

# **STATIONARY AND TRANSIENT DYNAMIC VERTICAL RESPONSES OF STRUCTURES INTERACTING WITH DISTINCT SOIL-FOUNDATION ARRENGEMENTS USING CORRECTED STRUCTURAL MODAL DATA**

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**Abstract.** This work investigates stationary and transient vertical response of multidegree of freedom structures interacting with different soil profiles through an extended modal analysis procedure. The methodology was originally proposed by Wu and Smith [1] to analyze the dynamic behavior of structures subjected to an incident wave fields. The present article revisits the previously mentioned article and expand it to consider external force excitations acting upon the structure. The original formulation considered the soil as a homogeneous half-space. In the present article, the response of structures supported by the half-space, layered soil profiles and also by piled structures is addressed. The stationary soil and pile response were synthetized through Boundary Element Method. The main characteristic of the method stated in this work is to modify the modal original parameters of the structure in other to incorporate in these parameters the dynamic response of the soil-foundation arrangements. Numerical studies will address the quantity of modified structural modes necessary to describe, with certain precision, the transient response of structure as a function of the soil-foundation supporting scheme. The numerical examples presented will also investigate the role of the soilfoundation system properties, like foundation mass, pile mass, density and length and the soil profile (half-space, layered) on the structural response. The methodology used in this work will allow to determine the transient response of multidegree of freedom structures interacting with several soil profiles with a low computational cost compared to concurrent methodologies.

**Keywords:** Dynamic Soil-Structure Interaction, Modified Modal Analysis, Foundation Dynamics.

## **1 Introduction**

The topic of Dynamic Soil-Structure Interaction (DSSI) has received the attention of several research areas in the last decades. This phenomenon can be observed when the structure receives vibratory waves from the operation of nearby placed machinery or from seismic events, resulting in an interdependent displacement of the structure and the soil.

Nowadays there is a large amount of information about DSSI, covering many aspects of the field. One of them is the transient and stationary dynamic response of a structure interacting with the soil that can be modelled in different ways using different numerical methods, such as Finite element method (FEM), Boundary Element Method (BEM) or Discrete Element method (DEM). Menglin et al. [2] and Dutta and Roy [3] did a bibliographic review showing different techniques used to model the DSSI.

Most DSSI procedures require a simulation scheme for the stationary or transient response of the soil domain. According to Schanz [3] a complete computational description of the transient soil response is very difficult to obtain and also presents a high computational cost. A possible way to achieve transient responses for DSSI problems is to obtain frequency domain solutions for a relative large frequency range and in the sequence apply the Inverse Fast Fourier Transform (IFFT) to synthesize the transient response. It is a possible strategy, but it is frequently very expensive computationally, although, today it can be programmed on parallel GPU systems, as shown by Labaki, Ferreira and Mesquita [5].

Time domain solutions were obtained by applying the IFFT of frequency domain responses of DSSI by Mesquita et al. [6] and by Lima, Labaki and Mesquita [7]. These transient solutions can be applied to the whole system, composed of coupled soil and structure, or it can be applied separately to obtain first the soil dynamics response and latter couple it to the structure by an interactive procedures, as shown by Damasceno, Labaki and Mesquita [8].

The principal objective of this paper is to expand the modified modal analysis proposed by Wu and Smith [1] to consider systems with external excitation forces interacting with different soil profiles. This method allows to incorporate the DSSI effects on structure response through a modification of its modal shapes and natural frequencies. The stationary soil and pile response, used in this article, were synthetized through Boundary Element Method by Labaki, Mesquita and Rajapakse [9, 10].

## **2 Statement of the problem**

Considering a DSSI problem such as Fig.  $1(a)$ , a mass-spring-damper system with N degrees of freedom (DOF) submitted to a generic vertical external force. This system represents a N-storey building resting on a rigid foundation of mass  $m<sub>o</sub>$  and a soil, modeled here as a homogeneous transversely isotropic half-space, with Young's modulus  $E_s$ , Poisson's ratio  $v$ , mass density  $\rho_s$  and material damping coefficient η. A case with the pile embedded in the soil is also considered in this paper. Pile considered has Young's modulus  $E_p$ , density  $\rho_p$ , radius  $a_p$ , and length  $h_p$ .

This model only investigates vertical translation motion of the foundation and each floor (relative to foundation), both in X-Z plane. Therefore, this DSSI system has N+1 DOFs. Proportional viscous damping is assumed for structure in such a way that is possible to apply orthogonality conditions for damping using the classic normal modes.

The Fig. 1(b) shows a schematization applied on model to be solved via modified modal analysis. In this method the system is divided into two subsystems. Subsystem I represents N-storey structure with an external excitation force, F, and moving base. Subsystem II represents the soil-foundation system with an external force excitation,  $F_{str}$ , coming from structure. The coupling of whole system is done both by vertical translation of the foundation,  $Z_0$ , is the same of subsystem I base motion, and applied force on foundation,  $F_{str}$ , that represents dynamic structure influences on soil response. The problem treated here is also valid to systems with incident waves as excitation mechanism.



**Figure 1. (a) Dynamic Soil-Structure interaction model, (b) Schematization of DSSI model for application of modified modal analysis**

## **3 Formulation**

The time-harmonic vertical displacement Z of N-degree-of-freedom Subsystem I in Fig. 1b under external time-harmonic force  $F_{ext}$  or imposed time-harmonic base displacement  $Z_0$  of circular frequency  $\omega$  can be written in terms of a superposition of its mode-shape matrix  $[\phi]$  such that:

$$
Z \omega = [\phi][H \omega] [\phi]^T F_{ext} \omega + \omega^2 \Gamma Z_o \omega + Z_o \omega , \qquad (1)
$$

in which

$$
H \omega^{-1} = [\phi]^T - \omega^2 [M] + i\omega [C] + [K] [\phi], \qquad (2)
$$

is the frequency response receptance function of the structure,

$$
\Gamma = \left[\phi\right]^T \left[M\right] \ 1 \ \ (3)
$$

is a vector containing the coefficients of generalized modal load associated with the vertical motion of the structure, and [M], [C], and [K] are respectively classical  $N\times N$  mass, damping, and stiffness matrices of the system. As for the soil part (Fig. 2), the equilibrium condition yields

$$
F_s \quad \omega \quad -F_{str} \quad \omega \quad = \omega^2 m_o Z_o \quad \omega \quad , \tag{4}
$$

in which  $F_s=K_{VV}Z_0$ , where  $K_{VV}=K_{VV}(\omega)$  is the impedance matrix of the soil in the vertical direction, and  $F_{str}(\omega) = \{1\}^{T} \{F_{ext}(\omega)\} + \omega^2 \{1\}^{T} [M] Z(\omega)$  is the force resulting from the interaction with the structure.

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The expansion of Eq. (4), in view of Eq. (1) yields the total modified response of the structure with the incorporation of the effect of the soil:

$$
Z^{\text{mod}} \omega = [\phi][H \omega] [\phi]^T + \omega^2 \Gamma \quad R \omega + 1 \quad R \omega \quad F_{ext} \omega , \qquad (5)
$$

in which

$$
R \quad \omega = \frac{1 + \omega^2 \quad \Gamma \quad \left[ H \quad \omega \right] \left[ \phi \right]^T}{K_{vv} \quad \omega \quad -\omega^2 m_T - \omega^4 \quad \Gamma \quad \left[ H \quad \omega \right] \quad \Gamma},\tag{6}
$$

and  $m_t = m_0 + m_1 + ... + m_N$  is the total mass of the structure. The impulse response of the coupled soilstructure system can be obtained by applying direct FFT to this time-harmonic solution. The transient response to any other type of excitation can be obtained through convolution of the impulse response to the new transient load.



**Figure 2. Soil-foundation interaction with equilibrium of force.**

#### **4 Numerical results**

The modified modal analysis proposed by [1], and expanded in this paper to structures with external force as excitation mechanism, was validated through the solution of the system shows in Fig. 3 via classic modal analysis and modified modal analysis.

The number of used dofs in numerical example were five dofs. A physical rate damping used in this example was  $\xi = 2\%$  and from it was defined proportional viscous damping. The values of mass (kg) and stiffness (N/m) of the system were, respectively,  $m_0 = 500$ ,  $m_1 = 350$ ,  $m_2 = 270$ ,  $m_3 = 180$ ,  $m_4$  $= 130$ ,  $k_0 = 6$ ,  $k_1 = 5$ ,  $k_2 = 4.5$ ,  $k_3 = 4$  e  $k_4 = 3.5$ . The excitation force, F, only applied in the last mass,  $m<sub>4</sub>$ , is:

$$
F \ t = \begin{cases} 0 < t < 1 \to \ 5t \ N \\ 1 < t < 4 \to 5N \\ 4 < t < 5 \to -5t + 25 \ N \\ t > 5 \to 0N \end{cases} \tag{7}
$$



**Figure 3. System for validation of modified modal analysis.**

For validation of numerical solution in this paper, the transient and stationary responses will be compared with classic modal analysis for the first and the last floor. The results are shown in Fig. (4).<br>FRF of  $Z_4(\omega)$ 



**Figure 3. Stationary and transient response of (a) bottom floor and (b) top floor**

The Fig. 4 shows the transient and stationary responses for bottom and top floors of the system via classic and modified modal analysis. Observing these numerical results, is possible to see a good convergence between the methods.

With the problem already validated for a generic mass-spring-damper system, now this modified modal analysis will be validated for a 1-DOF system coupled with a soil modeled as a homogeneous half-space. The response obtained via the present modified method will be compare with the

corresponding solution presented by Labaki, Mesquita and Rajapakse [9]. In Fig. 5, System I was solved via present methodology and System II by Labaki, Mesquita and Rajapakse [10].



**Figure 5. System used for validation of the present methodology and a direct coupling of the soil modeled as a homogeneous half-space**

In order make both systems equivalent, the adopted a high value of stiffness,  $k^1=1e^{15}$  N/m, and a low rate of damping, ξ=1e-<sup>7</sup>, ensuring that mass-spring-damper of System I behaves like a rigid body. The value of foundation mass was m<sub>0</sub>=157,08kg (for a density of  $p=5000\text{kg/m}^3$ ) and the amplitude of the applied force was F=1N. The parameters for a half-space were, E=2.5Pa,  $c_s$ =1m/s,  $\rho_s$ =1kg/m<sup>3</sup>, and  $\eta$ =0.01. Normalized frequency  $a_0$  is defined as  $a_0 = \omega/c_s$ . The obtained results are shown in Fig. 6.



**Figure 6. Validation for system in figure 5 in stationary and transient response**

Figure 6 shows a good convergence between the method presented in this paper and the method developed by Labaki, Mesquita and Rajapakse [9].

Figure 7 shows a 4-DOF system solved via modified modal analysis. System II, III and IV were solved via modified modal analysis and System I via classic modal analysis. The goal here is to verify the influence of the soil in the response of the structure. This solution is compared to a system with a fixed-base and a system with a soil approximated as a spring.



**Figure 7. Structure over a fixed base, spring K, half-space and pile embedded in a half-space**

The values of mass (kg) and stiffness (N/m) were, respectively,  $m_1 = m_2 = m_3 = m_4 = 706860$ ,  $K=k_1=k_2=k_3=k_4=2.375e^9$ . The soil properties without the presence of the pile are:  $G_s=450Mpa$ ;  $p_s=11250\text{kg/m}3$ ,  $v=0.3$  and  $\eta=0.01$ . The soil properties with the presence of the pile are:  $G_s=90\text{Mpa}$ ;  $\rho_s$ =2700kg/m3, v=0.3 and  $\eta$ =0.01. The pile properties are: .The damping ratio of the structure was  $\xi$ =0.02 and static stiffness of half-space was equal to K. The dynamic flexibility of the half-space and the pile are illustrated in Fig. 8.



**Figure 8. Dynamic flexibility of the half-space and the pile embedded in half-space**

The transient excitation force F is applied in the fourth mass,  $m_4$ , and is shown in Fig. 9:



Força de excitação aplicada em m<sub>4</sub>

**Figure 9. Excitation force applied in fourth mass.**

The stationary responses for the bottom,  $m_1$ , and top,  $m_4$ , floors is shown in Fig. 10.



#### **Figure 10. Stationary response of bottom floor, zo, and top floor, z4 interacting with distinct soil profiles.**

Figure 10 shows that System I (fixed base) has higher resonance frequency if compared with System II, III and IV (K, half-space and a pile embedded in a half-space), showing the modification of modal shapes and natural frequencies of the system when incorporating the SSI effects. In addition, System II, III have the same resonance frequency, however the amplitude displacement of the System III is smaller than the amplitude of displacement of the System II because the soil, as an unbound medium, radiates part of the energy of vibration away from the excitation force. System 4, even with the same static stiffness as System 2 and 3, has slightly higher resonance frequencies and smallest



displacement. As noted by Wu and Smith [1], the effect of the soil is more significant in the first modes. The transient responses for the bottom,  $m_1$ , and top,  $m_4$ , floors is shown in Fig. 11.

#### **Figure 11. Stationary response of bottom floor, zo, and top floor, z4 interacting with distinct soil profiles.**

Figure 11 shows that System I (fixed base) has shorter oscillation period if compared with System II, III and IV (K, half-space and a pile embedded in a half-space), showing the influences of the SSI. In addition, System II, III have almost the same oscillation period; however, the amplitude displacement of the System III is smaller than the amplitude of displacement of the System II. System 4 has a faster amplitude decay than other systems. The degree of freedom closer to the soil are more influenced than the degree of freedom farther from the soil

### **5 Conclusions**

This paper presented an extension of the modified modal analysis scheme to model dynamic soilstructure interaction problems. The scheme was illustrated with a system in which a structure was modeled by mass-spring-damper systems with multiple degrees-of-freedom. Different structure supports schemes were considered: a fixed base, a linear spring, a homogeneous half-space and a pile embedded in the half-space. The proposed methodology, which uses modified modal quantities was able to determine transient response of linear structures interacting with districting supported schemes. The results are very coherent and clearly indicated the dynamic characteristics of each supporting mechanism. The set of present results are an original contribution to transient DSSI studies.

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