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A RESERVOIR SIMULATOR BASED ON FORMULATIONS OF ISOGEOMETRIC BOUNDARY ELEMENTS

Lúcio G. Nascimento
Gustavo S. V. Gontijo
Éder L. de Albuquerque
lucio.engmec@gmail.com
ggontijo@gmail.com
eder@unb.br
Affiliation
Universidade de Brasília, 70910-900, Brasília/DF, Brasil
Lucas S. Campos
ufesisc@gmail.com
Affiliation
Av. Fernando Ferrari 514 Goiabeiras, 29075-910, Vitória/ES, Brasil
Jon Trevelyan
jon.trevelyan@durham.ac.uk
Affiliation
Durham, DH1, United Kingdom

Abstract. The development of the reservoir simulator for the study of water and gas cone phenomena in the producer well (represented by a sink) is obtained by combining the Boundary Element Method (BEM) and the Isogeometric Formulation using the NURBS (Non Uniform Rational B-Spline) as shape functions. Through the Isogeometric Formulation, the discretization of the geometric model (mesh generation), which is the step of the numerical analysis that requires more time for the engineer, is no longer necessary, since the same functions that describe the geometry can also approximate the field variables in the BEM, making it a great advantage in its use. The same discretization used in the geometric model, generated in CAD (Computer Aided Design) modeling programs, can also be used by the BEM. From the modeling of the potential flow, i.e. the study of multiphase flow in porous media from the Laplace Equation, the isogeometric formulation can be coupled to the problem and, therefore, to obtain the reservoir simulator for the single phase case. The determination of the boundary conditions for the model, including the analysis of fluids interface movement, is also presented. The final code is able to present itself as an efficient tool for the analysis of oil extraction in reservoirs in the presence of undesirable fluid production. Validation of the results is carried out by comparing to the conventional Boundary Element Method.

Keywords: Boundary Element Method (BEM), Isogeometric formulation, Non Uniform Rational B-Spline (NURBS), Reservoir simulator, Potential flow.

1 Introduction

The cost of implanting an oil production plant (onshore or offshore) is quite high, even by the standards of a consolidated member company of this market such as PETROBRAS. High productivity is always desired, regardless of the prospecting conditions. This means production in the largest volume possible; a "clean" oil and gas with lower concentration of methane. Only for optimum production conditions, however, it is possible to obtain extraction at the desired speed and purity. Far from the utopian case, there are several factors that limit production, from the extraction through the well in the reservoir characterized by the multiphase flow in porous media, as in the multiphase flow in the transport line, the pipeline-riser system, as well as insurgent problems in separators, manifold and others.

The present work is directed to a reservoir simulator in order to study the formations of the water and gas cones, conditions detrimental to the production since there is infiltration of other fluids with greater mobility than petroleum, flowing toward the well at which takes the form of a cone, according to the pressure gradient. The simulator is constructed using the Isogeometric Boundary Element Method (IGA-BEM), which has numerical advantages not only in relation to other numerical methods such as the Finite Difference Method (FDM) and Finite Element Method (FEM), but also to the conventional Boundary Element Method (BEM), since the IGA-BEM uses NURBS (Non Uniform Rational B-Spline) geometries that bring greater smoothness to the curves of the elements in question and ensure the continuity of the variables of field and geometry.

The phenomena of the water and gas cones are limiting factors in the productivity of an oil well, as they occur due to the pressure gradient applied by the well in order to extract the oil from the reservoir. Figure 1 shows an oil reservoir coexisting water and gas cones. The pressure gradient reaches all the fluids present in the reservoir (oil, water and natural gas) and as water and gas have greater mobility than oil, they tend to flow towards the well, each taking the form of a cone. The water cone and/or gas cone reach the end of the well for certain extraction flow values, causing water and/or gas production along with the oil, thereby reducing productivity.

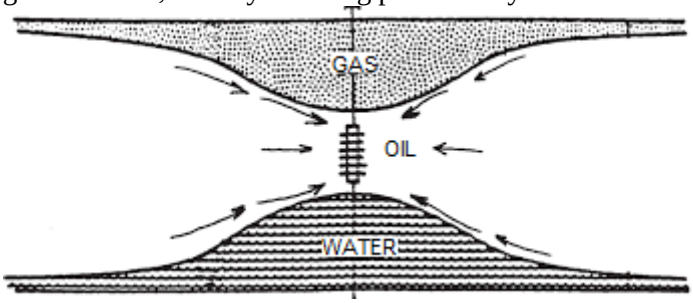

Figure 1. Oil reservoir coexisting water and gas cones

Analytical treatments with severe approximations and restrictions were used in the formulation of several models and correlations that are now used to estimate the behavior of the water and gas cones in order to overcome the great complexity of the problem (Rosa et al., [1]). Nowadays there are advanced numerical analysis tools that allow the study of these phenomena through computer simulation, minimizing the simplifications in the models used and increasing the accuracy of the results obtained. The tool used for the reservoir simulator, the theme of this work, it is the IGA-BEM.

2 Theoretical model

In this work the formulation of the Isogeometric Boundary Element Method (IGA-BEM) is presented for problems governed by the Laplace equation; model adopted - the potential flow. The basis of the IGA-BEM formulation is the fundamental solution of the governing differential equation.

The fundamental solution applied in this work corresponds to the potential response in an infinite medium when the sink (or source) is concentrated at a point. A simple yet powerful tool is used in the qualitative evaluation of the factors affecting the multiphase flow in porous media, Darcy's Law. Figure 2 shows the Darcy's experiment.

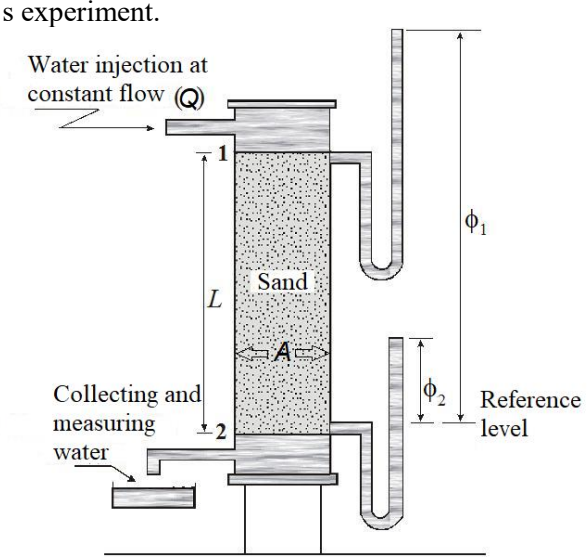

Figure 2. Darcy's experiment

Darcy (1856) arrived at the following equation for the flow in porous media:

$$Q = \frac{KA}{L}(\Phi_1 - \Phi_2) \tag{1}$$

From Eq. (1), Q is the volumetric flow rate of the fluid, A is the cross-sectional area of the apparatus, L is the length to be traversed by the fluid or "sand height", Φ_1 and Φ_2 are the heights and K is the hydraulic conductivity of the medium, defined by Bear [2] as:

$$K = \frac{\kappa \rho g}{\mu} \tag{2}$$

From Eq. (2), κ is the absolute permeability of the porous media, ρ is the specific mass of the fluid, μ is the dynamic viscosity of the fluid and g is the gravity acceleration. The permeability κ is a characteristic intrinsic to the porous media and independent of the fluid that is filling it. The hydraulic conductivity K, due to it involves the specific mass and the dynamic viscosity of the fluid, ρ and μ respectively, is therefore dependent on the fluid that is flowing.

Dividing both sides of Eq. (1) by A, one arrives at the expression for the apparent velocity or volumetric flow per unit cross-sectional area, q:

$$q = -\frac{K}{L}(\Phi_1 - \Phi_2) \tag{3}$$

or yet

$$q = -\frac{K}{L}\Delta\Phi \tag{4}$$

where q is still referred as the average flow (macroscopic) velocity.

When applying the property of the calculation making the limit of the length L tends to zero, one has:

$$q = -K\frac{\partial \Phi}{\partial x} \tag{5}$$

where x is the coordinate of the flow direction.

For two-dimensional or three-dimensional flow, one has:

$$q = -K\nabla\Phi \tag{6}$$

By Eq. (6) it can be seen that the apparent velocity of the fluid, q, is linearly related (through the hydraulic conductivity of the medium) to the gradient of a potential function. This function is called the flow velocity potential. Liggett and Liu [3] define it through the piezometric height, given by the following expression:

$$\Phi = \frac{p}{\rho g} + z \tag{7}$$

The first term of Eq. (7) is therefore known as piezometric height; the second term being the height that is associated with a referential or, still, as the energy associated with the position of the fluid in the gravitational field.

Liggett and Liu [3] further define the real average velocity of the fluid inside the pores as:

$$q = \Omega u \tag{8}$$

where u is the real average velocity of the fluid and Ω is the porosity of the medium.

The fluids, as well as the medium, are considered incompressible in the adopted potential flow model. The porous media is homogeneous with constant permeability and isotropic in the region of the flow, and also, the dynamic viscosity of the fluids is considered to be constant. In this model, the pores are saturated with only one fluid at a time.

From the Eq. (6) and the principle of mass conservation, it has:

$$\nabla.(-K\nabla\Phi) = 0 \tag{9}$$

Considering the medium as being homogeneous, the hydraulic conductivity K is assumed to be constant. Therefore, Eq. (9) becomes:

$$\nabla^2\Phi = 0 \tag{10}$$

Equation (10) is defined as the Laplace Equation, which governs the flow in a porous media. In the present work a point sink must be added to be counted in the hydrodynamics of the problem, so applying the principle of mass conservation results in:

$$\nabla.\,\mathrm{q} = -Q\delta(x - x_s) \tag{11}$$

Rearranging the Eq. (11), it becomes:

$$\nabla^2\Phi = \frac{Q}{K}\delta(x - x_s) \tag{12}$$

where δ is the Dirac delta function and x_s is the position vector of the sink.

Two works are used as discretization model of the potential flow to study the phenomena of the water and gas cones, through the conventional BEM formulation: Dias Jr. [4] and Gontijo [5]. Dias Jr. [4] studied the phenomenon of the water cone during the extraction (recovery) of oil by the producing well. He performed a numerical simulation of the oil reservoir in order to study the amount of water produced when the oil is being extracted using the conventional Boundary Element Method (BEM).

Gontijo [5] presents a study on the phenomena of water and gas cones in horizontal oil wells. He performs a two-dimensional numerical modeling of the phenomena using the conventional Boundary Element Method (BEM) and then develops three different simulators among them by the configuration of the fluids in the reservoir and the accounting of the compressibility factor in the porous media. The determination of the boundary conditions for the three models, including the conditions of compatibility and equilibrium at the interface between fluids, is presented. The proposed simulators were developed and validated in comparison to analytical results available in the literature.

2.1 Isogeometric formulation

The Isogeometric Analysis is an attempt to eliminate the generation of the mesh using the discretization of Computer Aided Design (CAD) software, where the BEM has significant advantages. The predominant technology used by CAD in the representation of complex geometries is the Non-Uniform Rational B-Spline (NURBS). This makes it possible to accurately reproduce entities that would only be approximated by polynomial functions, including circular conic sections, cylinders,

spheres, ellipsoids, paraboloids, etc. There is a vast literature dealing with the different aspects of NURBS (e.g. Piegl and Tiller [6], Rogers [7]) and the decades of research have resulted in several efficient algorithms for fast assessment and refinement.

The main concept presented in Hughes, Cottrell and Bazilevs [8] is to use NURBS not only as a discretization of geometry, but also as a discretization tool in the analysis, assigning the denomination of isogeometric analysis to this characteristic of methods. In the BEM, a formulation using B-Splines is presented initially in Cabral et al. [9] and Cabral, Wrobel and Brebbia [10], but still without concern for integration into CAD. This integration was later observed in Simpson et al. [11] and Simpson et al. [12]. The Isogeometric concept is also present in mesh-free methods, creating spline-based methods, as presented in Wang and Zhang [13], Shaw and Roy [14], Kim and Youn [15].

Two other works of extreme importance for the isogeometric formulation are used as the basis for this study, Campos [16] and Sena [17]. Campos [16] develops an isogemetric formulation of the BEM, in which the polynomial form functions are replaced by the NURBS functions. The developed formulations are implemented and applied in the analysis of several numerical examples and their results are compared with the Boundary Element Method with the use of polynomial form functions. Sena [17] proposes an isogeometric formulation of the BEM for potential two-dimensional problems. Unlike conventional BEM, the boundary conditions in the isogeometric formulation can't be applied directly to the problem, since the control points are typically outside the boundary. To overcome this problem, it uses an E-transformation matrix for B-Splines that can relate the values between the control points and the collocation points. Numerical solutions for two-dimensional isogeometric problems were obtained and compared to analytical solutions and satisfactory results for errors were found.

NURBS can represent exact conic sections: circles, hyperboles, ellipses, and parables; as for example, in the application of the case addressed in this work - the phenomena of water and gas cones. Since the usual Lagrangian shape functions are not able to describe more complex geometries in a precise way. NURBS are better suited to represent the geometries and can also be used to approximate the field variables. When complex geometries are used in a problem, using the analytical representation in the BEM results in a greater precision for the same number of elements.

The NURBS can be calculated by the following expression:

$$\boldsymbol{C}(\boldsymbol{t}) = \frac{\sum_{i=1}^{n+1} \boldsymbol{B_i} N_{i,k}(t) w_i}{\sum_{i=1}^{n+1} N_{i,k}(t) w_i} = \sum_{i=1}^{n+1} \boldsymbol{B_i} R_{i,k}(t) \tag{13}$$

where $R_{i,k}(t)$ is the rational basis function given by

$$R_{i,k}(t) = \frac{N_{i,k}(t) w_i}{\sum_{i=1}^{n+1} N_{i,k}(t) w_i} \tag{14}$$

and $\sum_{i=1}^{n+1} \boldsymbol{B_i} N_{i,k}(t)$ is the B-Splice curve, where $\boldsymbol{B_i}$ are the control points and $N_{i,k}(t)$ are the base functions of order k and degree k-1. B-Splines are a generalized form of the Bezier curves. Therefore, NURBS is a generalization of B-Spline, replacing the polynomial base with a polynomial ratio. Its characteristics are basically the same for this reason.

2.2 Isogeometric Boundary Element Method (IGA-BEM)

By applying the NURBS to the shape functions, the BEM is now referred as Isogeometric Boundary Elements. The sum of all the Isogeometric Boundary Elements will then be a representation that, in most cases, it will be the actual representation of the contour. For the isogeometric formulation, the greater the discretization, that is, the greater the refinement of a Bezier curve in which the control polygon was divided, the better the approximation made by the Isogeometric Boundary Elements. While the real contour is called S, the approximate contour is denoted by Γ.

Each part of the real contour S is approximated by one or more Bezier curves with a continuity mechanism between the segments. Knowing that $R_{i,k}(t)$ is the rational base function of order k (number of control points), the approximation with the isogeometric boundary elements becomes:

$$c\Phi_i^c(x_d, y_d) = \int_\Gamma \sum_{i=1}^{n+1} (\Phi_i^c R_{i,k}(t)) q^*(x_d, y_d)\, d\Gamma - \int_\Gamma \sum_{i=1}^{n+1} (q_i^c R_{i,k}(t)) \Phi^*(x_d, y_d)\, d\Gamma \tag{15}$$

where Φ_i^c and q_i^c are the potential and velocity, respectively, at the control point i. As these values are punctual (they do not vary along the contour), we can organize them as follows:

$$c\Phi_i^c(x_d, y_d) = \sum_{i=1}^{n+1}(\Phi_i^c \int_\Gamma R_{i,k}(t) q^*(x_d, y_d)\, d\Gamma) - \sum_{i=1}^{n+1}(q_i^c \int_\Gamma R_{i,k}(t) \Phi^*(x_d, y_d)\, d\Gamma) \quad (16)$$

Equation (16) rearranged with the terms of the matrix $\boldsymbol{H}$ on one side and the terms of the matrix $\boldsymbol{G}$ on the other, becomes:

$$\sum_{i=1}^{n+1} H_{i,k}\Phi_i^c = \sum_{i=1}^{n+1} G_{i,k} q_i^c \quad (17)$$

or by the matrix form, such as:

$$\boldsymbol{H\Phi^c = Gq^c} \quad (18)$$

The term c will have influence on k elements of matrix $\boldsymbol{H}$, which does not occur in conventional BEM. Equation (18) is the Boundary Integral Equation discretized, represented in matrix form. In the case where the governing equation has a point sink, Eq. (18) becomes:

$$\boldsymbol{H\Phi^c = Gq^c - s} \quad (19)$$

where $\boldsymbol{s}$ is the vector that holds the contribution of each point sink. Each element of this vector corresponds to the effect of the sinks at a given source point d, represented by:

$$s_d = \sum_{j=1}^{n_s} Q_j \Phi^* \quad (20)$$

where n_s is the number of punctual sinks of the problem, Q_j is the intensity of the sink, and Φ^* is the fundamental solution of the potential.

In order to separate the known variables from the unknown, it is necessary to reorganize the equation through a matrix system. Therefore, it has:

$$\boldsymbol{Ax = b - s} \quad (21)$$

or

$$\boldsymbol{x = A^{-1}(b - s)} \quad (22)$$

Through the linear system given by Eq. (22) it is possible to obtain the unknown values of the potential $\boldsymbol{\Phi}$ and the velocity $\boldsymbol{q}$ in the contour. However, it should be noted that the control points are typically outside the boundary, i.e. the boundary conditions cannot be applied directly. A transformation matrix $\boldsymbol{E}$ for B-Splines is then used to overcome this problem (Cabral et al., [9]). This matrix uses the base functions to relate the values at the control points to the values at the collocation points, through the following:

$$\boldsymbol{\Phi = E\Phi^c} \quad (23)$$

$$\boldsymbol{q = Eq^c} \quad (24)$$

where the vectors $\boldsymbol{\Phi}$ and $\boldsymbol{q}$ contain the values for potential and velocity, respectively, at the collocation points; while $\boldsymbol{\Phi^c}$ and $\boldsymbol{q^c}$ contain the values for potential and velocity, respectively, at the control points. The same idea can be applied to NURBS.

Replacing the Eq. (23) and (24) in the Eq. (18), it has:

$$\boldsymbol{HE^{-1}\Phi = GE^{-1}q} \quad (25)$$

Equation (25) can be solved in the usual manner as conventional BEM.

2.3 Interface movement

Through the phenomena of the water and gas cone, an interface movement between the fluids is present in the problem. In order to quantify this movement, the method of repositioning control points (Piegl and Tiller, [6]) is used. The repositioning of the control points takes place in a curve of type B-Spline or NURBS. From Eq. (13) for the NURBS curve, an arbitrary control point $\boldsymbol{B_x}$ will undergo a translation to $\widehat{\boldsymbol{B}}_{\boldsymbol{x}}$ through a vector $\boldsymbol{V}$, given by:

$$\boldsymbol{V} = \widehat{\boldsymbol{B}}_{\boldsymbol{x}} - \boldsymbol{B}_{\boldsymbol{x}} \tag{26}$$

Therefore, the new curve for the transferred arbitrary control point becomes:

$$\widehat{\boldsymbol{C}}(\boldsymbol{t}) = \boldsymbol{B}_{\boldsymbol{i}} R_{i,k}(t) + \cdots (\boldsymbol{B}_{\boldsymbol{x}} + \boldsymbol{V}) R_{x,k}(t) + \cdots \boldsymbol{B}_{\boldsymbol{n+1}} R_{n+1,k}(t) = \boldsymbol{C}(\boldsymbol{t}) + R_{x,k}(t)\, \boldsymbol{V} \tag{27}$$

Equation (27) expresses the functional translation of all points of the curve $\boldsymbol{C}(\boldsymbol{t})$ in which $\boldsymbol{t}$ (curve parameter) $\in [\boldsymbol{t_x}, \boldsymbol{t_{x+k+1}})$. All points on the curve outside this range are not affected. The maximum translation occurs at the maximum of the rational base function $R_{x,k}(t)$, as can be seen, for example, in Fig. 3.

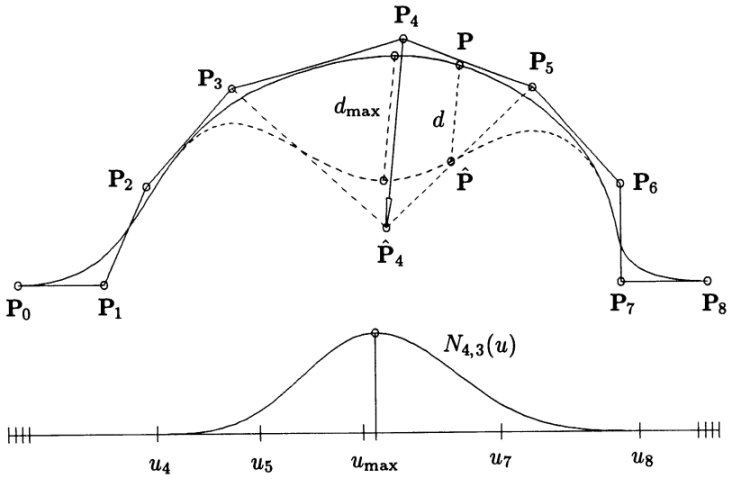

Figure 3. Maximum translation of an arbitrary control point. Reproduced from Piegl and Tiller [6]

Applying Eq. (27) to $n + 1$ control points, it is possible, therefore, to cause the total interface movement. It is worth mentioning that the first and last control points, which theoretically must undergo the translation in the interface movement, are fixed so that there is convergence of the translation of the curve as a whole.

The calculated speed at the interface is the speed of the material points that defines it. As a function of this calculated speed, it is possible to deduce the equation that defines its position. For this case it is considered that there is a function F, defined by:

$$F = z - \lambda(x, t) = 0 \tag{28}$$

where $\lambda(x, t)$ is the function that expresses the height of the interface for a given coordinate x and a certain time t.

The interface is a material line, so it can do:

$$\frac{DF}{Dt} = \frac{\partial F}{\partial t} + \frac{q}{\Omega}.\nabla F = 0 \tag{29}$$

where q/Ω is the real average velocity of the fluid inside the pores given by Eq. (8).

Replacing the Eq. (28) in the Eq. (29), it has:

$$\frac{DF}{Dt} = -\frac{\partial \lambda}{\partial t} + \frac{q}{\Omega}.(\nabla z - \nabla \lambda) = 0 \tag{30}$$

which can be written as

$$\frac{\partial \lambda}{\partial t} = \frac{1}{\Omega}(q.\nabla z - q.\nabla \lambda) \tag{31}$$

Equation (31) is the equation that defines the variation of the position (at z) of a given point of interface placement. This equation includes a linear portion, $q.\nabla z$, and a nonlinear portion, $q.\nabla \lambda$. The nonlinear contribution to the interface displacement is disregarded in this work. The information obtained is the speed normal to the collocation point (q), so the equation written in terms of this normal speed projected on the z axis is given by:

$$\frac{\partial \lambda}{\partial t} = \frac{1}{\Omega} \frac{q}{\cos \eta} \tag{32}$$

where η is the angle formed between the vector normal to the collocation point and the z-axis.

Equation (32) is the equation for the simulation of free surface movement or interface between two fluids. The implementation of this equation in the developed code must be made in such a way as to take into account the size of the time step used in the iterations. Writing Eq. (32) in terms of finite differences (Zhang et al., [18]), it has:

$$z_{m+1} = z_m - \frac{\Delta t}{\Omega \cos \eta} [\beta (q.\hat{e}_z)_{m+1} + (1-\beta)(q.\hat{e}_z)_m] \tag{33}$$

where β is the coefficient that defines how much the collocation point moves; z is the height of the interface; Δt is the size of the time step and the sub-indices m and $m+1$ are, respectively, the current and next immediate time step.

2.4 Boundary conditions

The reservoir simulator for single-phase flow is used to simulate an oil extraction subject to the occurrence of a gas cone, that is, for the reservoir configuration where one layer of oil and another layer of gas coexist. Figure 4 shows the general appearance of this type of problem.

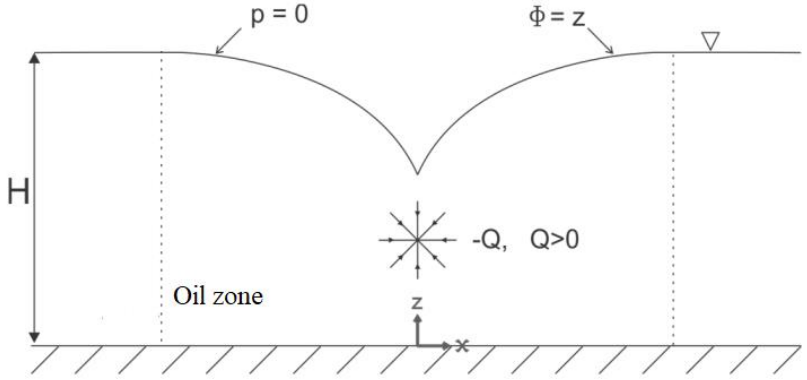

Figure 4. Gas Cone Phenomenon - Boundary Conditions

The geometry of the reservoir is rectangular for purposes of simulation simplification. As already described, the governing equation of the potential flow is the Laplace Equation - Eq. (10), written in terms of the potential Φ. The boundary conditions applicable to each of the knots are of known potential or known speed. The simulator contains only oil (single phase). All parameters used in the code are set out in Table 1 below.

Table 1. Parameters used in the code

Quantities	Nomenclature	Measurement unit (SI)
Specific mass	ρ	kg/m^3
Dynamic viscosity	μ	Pa.s
Medium's porosity	Ω	-
Absolute permeability	κ	m^2
Hydraulic conductivity	K	m/s
Potential	Φ	m
Well intensity	Q	m^2/s
Well coordinates	(x_p,z_p)	m
Total time	t_t	s
Running time	t_b	s
Time step size	Δt	s
Time step quantity	npt	-

In the potential single phase flow, the height of the oil zone undisturbed by the flow, H_o, is equal

to the height of the entire simulated portion of the reservoir, as shown in the Fig. 5 below.

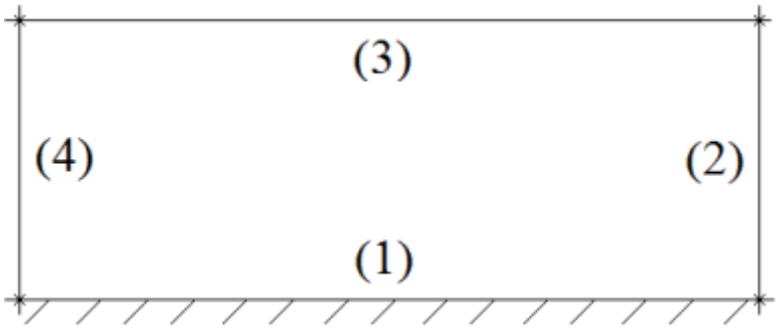

Figure 5. Rectangular reservoir containing only one fluid

The potential is known on the two sides (2) and (4) and on the free surface (3). Its value is constant and can be calculated through Eq. (7), where p is the total pressure at which a fluid particle located in the position of the knot in question is submitted, that is:

$$p = p_o + p_t \tag{34}$$

where p_o is the pressure relative to the weight of the oil column above the knot and p_t is the pressure at the top of this oil column, i.e. the pressure the gas makes on the free surface. Substituting Eq. (34) into Eq. (7) and setting the pressures as the relative heights, it has:

$$\Phi = \frac{p_t}{\rho g} + H_o \tag{35}$$

At the interface between the oil and the gas - free surface (3), the existing pressure is only that which the gas exerts on the oil (p_t). The pressure on the free surface can be assumed to be zero, without loss of generality, therefore, the entire domain has a constant potential and value equal to its height (H_o). Therefore, all knots of the lateral boundaries (2) and (4) and of the free surface (3) have potential known as:

$$\Phi_2 = \Phi_3 = \Phi_4 = H_o \tag{36}$$

where H_o is the height of the oil zone measured by the z-coordinate of the free surface undisturbed by the flow.

The lower limit (1) is the waterproof base of the reservoir; consequently, there is no flow of oil in the direction normal to it. The existing boundary condition is:

$$q_{(1)} . \hat{\boldsymbol{n}}_{(\mathbf{1})} = 0 \tag{37}$$

3 Results

In this work the results are presented for a single phase potential simulator, that is, to simulate only the phenomenon of the gas cone in order to be compared and validated to the work of Dias Jr. [4]. The pumping by the producing well occurs during 3/4 of the time, being switched off shortly thereafter.

The physical conditions of the problem are presented in Table 2, such as the simulator: well intensity, porous media properties and reservoir geometry.

Table 2. Physical conditions

Quantities	Nomenclature	Measurement unit (SI)
Hydraulic conductivity	K	0,0039 m/s
Potential	Φ	1 m
Well intensity	Q	$-1{,}0\text{x}10^{-2}$ m^2/s
Well coordinates	(x_p,z_p)	(0,5;0,1) m

Total time	t_t	400 s
Running time	t_b	300 s
Time step size	Δt	6,667 s
Time step quantity	npt	60

The reservoir has the dimension: 1 x 1 m^2. Figures 6 and 7 present the geometry and the physical characteristics as the mobile interface with potential gradient. Figure 6 shows the reservoir simulator developed in this work, that is, with mathematical formulation through the Isogeometric Boundary Elements Method, where the shape functions used are of the NURBS type.

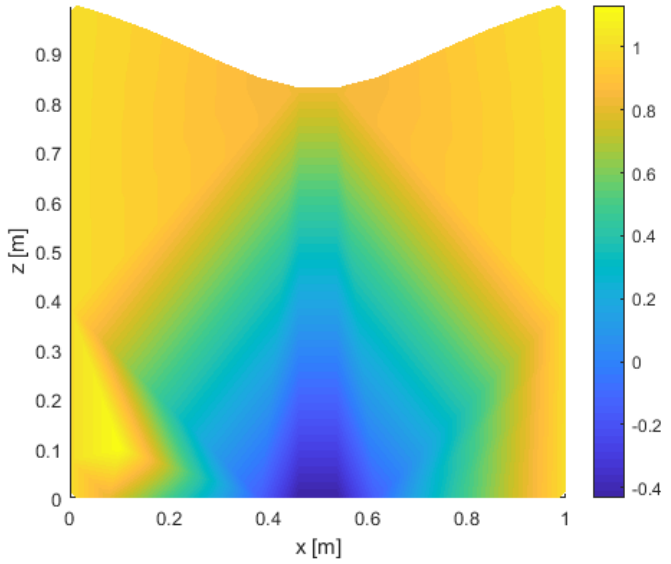

Figure 6. Potential gradient with mobile interface at time t = t_b = 300s (IGA-BEM)

Figure 7 presents the reservoir simulator developed by Dias Jr. [4] using conventional BEM. Comparing the two reservoir simulators, one can observe how NURBS can discretize the problem more precisely.

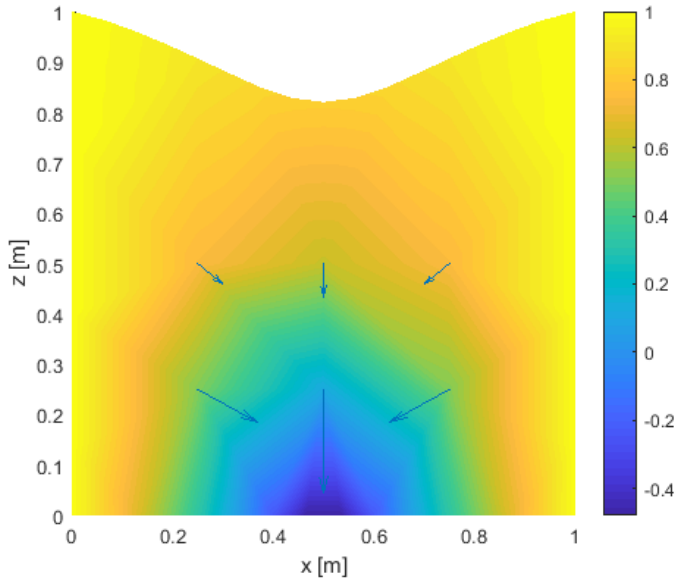

Figure 7. Potential gradient with mobile interface at time t = t_b = 300s (conventional BEM). Reproduced from Dias Jr. [4]

Figure 8 shows the variation of the vertical position of the midpoint on the free surface through

the implementation of the isogeometric formulation in the problem. Figure 9 shows the same analysis for the simulator of Dias Jr. [4], that is, through the conventional BEM.

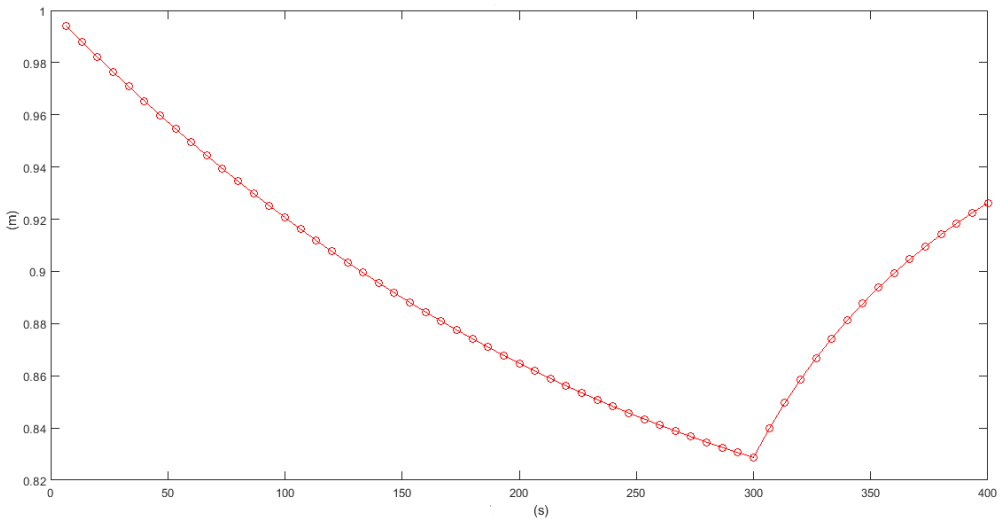

Figure 8. Vertical position of the midpoint on the free surface (IGA-BEM)

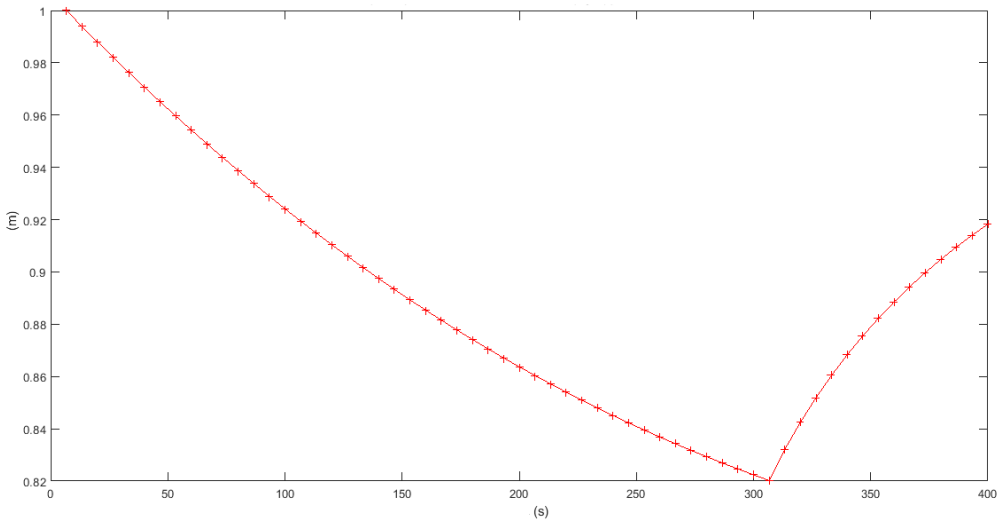

Figure 9. Vertical position of the midpoint on the free surface (conventional BEM). Reproduced from Dias Jr. [4]

The results presented, therefore, can validate the effectiveness of the Isogeometric Boundary Element Method formulated and implemented.

4 Conclusion

The main characteristics and information regarding the phenomena of the water and gas cones in horizontal wells were gathered in this work. The discretization was carried out for the single phase potential flow model, in which it was developed through formulations of the Isogeometric Boundary Elements Method for the study of the phenomenon of the gas cone, as well as the existing boundary conditions.

The study can prove that the IGA-BEM is a competitive tool not only for conventional BEM, but also for FDM and FEM. The method evidenced the efficacy in the treatment of problems such as the multiphase flow in porous media, especially when there are movable borders involved. It is based on the literature that the isogeometric formulation allied to the BEM is an effective strategy to increase the precision of the implemented algorithms, even if this type of method contributes to the increase of the computational costs involved.

The developed reservoir simulator was used in the simulation of the gas cone phenomenon, since the problem was modeled for the single phase potential flow. The main objective is to analyze the production of oil without the undesired production of gas from the gas layer just above the reservoir.

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