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INTERACTIVE GRAPHICS TOOL FOR STRUCTURAL ANALYSIS AND DESIGN OF CONTINUOUS PRESTRESSED BEAMS ACCORDING TO NBR 6118/2014

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Abstract. Nowadays, prestressed concrete structures have been increasingly used for covering long spans due to several advantages over conventional reinforced concrete structures. In the design of such structures, a fundamental step consists on the structural analysis to obtain the loads generated by the prestressing. There are few commercial software, however, that allow the analysis of statically indeterminate prestressed structures, such as continuous beams, widely used in bridges. In this sense, the current work presents an interactive graphics software developed in MATLAB, which allows the analysis of continuous beams for any cable layout. The analysis step is based on an association of the force method, used to obtain the equivalent loads generated by prestressing, with the direct stiffness method, used to obtain displacements, support reactions and diagrams, generated by the prestressing and other user defined loads. The software also allows to obtain the stresses generated by different load combinations over sections throughout the length of the beam, which can be used for performing structural checks according to NBR 6118/2014. In order to verify its correct functioning, results of tests carried out with several beams models are presented, varying the number of spans and the cable layout, which were compared with values obtained with a commercial finite element software. The results were satisfactory and show that the developed software is a useful and easy-to-use tool that can be used by engineers, teachers and students of the area.

Keywords: Prestressed concrete, Structural analysis, Force method, Direct Stiffness Method.

1 Introduction

Prestressing can be defined as the insertion of permanent internal stresses in a structure in order to improve its performance. In the case of concrete structures, which have low tensile strength, this mechanism happens through the imposition of compressive stresses in order to cancel out or reduce the tensile stresses caused by external loads, usually using steel cables, which allows the optimization of the use of the concrete in compression [1].

In the design of such structures, a fundamental step consists on the structural analysis to obtain the loads generated by the prestressing, which depends on the cable layout. In the case of statically indeterminate prestressed structures, additional stresses generated by additional restraints appear, which makes the structural analysis complex.

There are several analytical and numerical methods that help obtaining these loads, such as Equivalent Load Methods [2, 3], which are based on the substitution of the effects of prestressing by forces and equivalent moments, and Influence Line Methods [4], that propose to obtain the bending moments from the integration of the product between the influence lines of moments and the diagram of the isostatic moments generated by prestressing. These methods, however, are difficult to automate and don't advance in the calculation of the displacements and stresses generated by the prestressing along the structure. Furthermore, there are few commercial software, that allow the analysis of statically indeterminate prestressed structures, such as continuous beams, widely used in bridges.

In this sense, the current work presents an interactive graphics software developed in MATLAB, version 2018 [5], which allows the analysis of continuous beams for any cable layout, considering post tensioning. The analysis step is based on an association of the force method, used to obtain the equivalent loads generated by prestressing, with the direct stiffness method, used to obtain displacements, reactions and diagrams, generated by the prestressing and other user defined loads.

In order to verify its correct functioning, results of tests carried out with several beams models are presented, varying the number of spans and the cable layout, which were compared with values obtained with a commercial finite element software. The software also allows to obtain the stresses generated by different load combinations over sections throughout the length of the beam. In this sense, it will be shown how the software can be used in the design of such structures according to NBR 6118/2014 [6], considering bending moments and shearing forces.

2 Design according to NBR 6118/2014

According to section 3.1. of the Brazilian code NBR 6118/2014 [6], prestressed structures are those in which part of the reinforcement is previously stretched by special prestressing equipment in order to prevent or limit the cracking and displacements of the structure under service conditions, and provide the best use of high strength steel in the Ultimate Limit State (ULS).

In such structures, the passive reinforcement can be defined as any reinforcement that is not previously stretched, whereas the active reinforcement consists of wires, strands or bars, in which a previous stretching is applied, used to induce prestressing forces. The code also defines post tensioning as the prestressing method in which the tendons are stressed and anchored at the ends of the concrete after the member has been cast and has attained enough strength, as shown in Fig. 1.

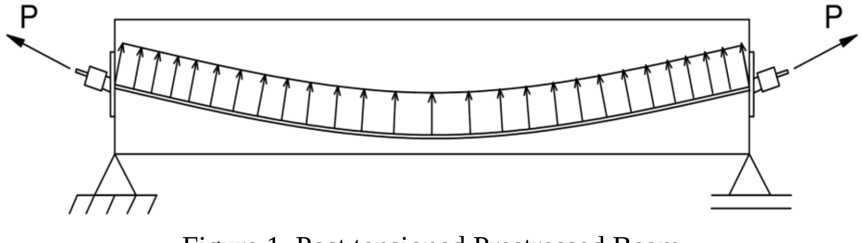

Figure 1. Post tensioned Prestressed Beam

According to Naaman [1], in such method, a mortar-tight metal pipe or duct, also called sheath, is placed along the member before concrete casting. The tendons could be placed loose inside the sheath prior to casting or could be placed after hardening of the concrete. After stressing and anchoring, the gap between each tendon and the sheath is filled with mortar grout, which subsequently gets hard. This technique implies using what are commonly called “bonded tendons”.

The forces generated by prestressing can be calculated directly from the eccentricity of the tendon in the cross-section of the structural element and the prestressing force or through a set of equivalent external loads or by the introduction of imposed deformations corresponding to the previous arrangement elongation of reinforcement.

NBR 6118/2014 also states that the forces generated by prestressing must be calculated considering the initial force and prestressing losses. As shown in the section 9.6.3 of the code, the losses can be divided into immediate losses due to the immediate shortening of the concrete, the friction between the reinforcement and the sheaths or the concrete, the sliding of the reinforcement and the accommodation of the anchoring devices, and in progressive losses, due to creep, retraction and relaxation. According to section 9.6.1, the mean prestressing force in the position x and time t, $P_t(x)$, can be calculated by Eq. (1), where $P_0(x)$ is the initial prestressing force, $\Delta P(x)$ are the immediate losses and $\Delta P_t(x)$ are the progressive losses.

$$P_t(x) = P_0(x) - \Delta P(x) - \Delta P_t(x). \tag{1}$$

As a simplification, in the present article the progressive losses will be taken as equal to 15%, as estimated by Pfeil [7] for continuous prestressed beams with bonded tendons. In this way, only the immediate losses will be calculated by the developed software. In post tensioned elements, the force along the tendon considering losses due to friction can be obtained by Eq. 2, which is shown below.

$$\Delta P(x) = P_i\left[1 - e^{-(\mu\sum\alpha + kx)}\right]. \tag{2}$$

Where $\Delta P(x)$ is the prestressing force along the cable considering friction losses, x is the position where $\Delta P(x)$ is calculated, Pi is the initial applied force, μ is the coefficient of friction, taken as 0,2 between wires and metallic sheaths, $\Sigma\alpha$ is the sum of deflection angles between the anchorage and the point located at position x and k is the coefficient of losses caused by unintended cable bends, usually taken as $0{,}01\mu$ [7].

After calculating the force due to prestressing, the internal forces can be obtained through the calculation sequence shown in section 3 of this article. The next step is to perform the structural checks according to the Brazilian code NBR 6118/2014, as will be shown below.

2.1 Stress Checking

Firstly, the Serviceability Limit States (SLS) need to be checked. They are the limit states of decompression, crack initiation, crack opening, excessive compression and excessive deformation.

Table 1 briefly shows the combinations of actions that must be checked in order to satisfy the Serviceability Limit States and the stress limits for each combination, considering limited prestressing, where p_0 is the prestressing in t=0, g_1 is the weight of the structure, q is an accidental load and p_∞ is the prestressing in $t=\infty$.

Table 1. Combination of Action and Stress Limits [8]

Stage	Combination	σ_{min}(-)	σ_{max} (+)
t=0	$1.1p_0+g_1$	$-0.7f_{ck}$	$1.2f_{ctm}$
Quasi-Permanent	$p_\infty+g_1+0.3q$	$-0.5f_{ck}$	0
Frequent	$p_\infty+g_1+0.5q$	$-0.5f_{ck}$	f_{ct}
Rare	$p_\infty+g_1+q$	0	0

2.2 Bending Moments in Ultimate Limit State (ULS)

The structural design of prestressed beams considering bending moments in the Ultimate Limit State (ULS) can be performed by calculating the Required Prestressing Force (F_{req}), the Required Cable Cross-sectional Area (A_p) and the Initial Prestressing Force (F_0), which is the force that might be applied to the cables at the beginning of the prestressing process.

To perform this, the first step is to calculate the design bending moment (M_{sd}), which can be obtained by Eq. (3), according to NBR 8681 [8], where M_g is the bending moment due to self-weight and M_q is the bending moment due to accidental loads.

$$M_{sd} = 1,35M_g + 1,5M_q. \tag{3}$$

The design shear force V_d can be obtained by Eq. (4), where V_g is the shear force due to self-weight and V_q is the shear force due to accidental loads.

$$V_d = 1,35V_g + 1,5V_q. \tag{4}$$

The Required Prestressing Force (F_{req}) can be obtained by Eq. (5), where z is the moment arm, and θ is the angle of the compressed diagonals, which is usually taken as 35º.

$$F_{req} = \frac{M_{sd}}{z} + \frac{V_d \cdot \cot(\theta)}{2}. \tag{5}$$

Then, the cable's area A_p can be obtained by Eq. (6), where f_{pyd} is the design tensile strength of prestressing steel.

$$A_p = \frac{F_{req}}{f_{pyd}}. \tag{6}$$

The initial prestressing force (F_0) can be calculated from the cable's area through Eq. (7), where f_{tk} is the tensile strength of the cable, which must be limited to 74% of the characteristic strength of the steel for steels of normal relaxation class, according to section 9.6.1 of NBR 6118.

$$F_0 = -f_{tk} \cdot A_{cord}. \tag{7}$$

The force F_0 is the force that must be applied to the cables at the anchorage point, considering the design bending moments and shear forces.

2.3 Shear Force in Ultimate Limit State (ULS)

Finally, the shear force must be checked in the Ultimate Limit State (ULS), considering the Model II of the Brazilian code. To perform this, the first step is to calculate the reduce shear force $V_{d,red}$, given by Eq. (8), where V_d is the design shear force and $V_{p\infty}$ is the shear force due to prestressing in t=∞.

$$V_{d,red} = |V_d| - 0.9|V_{p\infty}|. \tag{8}$$

The next step is to obtain Vc_0, given by Eq. (9), where f_{ctd} is the design tensile strength of concrete, b_w is the width of the section and d is the useful height.

$$V_{c0} = 0.6 \cdot f_{ctd} \cdot b_w \cdot d. \tag{9}$$

Next, the diagonal compression can be checked by the calculus of the compressed diagonal V_{rd2}, relative to the ruin of the struts, given by Eq. (10).

$$V_{rd2} = 0.54 \cdot \alpha_{v2} \cdot f_{cd} \cdot b_w \cdot d \cdot sen\theta \cdot \cos\theta. \tag{10}$$

Where f_{cd} is the design compressive strength of concrete, θ is the angle of the struts (θ=35º, for model II) and α_{v2} is a dimensionless factor given by Eq. (11).

$$\alpha_{v2} = \left(1 - \frac{f_{ck}}{250}\right). \quad (11)$$

The portion of the shear force resisted by concrete V_c can be calculated using Eq. (12).

$$V_c = \beta \cdot \left(\frac{V_{c0} \cdot (V_{Rd2} - V_{d,red})}{(V_{Rd2})} \right). \quad (12)$$

Where β is a dimensionless factor given by Eq. (13). In this expression M_0 is the cracking moment and M_{sd} is the design moment.

$$\beta = 1 + \frac{M_0}{M_{sd}}. \quad (13)$$

Finally, the shear force resisted by stirrups V_{sw} can be obtained by Eq. (14), which can be used to obtain the area of stirrups per meter, given by Eq. (15), where f_{yd} is the yield stress of steel.

$$V_{sw} = V_d - V_c. \quad (14)$$

$$\frac{A_{sw}}{s} = \frac{V_{sw}}{0.9 \cdot d \cdot f_{yd} \cdot \cot(\theta)}. \quad (15)$$

Next, it will be shown how the internal forces due to prestressing can be obtained so that it can be used in the structural checks.

3 Structural Analysis

Prestressing can be defined as a self-balanced system of forces, which does not generate support reactions in statically determinate structures, generating only internal forces due to the eccentricity of the cable [9]. These forces can be determined by isolating an infinitesimal element of the beam, considering the action of the prestressing force P, and equating to zero the sum of moments and forces relative to the neutral line, as shown in Figure 2.

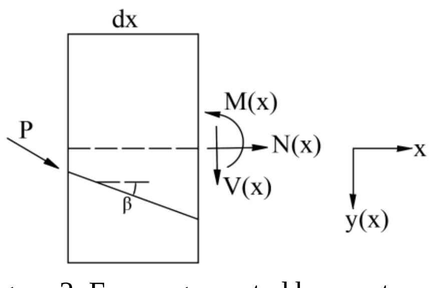

Figure 2. Forces generated by prestressing

In this sense, considering the sign convention shown in Figure 1 for the eccentricity of the cable in relation to the neutral line y(x) and considering that the cable is approximately straight on the infinitesimal length , the isostatic prestressing forces, normal N(x), bending moment M(x) and shearing force V(x) can be determined by the Eq. (16), (17) and (18), where β is the cable slope.

$$N(x) = -P \cdot sen(\beta). \quad (16)$$

$$V(x) = -P \cdot \cos(\beta). \quad (17)$$

$$M(x) = -P \cdot sen(\beta) \cdot y(x). \quad (18)$$

For statically indeterminate structures, however, the effect of the prestressing on the structure is composed of a determinate part, that can be obtained directly from the layout of the cable along the beam and of a statically indeterminate part [4]. Considering the example of a double cantilever beam containing a cable with force P, constant along the cable, as shown in Figure 3, a way of determining these forces is based on the application of the Force method, a classical method of structural analysis.

For the application of the Force Method, an auxiliary statically determinate system corresponding to a simply supported beam is adopted, with the virtual forces X_1, X_2 and X_3, whose positive directions are shown in Figure 3. Then, unity forces are applied in the directions of the virtual forces and the internal forces generated by the unit loads are obtained (E_1, E_2 and E_3 states, respectively), which will be used to calculate the flexibility coefficients. The internal forces generated by the prestressing along the auxiliary beam (named E_0 state) can be obtained by Eq. (16), (17) and (18).

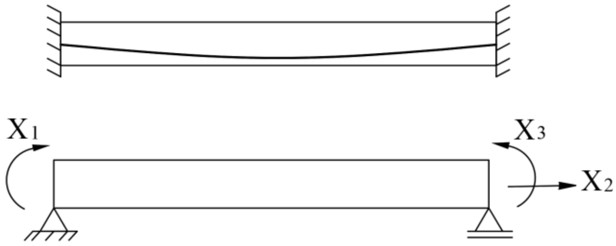

Figure 3. Double cantilever beam and its auxiliary statically determinate system

The flexibility coefficients and load coefficients δ_{ij} of the Force Method can be obtained by Eq. 19, for i=0,1,2,3, where N_i and M_i are the bending moments and normal forces generated by the E_i state and N_j and M_j are the bending moments and normal forces generated by the E_j state. In the calculation of the coefficients, the deformation energy due to the shear forces can be neglected, since the influence of this portion is very small for beams of large spans, common in prestressed structures [10].

$$\delta_{ij} = \sum_b \int \frac{M_i M_j}{EI} + \frac{N_i N_J}{EA} dx. \cdot \quad (19)$$

The method for numerical integration 1/3 Simpson's rule, given by Eq. (20) can be applied to solve the integral shown in Eq. (19), by dividing the beam into a pair number of intervals, where Δh is the interval size and x_i are the points where the function is calculated.

$$\int f(x)dx = \left(\frac{\Delta h}{3}\right) \cdot \left[f(\chi_0) + 4f(\chi_1) + 2f(\chi_2) + ... + 4f(\chi_{2n-1}) + f(\chi_n) \right]. \quad (20)$$

Once the flexibility coefficients are obtained, the virtual forces X_1, X_2 and X_3 can be determined from the displacement compatibility equations. Then, the support reaction due to prestressing can be obtained using equilibrium equations. This same logic can be used to obtain the equivalent nodal forces due to prestressing in a continuous beam. A practical way consists of dividing the beam into frame elements, each one corresponding to a span, and in the application of the force method for each element, as shown in Figure 4.

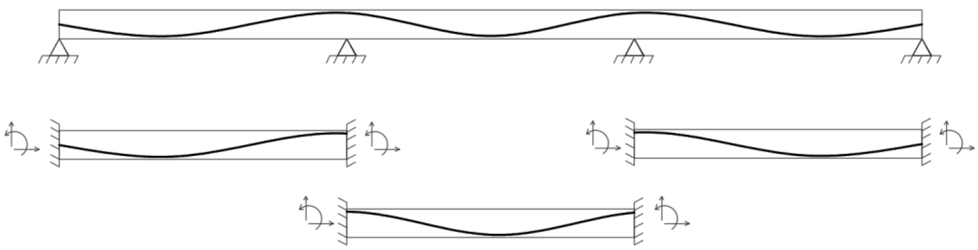

Figure 4. Continuous prestressed beam and its division into frame elements

Once the equivalent nodal forces are obtained, the next step is to apply the Direct Stiffness Method to obtain the displacements and nodal forces, according to the procedure described by Lima [11]. Initially, the stiffness matrices are obtained for each element, using Eq. (21), where E is the elastic modulus of the material, I is the Inertia Moment and A is the area of the section. In the calculation of the stiffness matrix, it is neglected the contribution of shearing deformation.

$$k_e = \begin{bmatrix} \frac{EA}{L} & & & & & \\ 0 & \frac{12EI}{L^3} & & & symmetrical & \\ 0 & \frac{6EI}{L^2} & \frac{4EI}{L} & & & \\ -\frac{EA}{L} & 0 & 0 & \frac{EA}{L} & & \\ 0 & -\frac{12EI}{L^3} & -\frac{6EI}{L^2} & 0 & \frac{12EI}{L^3} & \\ 0 & \frac{6EI}{L^2} & \frac{2EI}{L} & 0 & -\frac{6EI}{L^2} & \frac{4EI}{L} \end{bmatrix}. \tag{21}$$

The next step is to obtain the stiffness matrix of the structure from the contribution of the stiffness coefficients of each element and obtain the vector of combined nodal forces, and then use the equilibrium equation, shown in Eq. (22), where n is the number of degrees of freedom of the structure, to obtain the nodal displacements.

$$\begin{bmatrix} k_{11} & \cdots & k_{1n} \\ \vdots & \ddots & \vdots \\ k_{n1} & \cdots & k_{nn} \end{bmatrix} \cdot \begin{bmatrix} d_1 \\ \vdots \\ d_n \end{bmatrix} = \begin{bmatrix} f_1 \\ \vdots \\ f_n \end{bmatrix}. \tag{22}$$

In order to solve the system, the lines and columns corresponding to the directions with displacement equal to zero are eliminated and the displacements in the free directions are obtained from Eq. (23), where D is the displacement vector, K is the stiffness matrix and F is the force vector. Then, it's possible to return to Eq. (22) to obtain the forces in restrained directions.

$$D = K^{-1} \cdot F. \tag{23}$$

Finally, the forces along the beam can be obtained from the nodal loads. the moment for a section located at a distance x from the initial node of the element can be obtained from Eq. (24) where M_0 is the moment at the initial node of the element, F_0 is the vertical force at the initial node and M_p (x) is the bending moment due to prestressing in the section, calculated by Eq. (18).

$$M(x) = M_0 + F_0 \cdot x + M_p(x). \tag{24}$$

Similarly, the shear force and normal force can be obtained by Eq. (25) and (26), respectively.

$$V(x) = V_0 + V_p(x). \tag{25}$$

$$N(x) = N_0 + N_p(x). \tag{26}$$

Next it will be shown the software developed for the application of this methodology.

4 Software Pstress

In order to apply the analysis method described above, a software called Pstress was developed, using MATLAB [5]. The idea of the program is to provide a user-friendly interface so that users, such as teachers, students and even designers, can perform structural analysis of prestressed continuous beams in a practical and fast way. Figure 5 shows the opening screen of the software.

Figure 5. Opening Screen of Pstress

The program consists of a screen that allows the user to define the geometric properties of the beam and its material, the position of the supports and the geometry of the prestressing cable. Cable geometry is interpolated using spline functions from user supplied points, which can be defined using the cursor or through their manually supplied coordinates. The screen also allows the user to apply external loads distributed along the beam. Figure 6 shows the initial screen of the program.

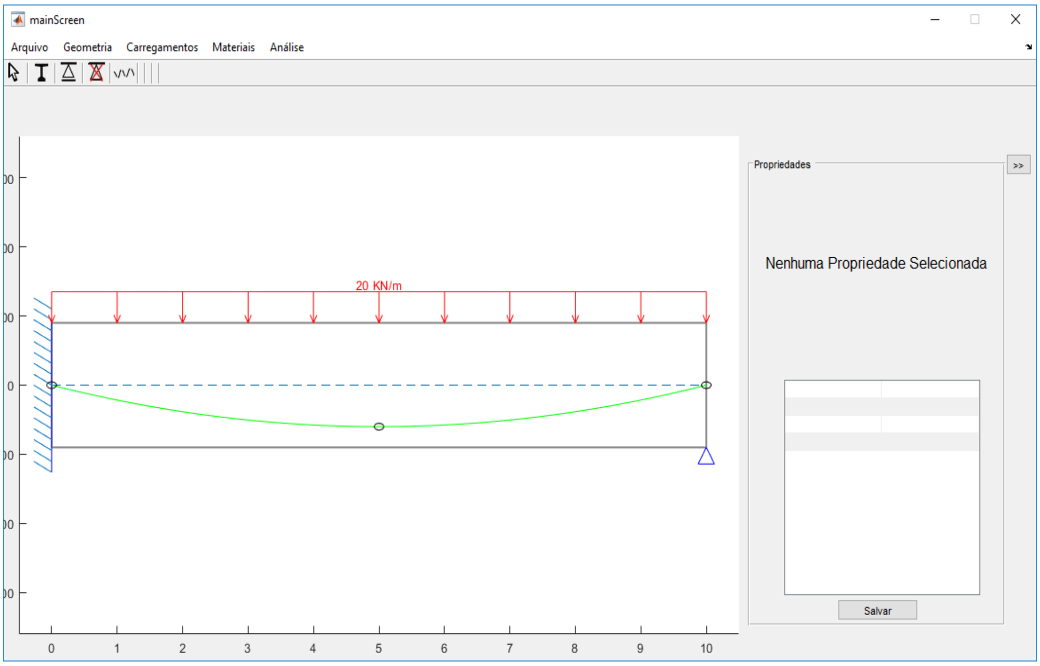

Figure 6. Initial Screen of Pstress

Once the beam and cable properties are defined, the user has the option to perform the analysis, which generates a results screen, which provides the user with bending moment, normal, and shear force diagrams along the structure generated by prestressing, external loading and their combinations. Figure 7 shows the screen with the results provided.

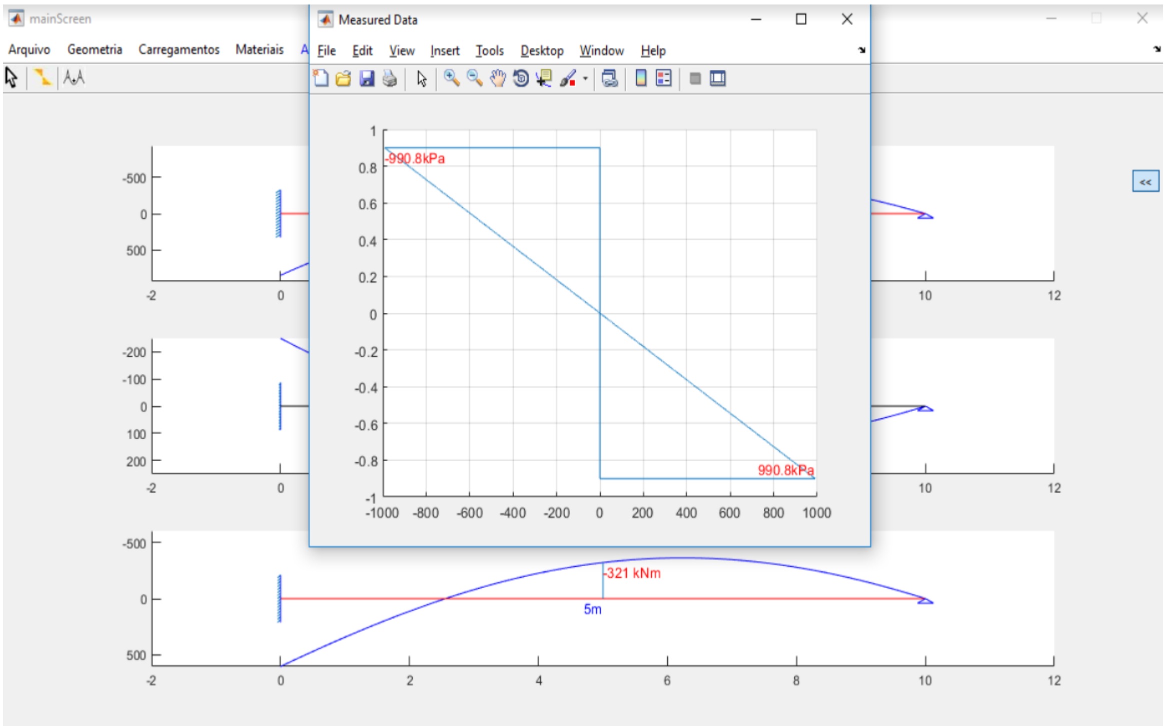

Figure 7. Screen showing analysis results

The software also allows to obtain the stresses generated by different load combinations over sections throughout the length of the beam, which can be used for performing structural checks according to NBR 6118/2014. Figure 8 shows the screen with the structural checking, which allows the user to choose a section, obtain the internal forces and verify the stresses, bending moments and shear forces.

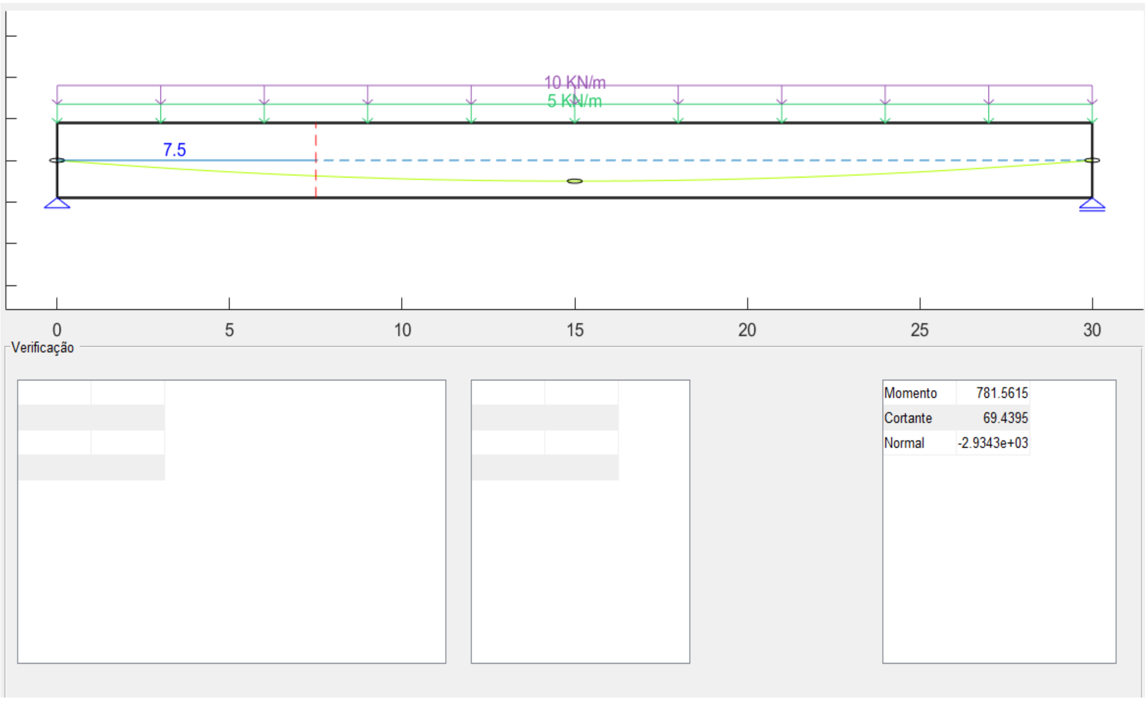

Figure 8 –Screen showing the structural checking

The flowchart shown in Figure 9 below presents a summary of the methodology of the program operation.

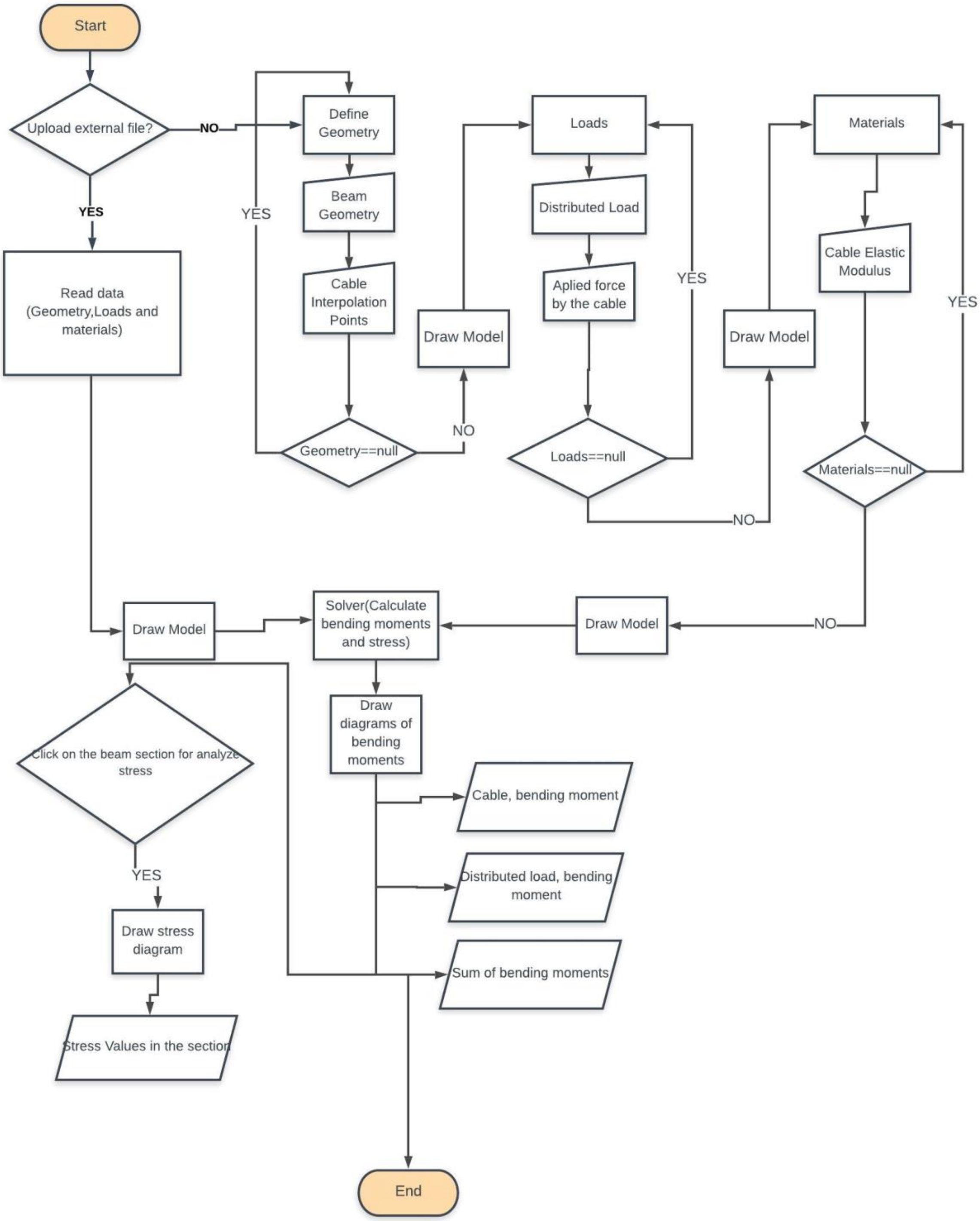

Figure 9. Flowchart of Pstress

5 Applications

To validate the software, three models of beams were used, which are shown in Figures 10, 11 and 12. It's also shown the cable layout and the coordinates of the points used to interpolate its geometry. It has been assumed by convention that downward distances, measured from the neutral line, are positive. The measurements are in meters. In these first models, prestressing losses were not considered, and no external load were applied.

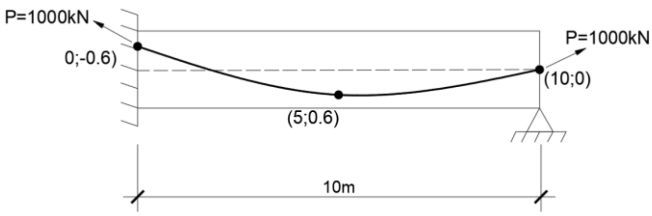

Figure 10. Model 1: Beam of 1 span

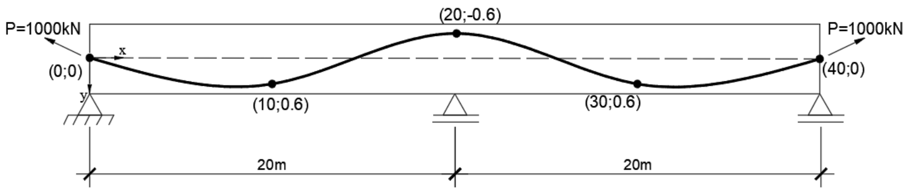

Figure 11. Model 2: Continuous Beam of 2 spans

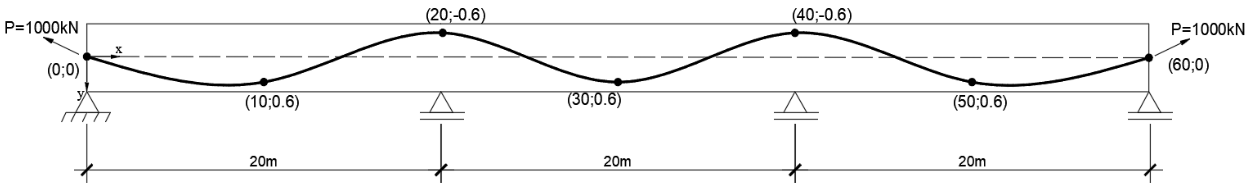

Figure 12. Model 3: Continuous Beam of 3 spans

Figure 13 shows the section properties and dimensions, which were adopted in all models. It was used concrete C40, with secant modulus of elasticity $E_{cs} = 3.19 \times 10^7$ kPa, calculated according to NBR 6118 (2014).

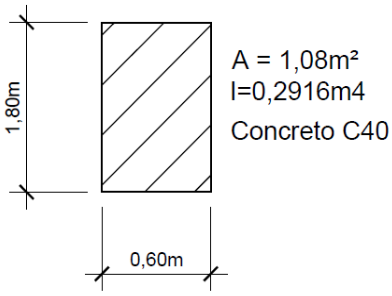

Figure 13. Section Properties

In order to show the structural checking according to NBR 6118/2014, a fourth model was used, as shown in Fig. 14. This model has two applied loads: g1(self-weight) and q (accidental load). In this model, the prestressing force is 3000kN and prestressing losses were considered (coefficient of friction μ=0,2).

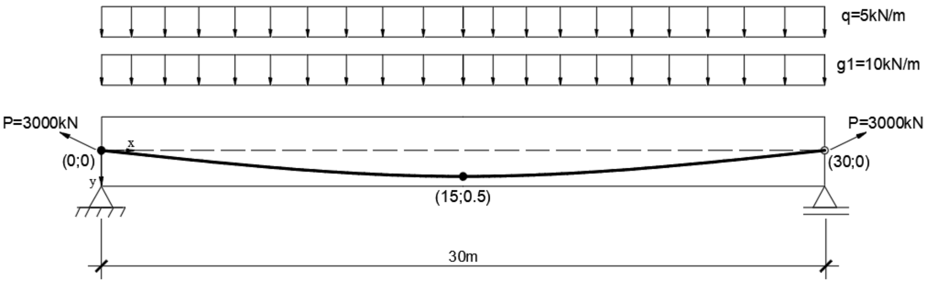

Figure 14. Model 4: Simply Supported Beam

Figures 15, 16, 17 and 18 show the cable layout for each model created in the software Pstress from the points provided.

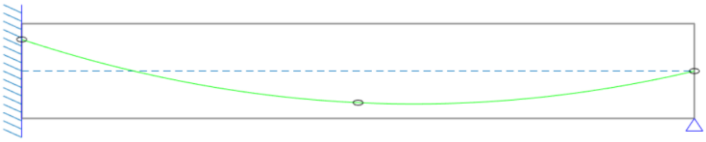

Figure 15. Model 1: cable layout in Pstress

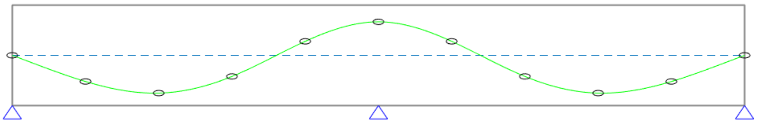

Figure 16. Model 2: cable layout in Pstress

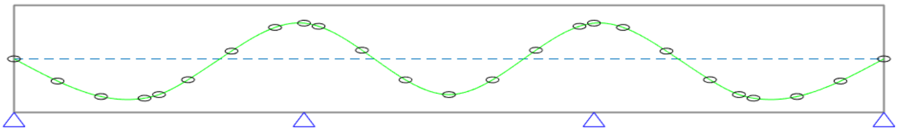

Figure 17. Model 3: cable layout in Pstress

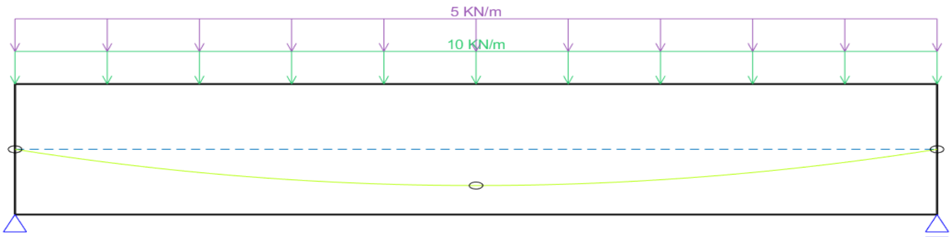

Figure 18. Model 4: cable layout and applied loads in Pstress

In order to compare and validate the results provided by the software, numerical models were developed using the commercial finite element software SAP2000 version 20 [12]. For comparison purposes, only the bending moment diagrams were analyzed. Figures 19, 20, 21 and 22 show the cable layout generated for each beam under study in SAP2000.

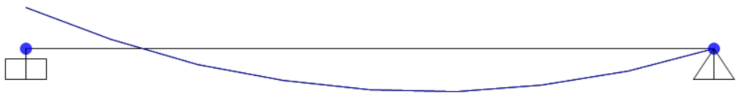

Figure 19. Model 1: cable layout in SAP2000

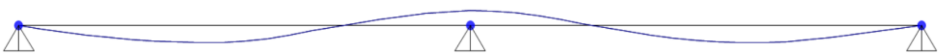

Figure 20. Model 2: cable layout in SAP2000

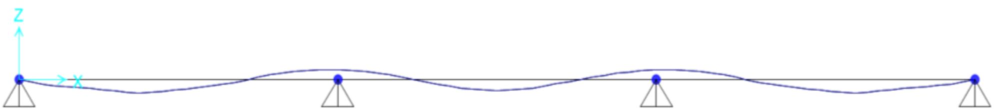

Figure 21. Model 3: cable layout in SAP2000

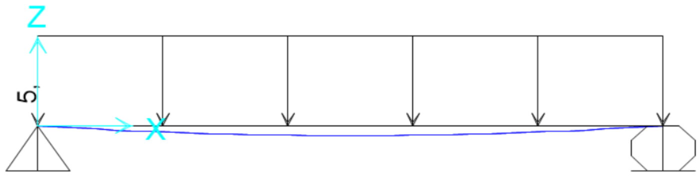

Figure 22. Model 4: cable layout in SAP2000

6 Results and Discussions

6.1 Results from Pstress

Figures 23, 24, 25 and 26 show the bending moment diagrams obtained in Pstress for models 1, 2 3 and 4. The distances are in meters and moments in kNm. In order to compare with SAP2000, only the bending moments were analyzed.

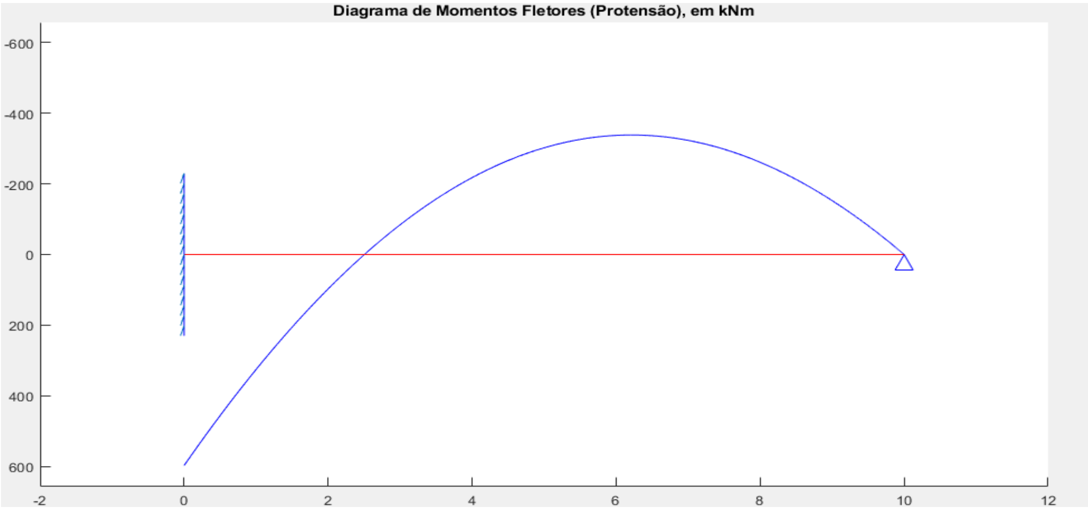

Figure 23. Bending Moment for Model 1 in Pstress (kNm)

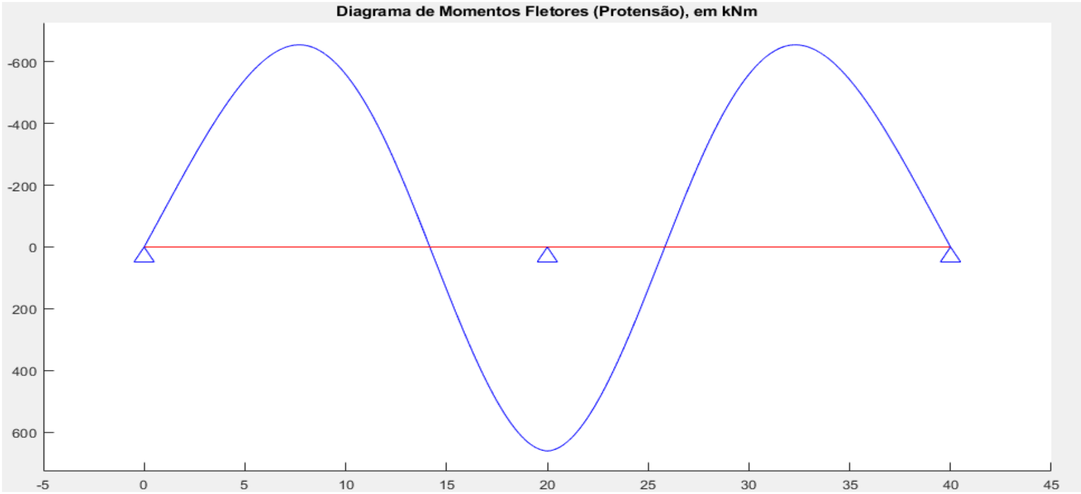

Figure 24. Bending Moment for Model 2 in Pstress (kNm)

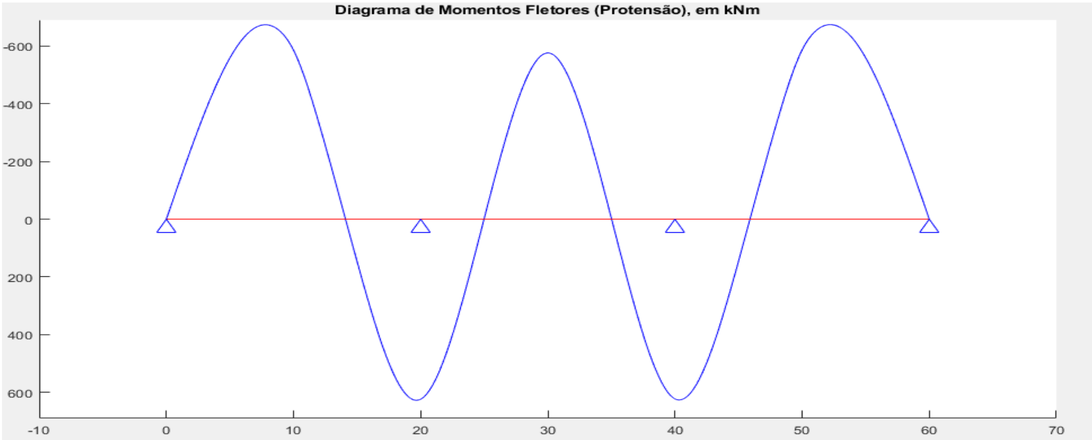

Figure 25. Bending Moment for Model 3 in Pstress (kNm)

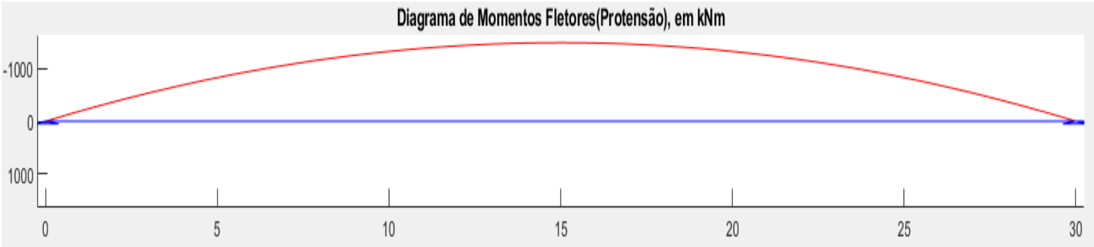

Figure 26. Bending Moment for Model 4 in Pstress due to prestressing force (kNm)

6.2 Results from SAP2000

Figures 27, 28, 29 and 30 show the bending moment diagrams obtained in SAP2000 for models 1, 2, 3 and 4. The distances are in meters and moments in kNm.

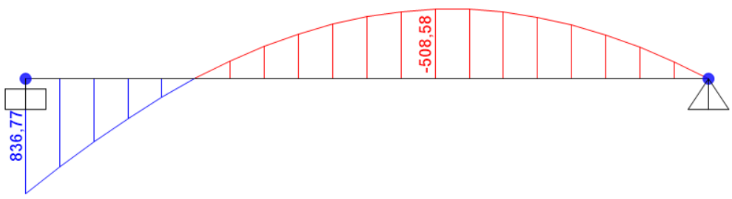

Figure 27. Bending Moment for Model 1 in SAP2000 (kNm)

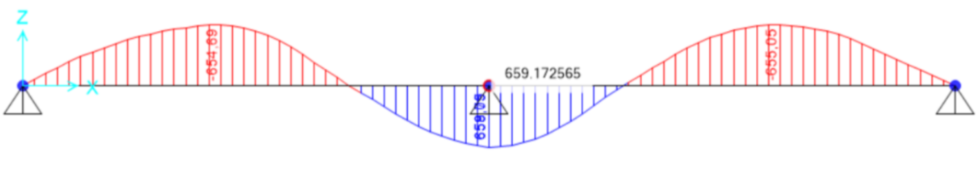

Figure 28. Bending Moment for Model 2 in SAP2000 (kNm)

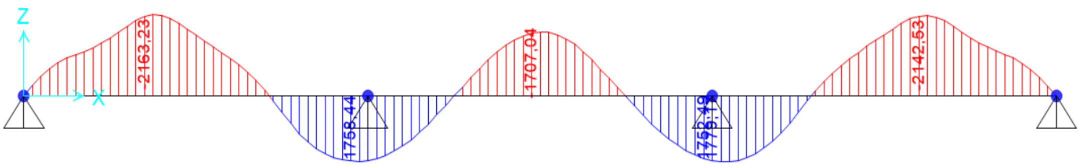

Figure 29. Bending Moment for Model 3 in SAP2000 (kNm)

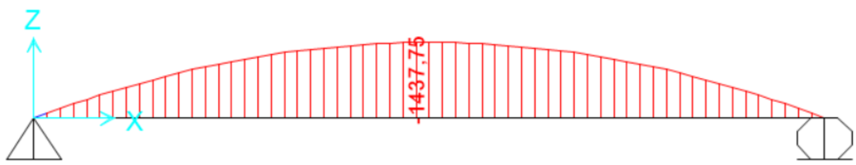

Figure 30. Bending Moment for Model 4 in SAP2000 due to prestressing force (kNm)

6.3 Comparison Between the Results

Table 2 shows the comparison between the results obtained for negative bending moments. In Model 4, the bending moment shown in the table is only due to the prestressing force. Variations between the results were calculated by taking as reference the values obtained in Pstress ($V_{Pstress}$), as shown in Eq. 27.

$$Var(\%) = \frac{(V_{SAP2000} - V_{Pstress}) \cdot 1000}{V_{Pstress}} \tag{27}$$

Table 2. Negative Bending Moments (kNm)

Model	Pstress	SAP2000	Var. (%)
Model 1	-501,6	-508,6	1,40
Model 2	-654,0	-655,1	0,17
Model 3	-674,3	-721,08	6,94
Model 4 (prestressing only)	-1436,6	-1437,8	0,09

Table 3 shows the comparison between the results obtained for negative bending moments.

Table 3. Positive Bending Moments (kNm)

Model	Pstress	SAP2000	Var. (%)
Model 1	853,7	836,8	-1,98
Model 2	659,9	659,2	-0,11
Model 3	627,7	593,1	-5,51
Model 4 (prestressing only)	0,0	0,0	0,00

The results obtained in Pstress are close to those obtained in SAP2000, with small variations, which may be associated with numerical errors, since the elements were divided into 1-meter intervals for the integration of the Force Method, using Simpson's numerical method. Another possible cause is a slight difference in cable layouts, since SAP2000 interpolates geometry using 2nd degree functions, whereas this program interpolates using cubic spline functions. The variations were greater for Model 3, since the interpolation of its cable layout by parabolic functions in SAP2000 proved to be inaccurate.

6.4 Structural Checking According to NBR6118(2014)

A structural checking was performed for model 4 according to NBR6118, as shown in section 2, considering the action of prestressing, self-weight and accidental load, in order to show this part of the software in operation. Figure 31 shows the prestressing force along the cable considering friction losses.

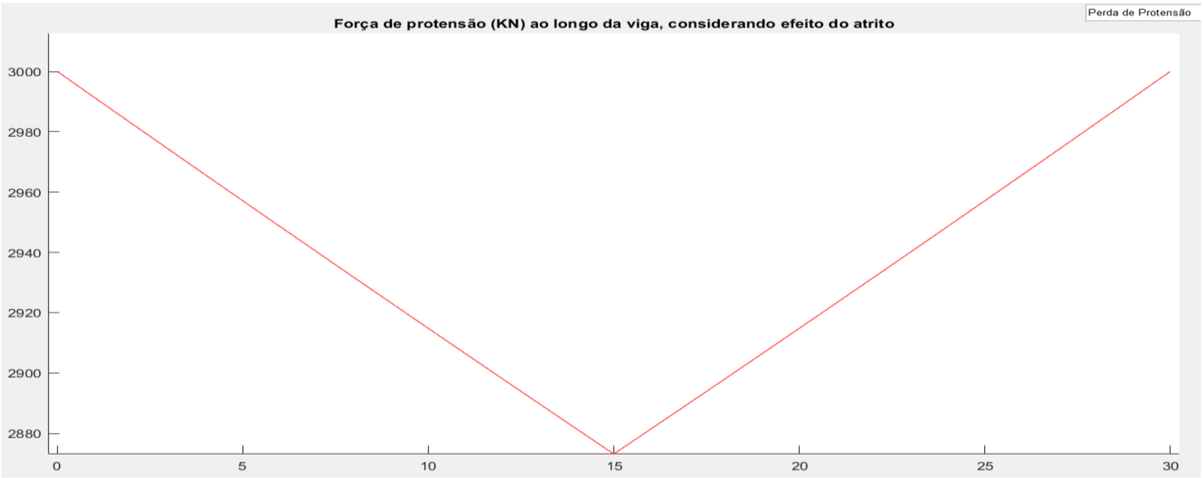

Figure 31. Prestressing force along the cable considering friction losses (kNm).

The design moment in Ultimate Limit State (ULS) was obtained from Eq. (28), according to NBR 8681 [8], where M_g is the bending moment due to self-weight, M_q is the bending moment due to accidental loads and M_p is the moment due to prestressing, which was considered as a favorable action.

$$M_{sd} = 1,35M_g + 1,5M_q + 0,9M_p. \tag{28}$$

Figure 32 shows the bending moments across the beam for each applied load and the total bending moment, considering the combination shown in Eq. (10).

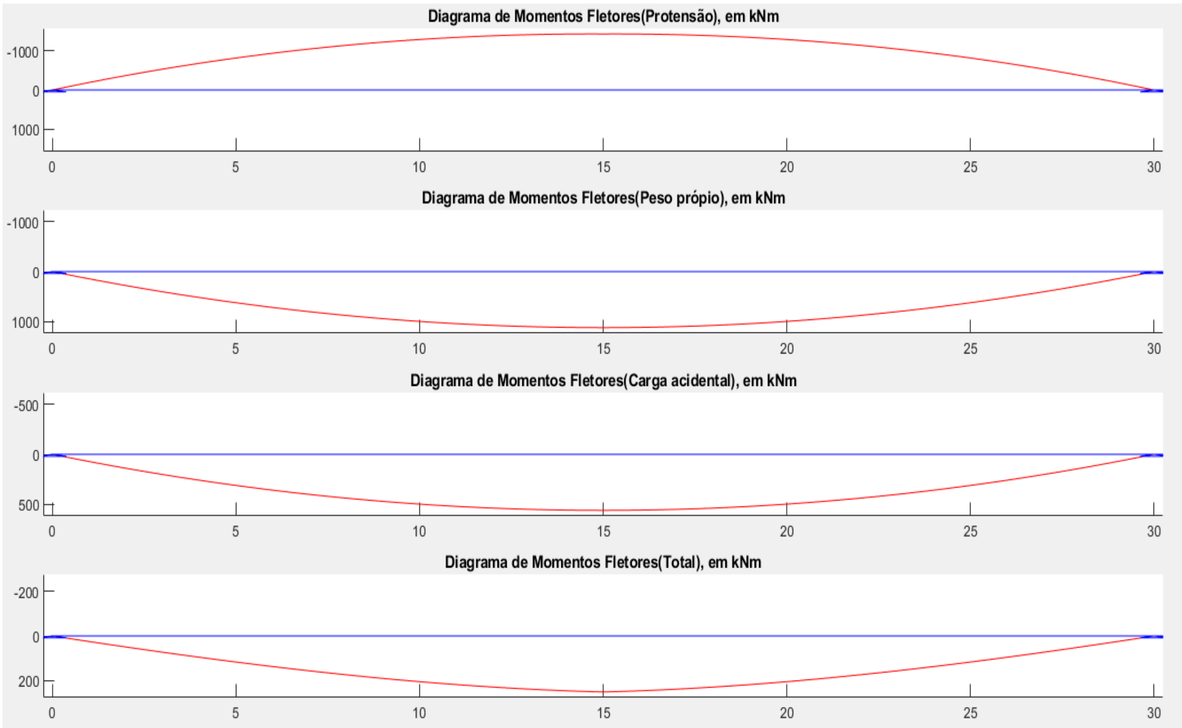

Figure 32. Bending Moments due to Prestressing, Self-Weight and Accidental Load

Figure 33 shows the shear forces across the beam for each applied load and the total shear force, considering a combination like that shown in Eq. (10), but considering the shear forces.

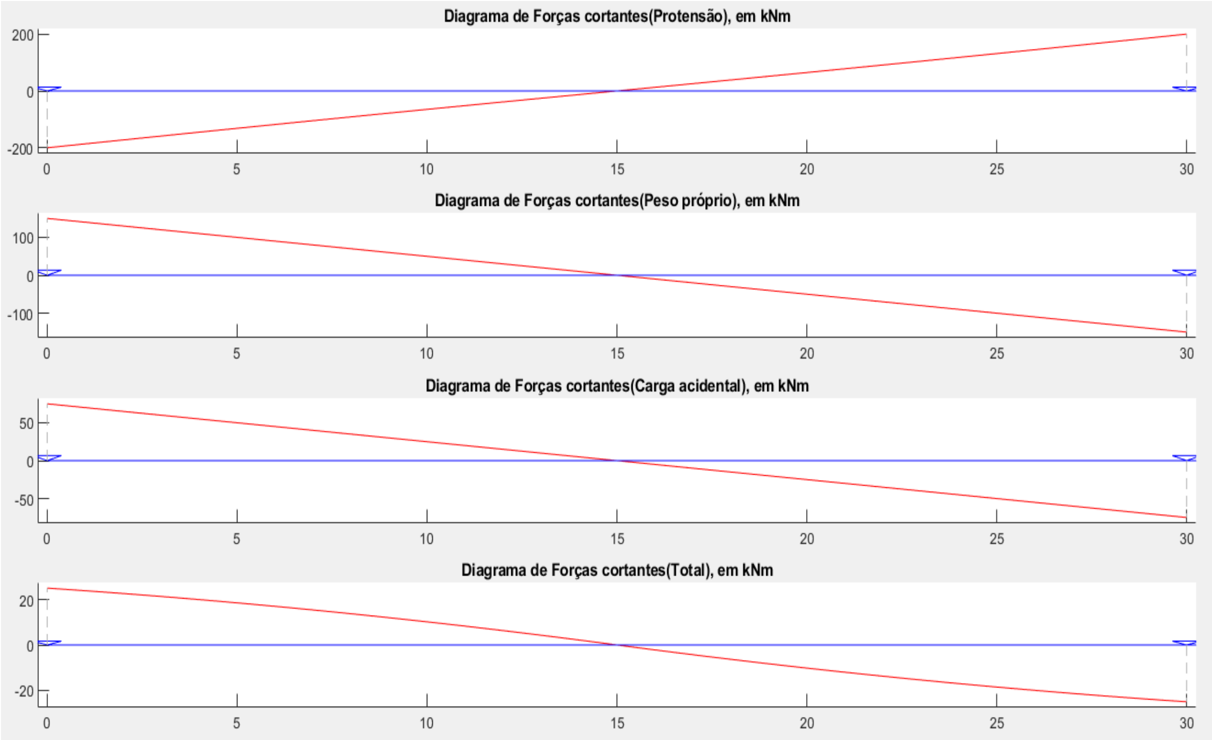

Figure 33. Shear Forces due to Prestressing, Self-Weight and Accidental Load

Figure 34 shows the structural checking in the middle of the beam according to NBR 6118, as presented in section 2. It's possible to see that the applied loads are in accordance with the stress limits presented in Table 1. According to the results, it's needed 8,4cm²/m of steel reinforcement to resist the applied shear forces.

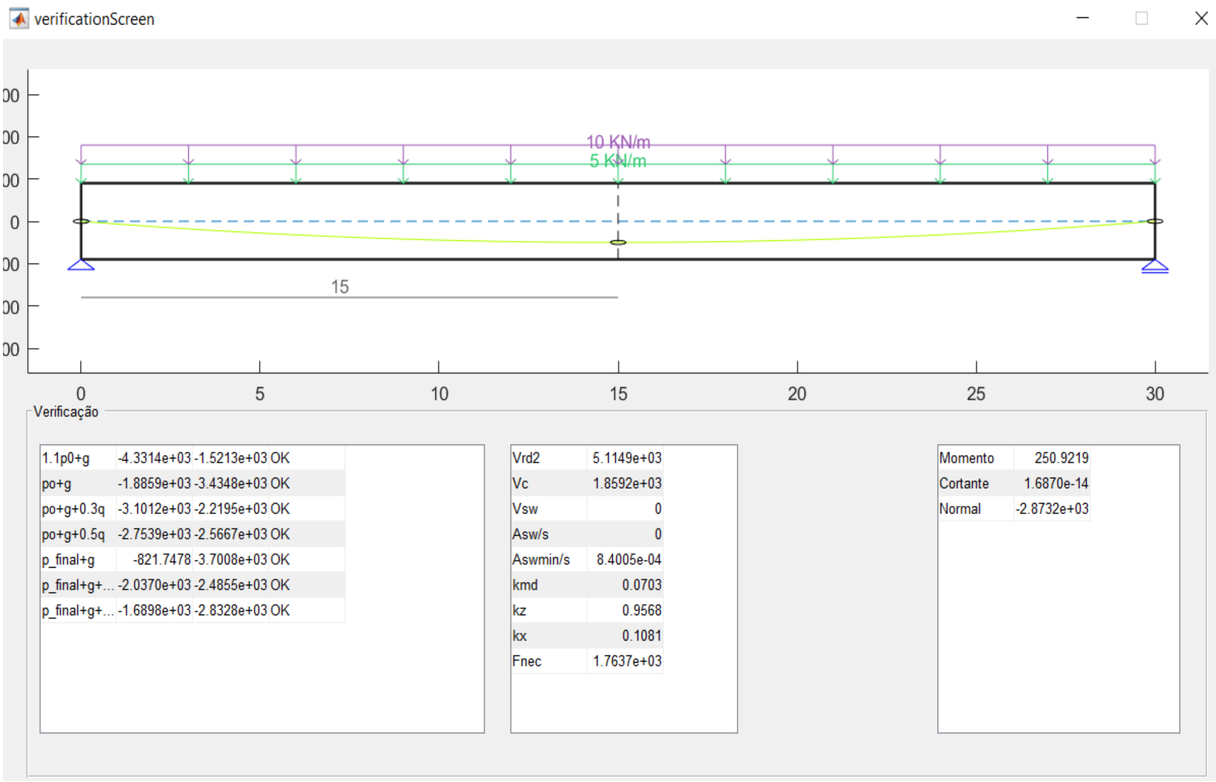

Figure 34. Structural checking according to NBR 6118

From the results presented in Figure 34, it can be concluded that the dimensions adopted for the beam and the applied prestressing force are in accordance with the criteria established by the Brazilian Code.

7 Conclusion

It can be concluded that Pstress is an effective tool for calculating the stresses generated by prestressing in continuous beams. Dividing the beam into sections for the numerical integration of the Force Method proved to be a viable option as it has led to results close to the commercial software. The division of the beam into elements and the consideration of the prestressing effect through equivalent nodal forces also proved to be an effective strategy.

The values obtained by the software show slight variations in relation to the commercial software, which may be associated with numerical errors or slight differences in the cable layout, as shown above. Therefore, it can be concluded that the developed software is a practical, easy-to-use tool, valid for any number of spans and for any curved cable layout, being useful for the calculation of the forces in this kind of structure.

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