape, reinforcement positioning, covering dimensions, among others, so that the design is limited to their availability. The alternative described employs Newton-Raphson's method to solve the nonlinear system of equations that arises from section equilibrium, keeping the neutral axis position and reinforcement diameters as variables. The nonlinear constitutive behavior of both materials involved is considered so that the equilibrium is expressed as a system of two nonlinear equations for bending normal to a symmetry axis. The concrete stress distribution proposed by Brazilian standard NBR 6118:2014 was used and a perfect elasto-plastic model was adopted for steel. The algorithm was validated by comparing the steel area and modes of failures obtained with traditional design methods, for several columns on different limit states.

Keywords: Reinforced concrete, Newton-Raphson's method, Nonlinear structural mechanics

1 Introduction

The use of computer software in civil engineering, just as in many areas of study, allows for an improved data analysis and better design, as structural behavior can be simulated and analyzed for many different applications. This work aims at implementing an algorithm in MATLAB to find the necessary steel area in a reinforced concrete column section, subject to ultimate limit state design criteria, for a given loading condition.

Mosley, Bungey and Hulse [1] states that the area of steel reinforcement needed for a cross-section is determined by (i) using design charts or constructing $M - N$ interaction diagrams (ii) a solution of the basic design equations, or (iii) an approximate method. The first approach is the most usual for hand calculations of common geometries, but is subject to the availability of design charts as in Montoya, Messeguer and Cabré [2] or Venturini and Rodrigues [3], thus not suitable for computer implementation.

This works describes a computer implementation for the design of reinforced concrete sections subject to normal forces and bending moment based on the solution of the section's equilibrium equations using Newton-Raphson's method (NRM). As a computer code, it is an automatic process that makes possible the easy manipulation for different sections.

2 Section equilibrium

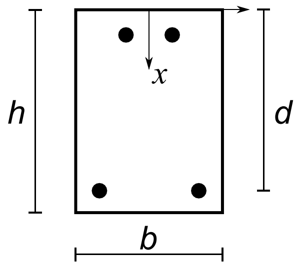

Figure 1. Concrete cross-section

Figure 1 shows a reinforced rectangular cross-section with sides $b \times h$ and symmetric by the vertical x axis and with centroid coordinate x_{CG}. Let this section have n reinforcement rods each with area A_s and position x_i, with the lowest bar having coordinate d. The section is subject to design normal force N_d and bending moment M_d that will cause stresses on the concrete that amount to the force F_c on the concrete and F_{si} in each steel bar. The section will be equilibrated if

$$\sum_{i=1}^{n} M_{si} + M_c - M_d = 0 \text{ and } \sum_{i=1}^{n} F_{si} + F_c - N_d = 0, \tag{1}$$

where M_c and M_{si} are respectively the moments of the force acting on the concrete and steel in relation to the x_{CG} axis.

The Bernoulli beam hypothesis is adopted so that the section remain plane and both steel (ε_{si}) and concrete (ε_c) strains are proportional to the distance to the neutral axis. The reinforcement area will be determined in ultimate limit state, where section strains are described trough the deformation domains defined by ABNT [4] by the position x_{NA} of the neutral axis.

The constitutive relation for concrete neglect tension and adopts a rectangular approximation for compressive stresses and a limited elasto-plastic relation is assumed for steel, also according to ABNT [4]. The design strength of concrete is f_{cd} and the resulting force on concrete is

$$F_c = \begin{cases} 0, \text{ if } x_{NA} < 0 \\ -0.68 \cdot f_{cd} \cdot b \cdot x_{NA}, \text{ if } 0 \leq x_{NA} \leq 1.25\, h \\ -0.85 \cdot f_{cd} \cdot b \cdot h, \text{ if } x_{NA} > 1.25\, h \end{cases}. \tag{2}$$

The force on a steel rod of diameter ϕ_i depends on whether it has yielded, so that

$$F_{si} = \begin{cases} -A_s\, f_{yd}, \text{ if } \varepsilon_{si} \leq -\varepsilon_y \\ A_s\, E_s\, \varepsilon_{si}, \text{ if } -\varepsilon_y \leq \varepsilon_{si} \leq \varepsilon_y \\ A_s\, f_{yd}, \text{ if } \varepsilon_{si} > \varepsilon_y \end{cases}. \tag{3}$$

The moments due to these forces with respect to the centroid of the concrete area are

$$M_c = F_c \cdot (0.4 \cdot x_{NA} - x_{CG}) \text{ and } M_{si} = F_{si} \cdot (x_i - x_{CG}). \tag{4}$$

2.1 Compatibility conditions

Brazilian standards by ABNT [4] limit the strains on concrete $\|\varepsilon_c\| \leq 3.5‰$ and steel $\|\varepsilon_s\| \leq 10.0‰$. Along with the linear strain distribution hypothesis, these limitations generate a compatibility statement of the strains on a section under failure described as five domains and governed by the position of the neutral axis x_{NA}.

For Domain 1, the neutral axis is over the cross-section so that $x_{NA} \leq 0$, the whole section is under tension and the strains will be limited by the lowest steel rod. The strains on the other rods and on concrete will be respectively

$$\varepsilon_{si} = 10 \cdot \frac{x_i - x_{NA}}{d - x_{NA}} \text{ and } \varepsilon_{cu} = 0. \tag{5}$$

On Domain 2, compressive stresses arise. The neutral axis will be on $0 < x_{NA} < 0.259d$, the strains will be $10.0‰$ on the lowest steel rod and $0 < \varepsilon_{cu} \leq 3.5‰$ on the most compressed concrete fiber.

$$\varepsilon_{si} = 10 \cdot \frac{x_i - x_{NA}}{d - x_{NA}} \text{ and } \varepsilon_{cu} = -10 \cdot \frac{x_{NA}}{d - x_{NA}}. \tag{6}$$

When the neutral axis is in a position between $0.259d < x_{NA} \leq 0.628d$, the section is said to failure on Domain 3. The largest strain in concrete will be $\varepsilon_{cu} = 3.5‰$ and the most strained steel rod will still be under yield $\varepsilon_{yd} < \varepsilon_{si} \leq 10‰$. When the neutral axis is on Domain 4, its position will vary between $0.628d < x_{NA} \leq h$, the maximum strain on concrete will be $\varepsilon_{cu} = -3.5‰$ and the lowest reinforcement will be on the elastic regime. For both cases, the strain on the other rods will be

$$\varepsilon_{si} = 3.5 \cdot \frac{x_i - x_{NA}}{x_{NA}}. \tag{7}$$

Lastely, on Domain 5, the section is fully compressed, the neutral axis will be in a position such that $x_{NA} > h$ and the strain on the most compressed concrete area will vary between $-2‰ < \varepsilon_{cu} \leq -3.5‰$. Nevertheless, the whole section cannot be on the plastic region at the same time, so strains are limited around a point $c = 3h/7$ below the most compressed fiber. The strains are, then

$$\varepsilon_{cu} = -2 \cdot \frac{x_{NA} - d}{x_{NA} - c} \text{ and } \varepsilon_{si} = 2 \cdot \frac{x_i - x_{NA}}{x_{NA} - c}. \tag{8}$$

3 Nonlinear system

The nonlinear system of equations arises from Eq. (1) and (2) with the appropriate substitutions for each domain. As described in Ruggiero and Loppes [5], in order to use NRM, there is the need to find the Jacobian of the equations, composed by the derivative of the equilibrium equations by the system variables, conveniently chosen to be the position of the neutral axis x_{NA} and the diameter of a rod ϕ.

$$J = \begin{bmatrix} \sum_{i=1}^{n} \frac{dM_{si}}{dx_{NA}} + \frac{dM_c}{dx_{NA}} & \sum_{i=1}^{n} \frac{dM_{si}}{d\phi} + \frac{dM_c}{d\phi} \\ \sum_{i=1}^{n} \frac{dF_{si}}{dx_{NA}} + \frac{dF_c}{dx_{NA}} & \sum_{i=1}^{n} \frac{dF_{si}}{d\phi} + \frac{dF_c}{d\phi} \end{bmatrix}$$

The derivatives of concrete stresses (M_c and F_c) are all zeros if the section is on Domain 1 or $x_{NA} \geq 1.25\ h$. If the section is in $0 \leq x_{NA} \leq 1.25\ h$,

$$\frac{dM_c}{dx_{NA}} = F_c \cdot 0.4 + 0.68 \cdot f_{cd} \cdot b \cdot x_{cg};\ \frac{dF_c}{dx_{NA}} = -0.68 \cdot fcd \cdot b;\ \frac{dM_c}{d\phi} = 0 \text{ and } \frac{dF_c}{d\phi} = 0.$$

The following are the derivatives pertaining to the steel forces, for a rod is in its elastic regime.

In Domains 1 and 2: $\frac{dF_{si}}{d\phi} = \frac{\phi \cdot \pi}{2} \cdot E \cdot 10 \cdot \frac{x_i - x_{NA}}{d - x_{NA}}$ and $\frac{dF_{si}}{dx_{NA}} = \frac{\phi^2 \cdot \pi}{4} \cdot E \cdot 10 \cdot \frac{x_i - d}{(d - x_{NA})^2}$

In Domains 3 and 4: $\frac{dF_{si}}{d\phi} = \frac{\phi \cdot \pi}{2} \cdot E \cdot 3.5 \cdot \frac{x_i - x_{NA}}{x_{NA}}$ and $\frac{dF_{si}}{dx_{NA}} = \frac{-\phi^2 \cdot \pi}{4} \cdot E \cdot 3.5 \cdot \frac{x_i}{(x_{NA})^2}$

For Domain 5: $\frac{dF_{si}}{d\phi} = \frac{\phi \cdot \pi}{2} \cdot E \cdot 2 \cdot \frac{x_i - x_{NA}}{x_{NA} - c}$ and $\frac{dF_{si}}{dx_{NA}} = \frac{-\phi^2 \cdot \pi}{4} \cdot E \cdot 2 \cdot \frac{x_i - c}{(c - x_{NA})^2}$

For the moment, the derivatives are, for all Domains,

$$\frac{dM_{si}}{d\phi} = \frac{dF_{si}}{d\phi} \cdot (x_i - x_{cg}) \text{ and } \frac{dM_{si}}{dx_{NA}} = \frac{dF_{si}}{dx_{NA}} \cdot (x_i - x_{cg}).$$

When the steel rod is on the plastic region,

$$\frac{dF_{si}}{d\phi} = \frac{\phi \cdot \pi}{2} \cdot (\pm f_{yd});\ \frac{dM_{si}}{d\phi} = \frac{dF_{si}}{d\phi} \cdot (x_i - x_{cg});\ \frac{dF_{si}}{dx_{NA}} = 0; \text{ and } \frac{dMs_i}{dx_{NA}} = 0$$

4 Results

To validate the algorithm, some examples of design of steel reinforcement where carried out using design charts. The physical and geometrical characteristics of the sections where fixed (f_{cd}, reinforcement position, section and cover sizes), just as the design stresses. From an initial guess for the neutral axis position and the rod diameters, the program solves the equilibrium equations.

4.1 First set

The first set of sizing examples comes from Chust and Pinheiro [6], imposing different stresses for a fixed size $20 \times 30\ cm$ section with $3\ cm$ cover and 4 reinforcement bars. The results are in Table 1.

4.2 Second set

This second set is proposed by Araújo [7] for different concrete strength $f_{cd} = f_{ck}/1.4$. The section consists of a $20 \times 40cm$ rectangle, with $4\ cm$ cover and 4 rows of 2 bars each, subject to a normal force of $N_d = 574kN$ and bending moment $Md = 143.5\ kNm$. The results are shown in Table 2.

Table 1. Results for the first set of examples

$N_d\ (kN)$	$M_d\ (kNm)$	ϕ chart (cm)	ϕ algor. (cm)	Difference
276	0	1,683139	1,6819	0,0736%
0	110	3,069523	3,0554	0,4601%
-367	110	2,640263	2,6487	0,3196%
-643	55	1,938055	1,9716	1,7309%
-1010	55	2,746624	2,6599	3,1575%

Table 2. Results for the second set of examples

$f_{ck}(MPa)$	ϕ chart (cm)	ϕ algor. (cm)	Difference
25	1,841627	1,8362	0,2947%
30	1,717306	1,7117	0,3265%
40	1,485223	1,4787	0,4392%
50	1,318925	1,3186	0,0247%

5 Conclusions

It is important to note the need for a good initial guess for the problem, as convergence of NRM is only guaranteed close to the solution.

For the cases of pure tension (Domain 1) and pure compression (5), the algorithm converges slowly, as the neutral axis will approximate $\pm\infty$. This consideration can be explored in future works.

The difference between the results described herein and those found in the reference works is small and probably due to approximations both in the computational approach and the use of charts.

The algorithm introduced is then flexible, as it is readily adaptable for different reinforcement distributions, reliable and easy to implement in a computation environment. It can also be improved as to provide commercially available diameters as an output.

References

[1] B. Mosley, J Bungey and R. Hulse. *Reinforced concrete design to Eurocode 2*. New York: Palgrave Macmillan, 2007.
[2] P. J. Montoya, Á. G. Meseguer and F. M. Cabré. *Hormigón armado*. 14th edition. Barcelona: Gustavo Gill, 2000.
[3] W. S. Venturini and R. O. Rodrigues. *Dimensionamento de peças retangulares de concreto armado solicitadas à flexão reta*. São Carlos: EESC-USP, 1987.
[4] Associação Brasileira de Normas Técnicas (ABNT). NBR 6118 Design of concrete structures – Procedure. Rio de Janeiro: ABNT, 2014.
[5] M. A. G. Ruggiero and V. L. R. Loppes. *Cálculo Numérico: Aspectos Teóricos e Computacionais*. Pearson, 2006.
[6] R. C. de Carvalho and L. M. Pinheiro. *Cálculo e Detalhamento de Estruturas Usuais de Concreto Armado*. v.2, 1.ed. São Paulo: Pini, 2009.
[7] J. M. de Araújo: *Curso de Concreto Armado* – Rio Grande: Dunas, 2003. v.4, 2.ed.