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THERMAL DIFFUSION OVER A MASSIVE PORTLAND CEMENT CONCRETE STRUCTURE

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Abstract. The thermal diffusion in continuous solid media is of relevant once temperature and its oscillations influence over a wide diversity of natural phenomena, namely, the alkali-aggregate reaction, concrete Creep and Shrinkage, that affect the concrete structures mechanical performance. The physical model referring to phenomenon approached in this paper culminates in the Heat Diffusion Differential Equation. Despite its modest formulation, the Finite Difference Technique may be used to support the computational tools applied to the numerical analysis expeditiously, suitable to the endorsement of studies and designs. The subject of this work is the numerical simulation of te thermal diffusion across concrete massive structures focusing specially over the temperature fields evolution by time of its continuous soli mass. With a view to the full development of the proposed subject one-dimensional and two-dimensional models are implemented using an automatic language translated algorithm through the Finite Difference Approach on the Heat Diffusion Differential Equation. The obtained results confirm the proposed model suitability to simulate the studied phenomenon behavior, so that represents strategically promising tools to perform similar tasks.
Keywords: Temperature, Diffusion, Finite Difference, Differential Equations, Numerical Simulation.

1 Introduction

The thermal diffusion in continuous solid media is worth of regard to the extent that temperature and their oscillations represent factors that exerts significant influence in a wide diversity of natural phenomena, such as the alkali-silica reaction swelling effect the concrete creep and its shrinkage that affect cement Portland concrete members, interfering, even in the performance of civil construction structures.

In this way, the prediction of the thermal fields evolutions by time across the solid mass of structural members, are of particular interest as regards analysis involving its performance. In this sense, the inclusion of modules intended to the temperature field simulation is useful.

The temperature fields numerical analysis can be made from versatile approximated methods, such as the Finite Element Technique. However, in some cases, notably if the solid object of study presents low values thermal diffusion parameters, as in the case of concrete, the application of the Finite Element Method is hampered, that's why, in particular, disturbs arising in the numeric stability from its use, originated from the numeric behavior referring to the convergence and equilibrium criteria.

In the face of such a panorama, it may be suitable the support of prototypes developed according to a alternative numerical modeling such as the Finite Difference Technique that, despite its modest conception is able to represent an effective strategic resource to answer to the demand now highlighted.

The aim of this work is the numerical simulation of the thermal diffusion across a massive concrete structure, paying attention, in particular, to the analysis of its temperature field progress by time.

With the view to the fulfillment of the subject of this work it was applied a computational algorithm developed by using the C++ automatic language, based on the Finite Difference Approach upon both the one-dimensional and the two-dimensional versions of the Heat Diffusion Differential Equation.

2 Modeling

The rational study of the heat diffusion, through the analysis of its propagation forms, must involve the three process of thermal energy transfer, namely, conduction, convection and radiation, Holman [1]. In practice, however, according to the reality of note, it may be suitable to prioritize that process which, effectively, predominates over the other remaining.

The temperature distribution problem physical modeling over a solid mass according to the transient regimen by heat conduction can be performed from its correlated Diffusion Differential Equation application, since it is considered the initial and the boundary conditions regarding the situations that are been analyzed, Holman [1].

The thermal diffusion analysis that is proposed to perform in this work will take into account the artifice validated by Madureira at al [2], and, at this way, although the specimen studied is a three-dimensional body, the heat flow through the solid mass will be modeled as a two-dimensional version.

Once a temperature gradient in the analyzed sound body has been occurred it will settle a heat flow q(Watts/m^2). If a appropriated source generates thermal energy according to a specific rate represented by g = g(x, y, t)(Watts/m^3), the heat transfer in the system can be expressed from the heat balancing scheme of Figure 1.

Figure 1. Energy Balance

Such a scheme can be represented in mathematical terms by the heat diffusion equation which, in its two-dimensional version, can be expressed through Eq. 1.

$$\frac{\partial}{\partial x}\left(K\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(K\frac{\partial u}{\partial y}\right) + g = \rho c\frac{\partial u}{\partial t} \quad (1)$$

For those cases by which the material is homogeneous and that features uniformly distributed thermal conductivity throughout the body volume, Eq. 1 reduces itself to the form:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{1}{K} g = a^2 \frac{\partial u}{\partial t} \tag{2}$$

Since it may let:

$$a^2 = \frac{c\rho}{K} = \frac{1}{\alpha} \tag{3}$$

the "α" parameter is the material thermal diffusivity and indicates the heat propagation rate through a solid mass constituted by a similar kind of material.

If the heat diffusion process occurs free of an energy external source Eq. 2 takes the form:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = a^2 \frac{\partial u}{\partial t} \tag{4}$$

The numerical solution of the problem may be obtained from the finite difference approach. For such a purpose the continuous solid object of analysis should be subdivided into several elements, drafting in this way a discrete mesh of points. The total observation time of the phenomenon is also subdivided from the consideration of some suitable instants of time accompanying the development of the phenomenon.

For each instant of time and for every point of the solid body the analytical derivatives of the function u = u(x,y,t), that appear on the Heat Diffusion Equation, Eq. 4, is replaced by its corresponding numeric versions that are written in the forms:

$$\left.\frac{\partial u}{\partial t}\right|_k = \frac{u_{i,j,k+1} - u_{i,j,k}}{\Delta t} \tag{5}$$

$$\left.\frac{\partial^2 u}{\partial x^2}\right|_i = \frac{u_{i+1,j,k} - 2u_{i,j,k} - u_{i-1,j,k}}{\Delta x^2} \tag{6}$$

$$\left.\frac{\partial^2 u}{\partial y^2}\right|_j = \frac{u_{i,j+1,k} - 2u_{i,j,k} - u_{i,j-1,k}}{\Delta y^2} \tag{7}$$

If it may be considered:

$$\Delta x = \Delta y \wedge \beta = \Delta t \left(\frac{a}{\Delta x}\right)^2 \tag{8}$$

and combining Eq. 4, Eq. 5, Eq. 6, and Eq.7 it may result into the recurrence form:

$$u_{i,j,k+1} = (1 - 4\beta) u_{i,j,k} + \beta\left(u_{i+1,j,k} - u_{i-1,j,k} + u_{i,j+1,k} - u_{i,j-1,k}\right) \tag{9}$$

If the prior aim to be accomplished is the complete problem solution, the initial condition and the conditions recognized, clearly, at the boundary of the problem domain, that reflect its reality, must be applied to the Eq. 9. The numeric values of the temperature distribution at further instant of time are so obtained, and, in this way, the thermal fields throughout the solid mass may be draft.

The problem featured in this paper may be solved from the Diffusion Differential Equation analytical solution, too. For such a calculus journey it may be suitable to resource to the Bernoulli proposal apud Kreyszig[3] that, consider its bi dimensional version solution as the multiplication involving three functions each of them depending, solely, by everyone independent variable, x, y and t. By using such an artifice the Diffusion Differential Equation exchange itself on three Ordinary Differential Equations. According to Kreyszig[3] and Farlow[4], once the initial and the boundary conditions having been applied, the problem solution would be:

$$u(x,y,t) = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} B_{mn} sin(\vartheta x)\sin(\mu y)\, e^{-\lambda_{mn}^2 t} \tag{10}$$

since that:

$$\vartheta = \frac{m\pi}{L_x};\ \mu = \frac{n\pi}{L_y}; e,\lambda_{mn} = \left(\frac{\pi}{a}\right)^2\left[\left(\frac{m}{L_x}\right)^2 + \left(\frac{n}{L_y}\right)^2\right] \tag{11}$$

and, the L_x and L_y parameters, represent the solid dimension although two coordinate directions. The B_{mn} coefficients are obtained from the EULER's equation, in its form applicable to the Fourier's series coefficients definition, apud Kreyszig[3], represented by:

$$B_{mn} = \frac{4}{L_x L_y}\int_0^{L_y}\int_0^{L_x} f(x,y) sin(\vartheta x)\sin(\mu y)\, dxdy \tag{12}$$

Where, f(x, y) is a real function, previously known, that describes the temperature field distribution at the instant that the diffusion phenomenon is triggered.

3 Computational Support

With a view to the acquisition of results aimed at the support of the tasks affecting the numerical simulation performed in this paper, a computational algorithm written by using the C++ automatic language was elaborated. The computational code is based on the approximation by finite differences and by the analytical solution of the Heat Diffusion Differential Equation.

Such algorithm was structured in this mode to provide the reading of input data referring to the diffusivity coefficient of the material, the discretization mesh topology of the spatial domain of the problem, the problem initial and boundary conditions as well as the phenomenon observation instants of time. The computational code includes in your logical schedule a manager module to perform the output of result in a neutral file focused to supply demands of a certain graphic post-processor.

For the visual display of thermal fields it was used a graphic postprocessor, drawn up in language recognized by the "Embarcadero Delphi" compiler that is compatible to the Windows platform in the programming language "C", Madureira and Silva [5]

4 Program Validation

The program validation has been verified from the comparison of the results obtained by using diffusion equation analytical resolution and those one performed from its numerical version. The computational code was applied to solve the thermal diffusion through a two-dimensional square thin plate 3.00 m size, Figure 2.

By examining the Figure 3, one may constate a good agreement between de curves presented in it.

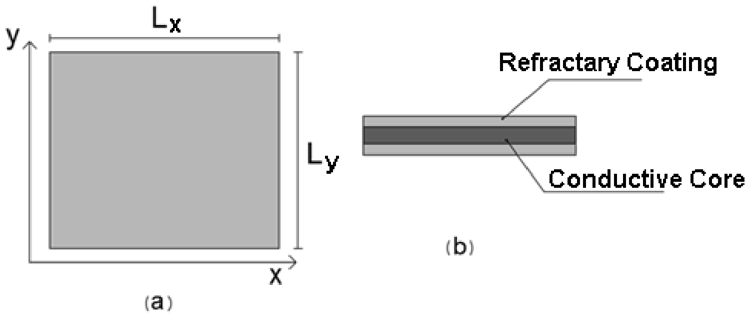

Figure 2. a - Plate; b – Cross section

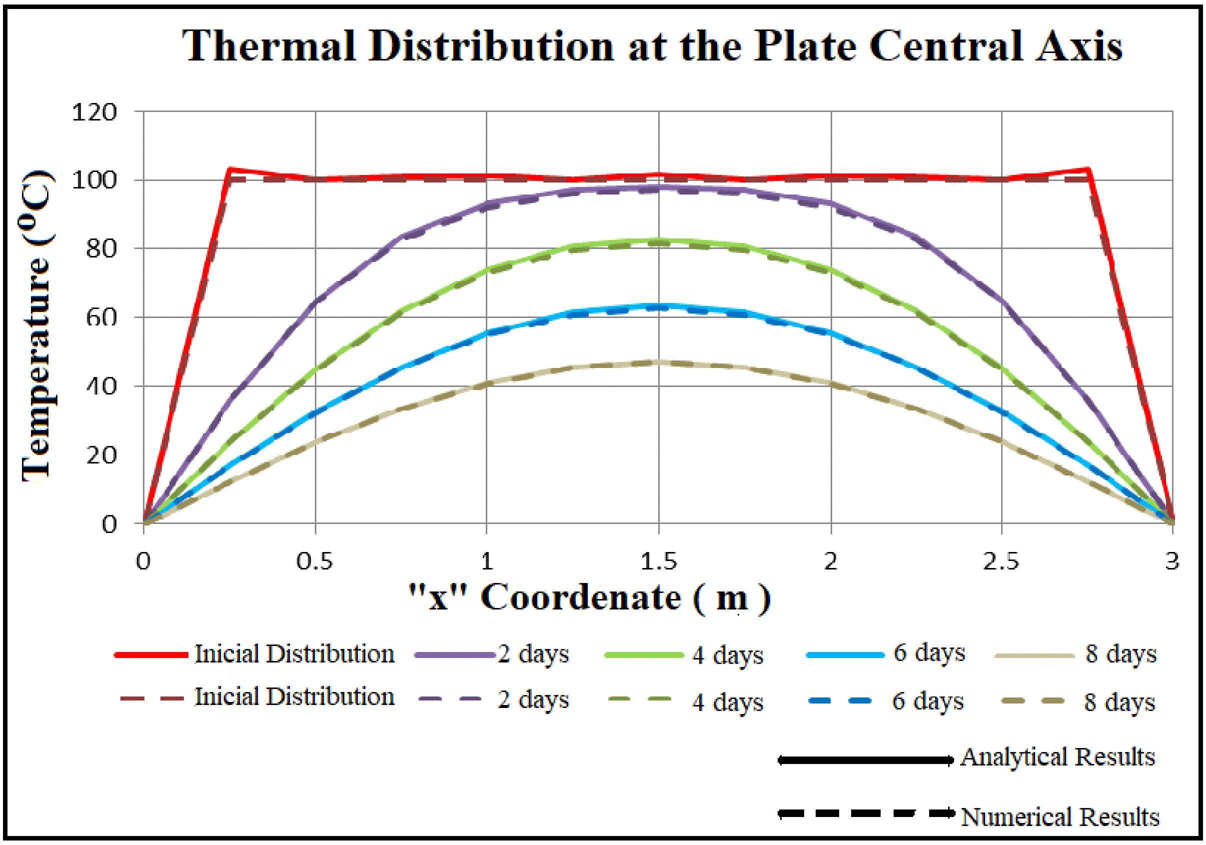

Figure 3. Analytical and numerical results comparison

5 Studied specimens

The constructive member that is analyzed in this paper is a dam presenting 400,00 m length, 70,00 m width at its bottom, 5,00 m width at its crest and 90,00 m height, vertical upstream surface and staggered at its downstream surface, Figure 4. The perimeter surface over the BA and the AG threads is exposed to the atmospheric air and to the sunny radiation and so is maintained at a temperature about 100°C. The perimeter surface over the GF, The FE and the AD threads is exposed to the contact with the soil foundation in which temperature changes from 100°C at the G point to 20°C at the D point. Over the BD line the dam surface is in contact with the mass of water. Along the CD line temperature is maintained at a temperature by the order of 20°C and Along the BC line the temperature changes from 20°C at the C point to 100°C. At the initial time, t = 0 (ZERO), the massive dam temperature presents itself about 0°C.

The sound mass of the studied specimen presents thermal diffusivity $\alpha = 8{,}1 \times 10^{-7}$ m²/s, Gambale and Guedes [6], corresponding to a thermal conductivity by 1,75 W/(m. °C), specific heat by 880 J/(Kg. °C) and specific mass about 24,0 kN/m³.

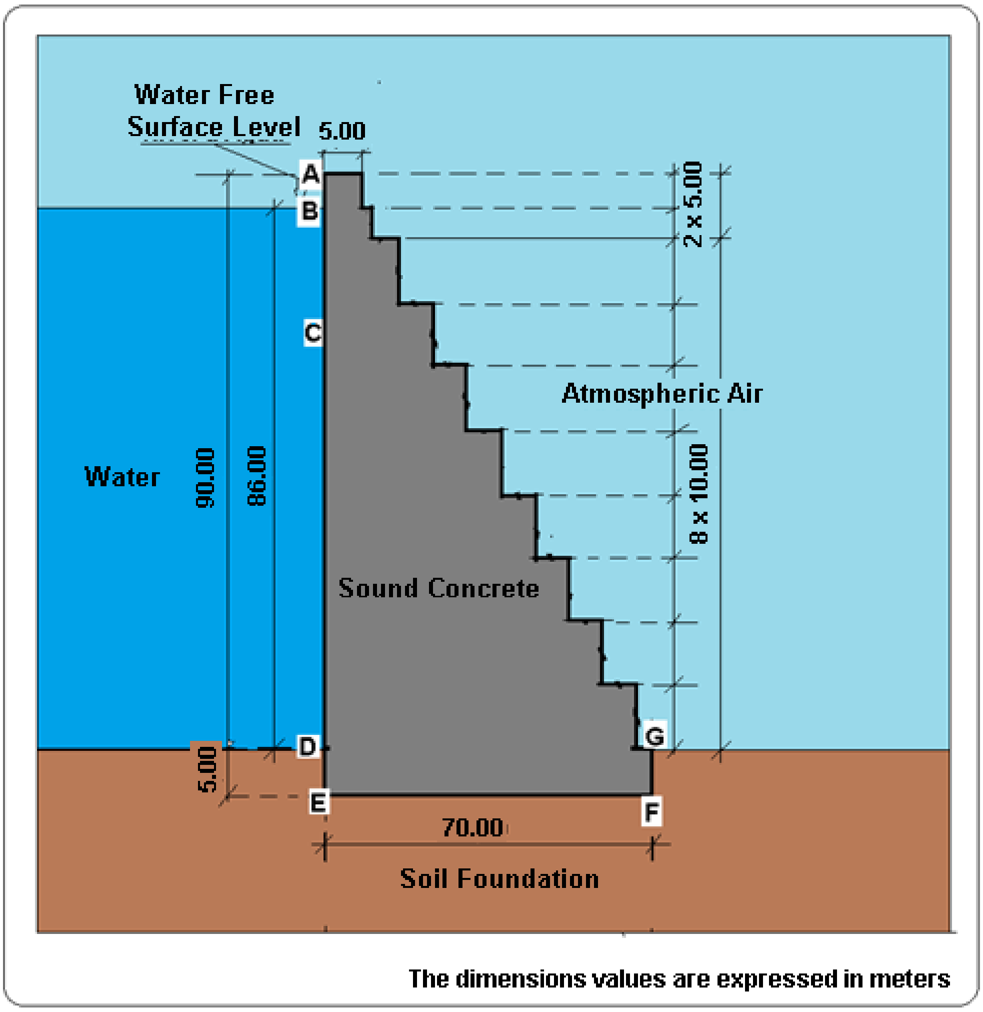

Figure 4. Studied specimens

In numerical simulation cases involving heat diffusion analysis, especially if the material, as the concrete, presents low values for the thermal diffusivity parameters, it may arise deficiencies regarding the accuracy of the results, characterized by instability of numerical nature, Madureira et al [2]. An effective solution for this kind of disturbance is to take care to get a suitable relation involving the discretization by time and the mesh discretization of the problem spatial domain, should be fixed a value between 0 and 0.5, for the "β" parameter of Eq. 8, Lima and Makino [6]. Once such proposal has been adopted and if it is considered the discretization that is practiced for the problem spatial domain, the adoption of time instants of the phenomenon observation about 6, 12, 30, 60, 120, and 240 months, proved to be adequate.

Although the problem that is being analyzed in this paper is, indeed, over the three-dimensional mode, by adopting a special device, it may be treated such a case as if it were over two-dimensional type. Such a device stems from the comparative thermal Diffusion Analysis involving a bar and a plate that present themselves certain correlated dimensions and the further extent of its completion to a solid three-dimensional body.

The models object of such analysis are a bar, figure 5.a, L_x = 3.00 m length while the plate has the same dimension L_x = 3.00 m in the "x" direction, figure 5.b, while in the "y" direction three distinct dimensions are considered, alternatively, namely, L_y = 3.00 m, L_y = 6.00 m and L_y = 12.00 m. Solely, the thermal diffusion results in the "x" direction are y considered. For those cases for which the dimension in 'y" direction present the values L_y = 3.00 m and L_y = 6.00 m, the temperature magnitude distribution across the "x" direction of the plate presented significant differences in relation to those found to the bar. For the case corresponding to de dimension Ly = 12.00 m, on the other hand, the referring results exhibit a good agreement, Figure 6. Thus, one may affirm that, for cases in which the greatest dimension of the plate presents value from four times the value of its smaller dimension, the thermal diffusion develops itself, solely, over the "x" direction. Once this reasoning is extended for a solid body that presents one of its dimensions much larger than the two others, in this case, the significant thermal diffusion manifests itself over the remaining directions whose dimensions be the smallest.

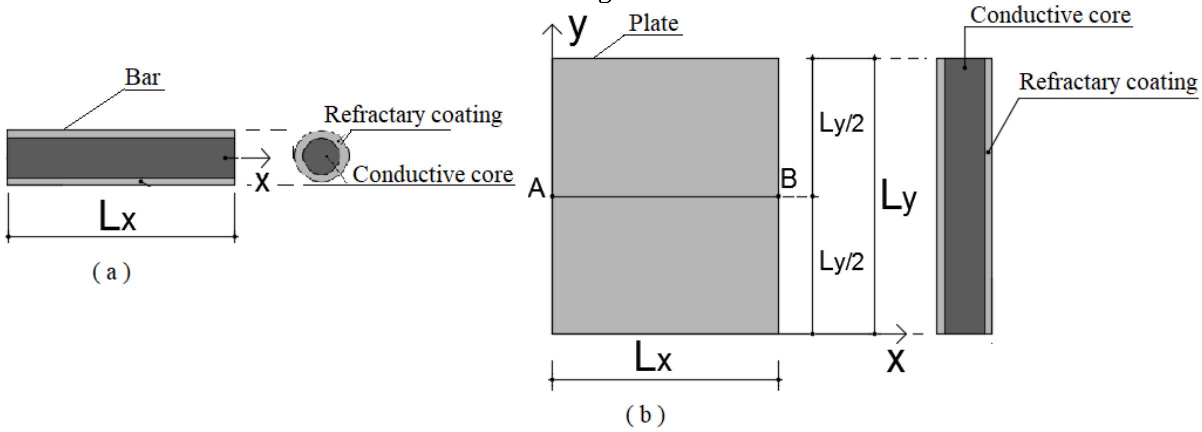

Figura 5. Characterization: a – Bar, b – Plate

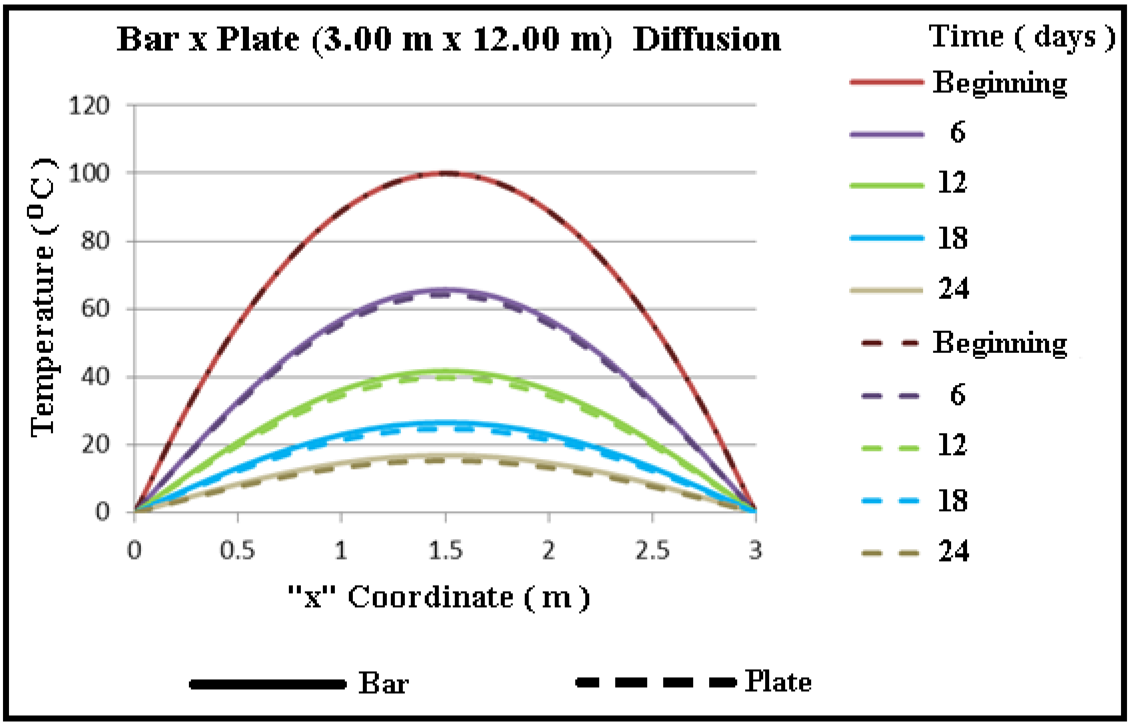

Figure 6. Thermal diffusion comparison bar x plate

6 Results

Figure 6 illustrates the evolution of the temperature fields by time over the dam body, that is the object of the thermal diffusion analysis proposed in this paper.

It is observed that figure 7.a, represents the instant of time in which, hypothetically, the phenomenon of thermal diffusion is started to be monitored, and shows the temperature distribution, so only, in the solid mass of concrete, omitting such information for the dam section contour.

On the other hand, it can be perceived from the sixth month, counted from the date when the diffusion monitoring in analysis began, the day in which the temperature field presents the distribution illustrated in Figure 7.b, the thermal status of the dam section contour, reflecting, in this way, the reality of the boundary conditions of the problem that is analyzed, as defined in section 5 of this work.

It is possible to constate, include, that at the sixth month, the phenomenon already exhibit some evolution, although according discreet amount, as well as, it is possible to indicate the tendency induced by the establishment of thermal gradients to promote heat flow solid bodies, from the regions at higher temperature levels, as in the case of the dam contour perimeter, for those regions at cooler thermal level, as in the case of the inner mass of the dam body.

By examining the set of all the thermal fields contained in Figure 6, one can concatenate a clear idea of its evolution over time, corroborating, even, the statement formulated in the previous paragraph of this work that the heat flow is processing of the surrounding regions to the boundary perimeter of the dam to its inner mass.

By performing an analysis of the temperature fields of Figure 7, in sequence, it is perceived that, from an instant of observation to its subsequently immediate, the thermal fields present distribution, notoriously, distinct, even, in the elapsed period from ten years to twenty years.

The behavior that is reported in the previous paragraph of this paper induces to the conviction that, after ten years, the heat diffusion phenomenon is yet in transient regimen.

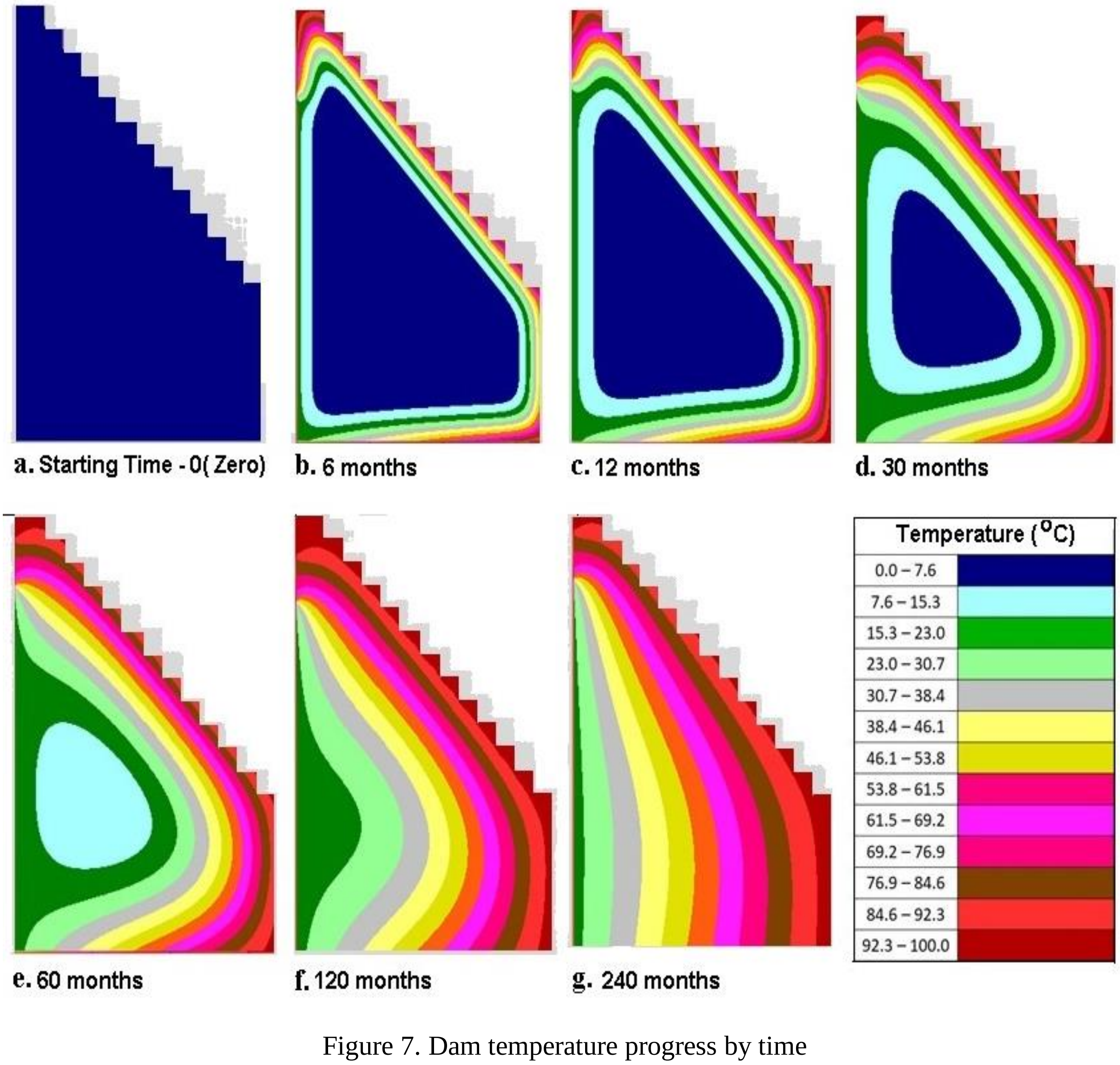

Figure 7. Dam temperature progress by time

7 Conclusions

This work refers itself to the numerical simulation of the thermal diffusion across a massive concrete structure, paying attention, specially, to the analysis of the temperature field progress by time.

The Analysis was performed from the application of a computational code developed according to the C++ automatic language, based on the Finite Difference Approach and the analytical solution, that are applied to the Heat Diffusion Differential Equation.

The studied specimen is a dam that is constituted by cement Portland concrete presenting 400,00 m length, 70,00 m width at its bottom, 5,00 m width at its crest and 90,00 m height, staggered at its downstream surface and vertical upstream surface.

It was adopted the instants of time of the phenomenon observation 6, 12, 30, 60, 120, and 240 months, that were defined in this fashion to attend certain device referring to numeric stability.

The thermal diffusion promoted the heat flow from the regions at higher temperature levels, as in the case of the dam contour perimeter, to those regions at cooler thermal level, as in the case of the inner mass of the dam body.

Due to thermal conductivity small value of the concrete, the heat diffusion phenomenon has endured itself over, at least, ten years, even at the absence of a heat energy source.

According the obtained results from the numeric simulation performed in this work the thermal diffusion process has developed consonant the expected behavior.

Thus, the proceedings and the computational support that was applied to perform the analysis object of this paper showed suitable.

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References

[1] Holman, J.P.. Heat Transfer, McGraw-Hill Book Company, 1972.

[2] Madureira, E.L. Medeiros, E.M. e SILVA, A.L.A. Difusão Térmica em Estrutura de Contenção de Concreto. 58º Congresso Brasileiro do Concreto. Belo Horizonte. MG. 2016.

[3] Kreyszig, E.. Matemática Superior. Volume 3. 2. ed. Rio de Janeiro. Livros Técnicos e Científicos, Editora S.A., 1978.

[4] Farlow, S.J.. Partial Differential Equations for Scientists and Engineers, John Wiley and Sons, 1982.

[5] Madureira, E.L. and Silva, A.L.A. (2013): Project1 – Programa para visualização de campos de tensões resultantes de analises não lineares de modelos bidimensionais de elementos finitos. Versão 1.0, Rio Grande do Norte: DEC/UFRN.

[6] Gambale, E.A. and Guedes, Q.M.. Difusividade Térmica do Concreto. Concreto Massa, Estrutural, Projetado e Compactado a Rolo - Ensaios e Propriedades. Ed PINI, 1997.

[7] Lima, K.L. e Makino, M.. Método das Diferenças Finitas Aplicado à Geotermia rasa em solos de pastagem em Marabá, Revista Científica da Universidade Federal do Pará. Edição n. 2, 2001.