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PROBABILISTIC MODEL OF HUMAN WALKING ACTION FOR DYNAMIC ANALYSIS OF PEDESTRIAN BRIDGES

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Abstract. The recent use of new materials and innovative structural designs in pedestrian bridges has been resulting in structures with low natural frequencies susceptible to excessive vibration problems, requiring a dynamic analysis in the design phase. In the present work, dynamic analyzes were performed considering initially deterministic models with two distinct formulations of the human loading simulation, Fourier series and biodynamic model, the latter to evaluate the relevance of the person-structure interaction in the structural response. To simulate the typical random characteristics of human walking loading, a probabilistic model was also implemented for the simulation of loading parameters. The theoretical results, considering the deterministic and probabilistic models for the two human loading formulations, were correlated with the experimental results of controlled human walking vibration tests (one and four persons walking on the fundamental frequency of the pedestrian bridge) and in normal operation. The best correlation between the theoretical and experimental results was obtained for the probabilistic model of the loading parameters considering the biodynamic formulation.

Keywords: Human loading simulation, walking, probabilistic model

1 Introduction

Computational simulation of human walking has been a challenge in current engineering due to a typical random characterization of this activity, influenced by factors such as posture and pedestrians' age. The main methods of simulation of human walking are the force model and the biodynamic model. These are based on a deterministic approach, often leading to conservative results in the analyzes.

In the probabilistic approach, the variables describing the force induced by human walking can be defined through density functions. Thus, parameters such as walking frequency, and step length vary at each iteration made, according to a probabilistic distribution.

The present paper carried out theoretical analyzes of the dynamic response of a flexible footbridge located in Rio de Janeiro. The structure was submitted to the action of pedestrian walking, simulated through the force model and the biodynamic model, in deterministic and probabilistic approaches. In this way, it is intended to evaluate which model has more consistent results with the experimental tests.

An experimental campaign, described by Rezende *et al.* [1], provided the necessary data for the elaboration of a three-dimensional model in finite elements. From this, a two-dimensional model was generated, considering that the structure has a similar behavior as a simply supported beam. The results obtained in this paper were compared with experimental signals and others acquired through deterministic simulations made by Rezende *et al.* [2] and obtained excellent correlations.

2 Simulation of human loading

According to Pimentel [3], human activities of walking, running, jumping and swinging laterally, in contact with the surface, are the movements of interest as sources of dynamic excitation on the runways. Walking is the most common activity considered in the project, since it is related to the normal use of a footbridge. Many two-dimensional and models of human loading were developed using walking laboratories (Medved [4]). Other measurements are based on the fixation of accelerometers in the person's waist, which reflect the kinematics of body movements, as in Solís *et al*. [5]and Toso *et al.* [6].

The sections below present the two models of human loading studied in this paper: The force model and the biodynamic model.

2.1 Force model

The most common model of human loading simulation consists of a variable force in position and time, whose intensity is periodic, given by a Fourier Series, shown in Eq.(1). Many sources in the literature present values for the coefficients of the series, as in Bachmann *et al.* [7], Murray *et al*. [8], Kerr [9], Young [10], Sétra [11] and ISO 10137 [12]. Due to reasons that will be discussed in the next sections, the present work uses only the coefficients obtained by Kerr [9].

Because of its simplicity, the force model is used extensively for the pedestrian walkway projects and has proved to be quite effective.

$$F(t) = G + \sum_{j=1}^{n} G\alpha_j \sin(2j\pi\, f_p t - \emptyset_j\ ) \tag{1}$$

G = weight of a single person;
f_p = step frequency;
α_j = coefficients of the jth harmonic;

ϕ_j = phase angles of the jth harmonic;
t = time;
$F(t)$ = force's magnitude.

2.2 Biodynamic model

Other analyzes consider the action induced by human walking by means of Biodynamic Models, simulating pedestrians by mass-spring systems of individual dynamic properties. This approach has produced more representative results, since it considers the human-structure interaction, allowing the study of the relevance in considering the coupled system in the final response.

Many sources in the literature have proposals for biodynamic models with varying degrees of freedom. Sachse *et al.* [13] used a biodynamic model with one single degree of freedom to simulate the interaction between pedestrians and crowded structures, and concluded that this can cause increased damping, variation in the value of natural frequencies and even the appearance of new modes of vibration. Kim *et al*. [14] used a biodynamic model with two degrees of freedom, whose properties were extracted from ISO 5982 [15] and compared the results with the force model, applied in a pedestrian footbridge model.

The applications of biodynamic models are not limited to the study and analysis of civil structures, but also the assessment of impacts on the human body. Liang & Chiang [16] developed biodynamic models with several degrees of freedom with an ergonomic approach to assess the impact of vibrations on sitting humans.

For this paper, the model developed by Toso *et al.* [6] was used. The Equations (2), (3) and (4) were generated to represent the dynamic properties of biodynamic models with one degree of freedom, using regression models and artificial neural networks from experimental results found in the literature. With the study, we tried to obtain a more accurate model to represent pedestrians loads based on kinetic and kinematic parameters of the individuals.

$$m(f_p, M) = -231.34 + 3.69M + 154.06f_p - 1.97Mf_p + 0.005M^2 - 15.25{f_p}^2 \tag{2}$$

$$c(M, m) = -1115.69 + 92.56M - 108.94m + 2.91Mm - 1.33M^2 - 1.30m^2 \tag{3}$$

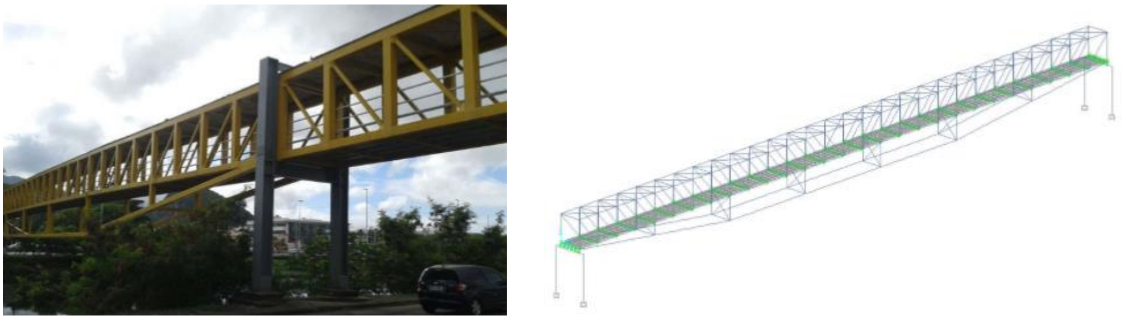
$$k(M, f_p) = 75601.45 - 1295.32M - 33789.75f_p + 506.44Mf_p + 3.59M^2 + 536.39{f_p}^2 \tag{4}$$

M = pedestrian's weight;
m = pedestrian's modal mass;
c = pedestrian's coefficient of damping;
k = pedestrian's stiffness coefficient;
f_p= pedestrian's step frequency.

3 Structure Description

The analyzed structure is a steel inverted queen post truss footbridge and has two spans, one with 68.5m and the other with 17.7m. Only the longest was instrumented because it is the most flexible and the most problematic in terms of vibration. The beams, and bracings are made of steel "U" profiles flange welded, forming a box section. The slab is pre-cast of reinforced concrete, simply supported on the transversal beams. The ceiling of the footbridge is asbestos tile. Fig.1-a presents a general picture of the structure, while Fig.1-b shows the model made in finite elements with SAP2000 [17].

The natural frequency of the structure, obtained experimentally, was 1.85 Hz. This value is within the range of the typical frequencies of human walking (between 1.5 Hz and 2.4 Hz). Therefore, the analyzed footbridge is very susceptible to vibration problems induced by human walking. This was verified during the experimental campaign by observing the pedestrians' discomfort when crossing the footbridge.



(a) (b)

Figure 1. Analyzed footbridge: (a) in situ (b) finite element model

3.1 Campaign of experimental tests

In order to validate the simulations with stride variability, it would be necessary to make a comparison of the data obtained numerically along with the experimental data. Due to this, the results obtained in the experimental campaign conducted by Rezende *et al.* [1] on the footbridge were used as a comparison.

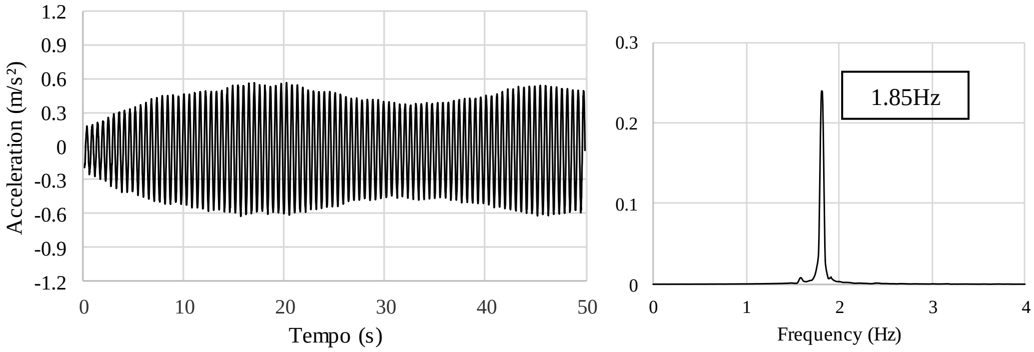
The free vibration tests consisted of the application of impulsive impacts with the heel on the footbridge. The forced vibration tests aimed to excite the first vertical mode of vibration, with experimental frequency of 1.85 Hz. For this, signals were produced with one and four people walking at 1.85Hz. The peak frequency was achieved with the help of a metronome.

It is important to highlight that during the tests, the traffic on the structure was not blocked and, due to the difficulty in synchronizing the steps of several people walking together, the excitation of the structure does not occur homogeneously.

The values of the natural frequencies of the structure obtained in the instrumentation in free vibration are found in Table 1. Fig.2-a and Fig.3-a present the signals obtained in the tests of forced vibration with one person and four people walking in the frequency of 1.85Hz, respectively, by means of a uniaxial accelerometer fixed in the middle of the span. Fig.2-b and Fig.3-b presents the spectra for one and four pedestrians crossing the footbridge, respectively. The peak on 1.85Hz represents the high energy of this frequency, and its predominance in the structural response.

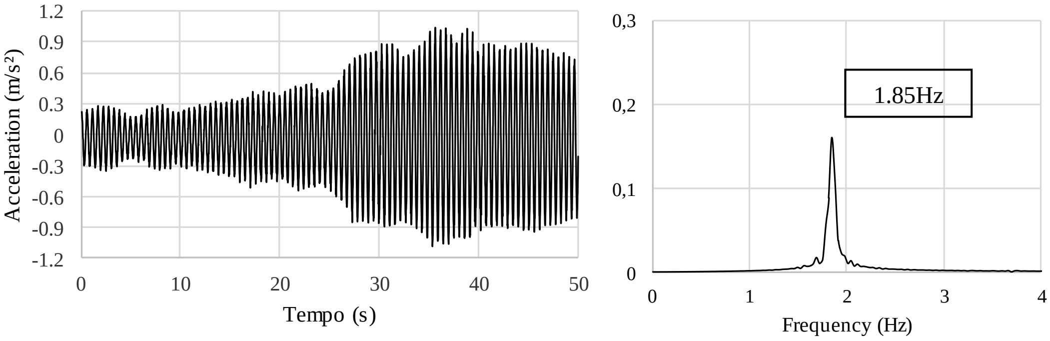
Table 1. Natural frequencies of the footbridge obtained by Rezende *et al.* [1]

Mode	Experimental Frequencies (±0,05Hz)	Modal Form	Damping Rate
1º	1.07	1st mode of lateral bending	1.53%
2º	1.85	1st mode of vertical bending	0.23%
3º	2.88	2nd mode of lateral bending	-
4º	4.20	2nd mode of vertical bending	0.56%



(a) (b)

Figure 2. Signals of acceleration in the vertical direction with one pedestrian walking at 1.85Hz in (a) time domain and (b) frequency domain



(a) (b)

Figure 3. Signals of acceleration in the vertical direction with four pedestrians walking at 1.85Hz in (a) time domain and (b) frequency domain

4 Implementation of the walking variability

Human walking is characterized as a periodic movement that includes parameters in continuous variation, like speed, pace, step length and asymmetrical patterns. Therefore, the deterministic approach proposed by the Force Model and the Biodynamic Model may lead to less consistent results. In a more accurate analysis, the human walking is a random process which can be modeled using statistical distributions.

To simulate the randomness of the pedestrian walking, the Monte Carlo Method (MCM) was used in this work. According to J. Hromkovic [18], the method consists of random samplings in which numerical results are obtained, which seek to simulate non-rational cognitive processes using a normal distribution.

In the next sections, it will be shown how the normal distributions were applied to the walking frequency, step length, and coefficients α and how a uniform distribution was applied to the phase angles of the Fourier Series' harmonics.

4.1 Frequency and step length

The step frequency f_p and step length c_p are independent variables in the simulation and the force applied to the footbridge's slab must be calculated using these two parameters. Several studies address the mean values μ of frequency and step length and their standard deviations for different crowd loads, as in Matsumoto *et al*. [19] and Živanović et al. [20]. It is remarkable that these studies considered that each one of the pedestrians acquires a different frequency, but without the variation of its values over time. In the case of this paper, it was assigned frequency values that vary along the crossing time.

Figure 4 and Fig.5 present examples of the normal distribution functions and that were used at the moment when sought to obtain resonance in the middle of the footbridge during the simulations.

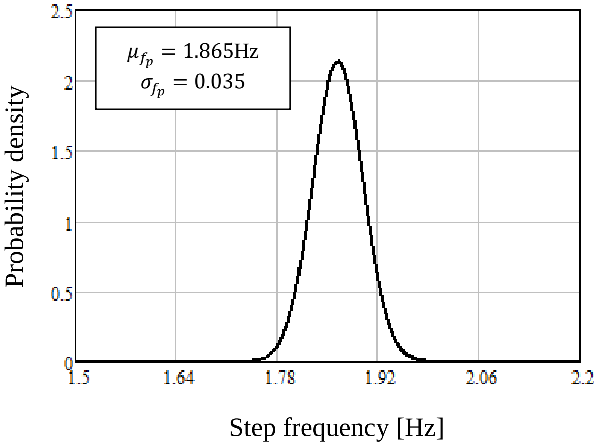


Figure 4. Density function for the step frequency

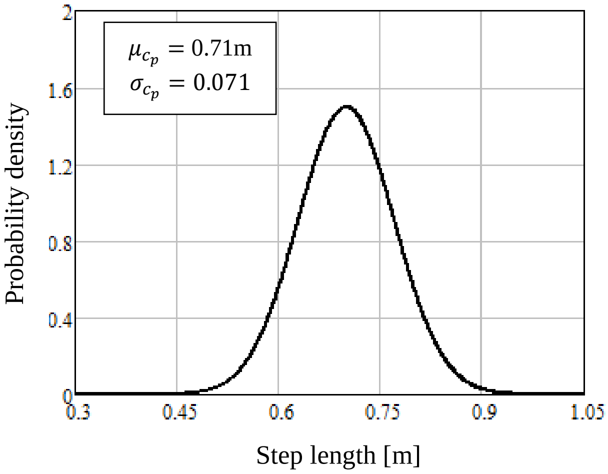


Figure 5. Density function for the step length

4.2 Phase angles

Živanović *et al.* [20] conducted a study of force models based on Fourier Series, considering harmonics and sub-harmonics for random crowd loads. Analyzing the frequency-generated spectra, it was possible to observe that the harmonics between range of 0.25 to 5.25Hz had the phase angles uniformly distributed over a range [-π, + π].

Similarly, in the present work, the phase angles $\emptyset_j$ were generated over time by means of random value choices present in the above-mentioned range, following a uniform distribution. The phase angle of the first harmonic has a zero value, since it is the reference among the other harmonics.

Therefore, only the second harmonic was submitted to the random choice of phase angles because it was the only harmonic frequency within the given range. Figure 6 shows the histogram obtained in the creation of the sampling of the values.

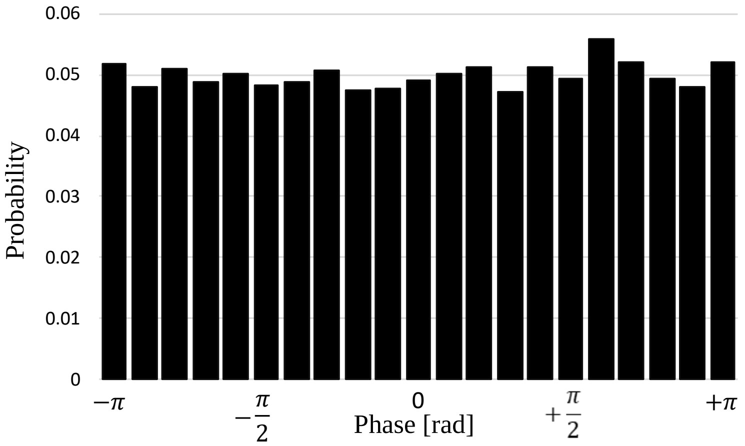


Figure 6. Histogram generated by uniform distribution of phase angles

4.3 Force's harmonic α coeffcients

Different people generate different values of dynamic load factors α, even when walking at the same frequency and Kerr [9] obtained equations, values and standard deviations for coefficients present in up to five harmonics, by means of experimental tests and numerical methods. For the first harmonic, α is dependent on the characteristic frequency of walking at a given moment, and can be calculated from the equation presented in Table 2 with a standard deviation of 0.16.

The method of obtaining the first harmonic α coefficient consisted in choosing a frequency for a time interval, following the normal distribution from Fig.4, and compute it by the equation from first harmonic at Table 2. Then a sampling of first harmonic α was created around the calculated value, following a normal distribution.

The second and third harmonics have a single coefficient and one standard deviation for any frequency. Their respective values can be seen in Table 2.

Table 2. Parameters of the normal distribution of α obtained by Kerr [9]

Harmonic	Harmonic coefficients [α]	Standard deviation [σ]
1º	$-0.2649{f_p}^3 + 1.3206{f_p}^2 - 1.7597f_p + 0.7613$	0.16
2º	0.07	0.03
3º	0.05	0.02

4.4 Force model with the applied stride variability

In Fig.7, it is possible to observe an example of force generated after the application of variability in all Fourier Series' parameters, which denote the force model, where step frequency, phase angle and harmonic coefficients vary over time. It can be observed some varied amplitudes and irregular tracings, which is more representative of the reaction force response of a person walking. For comparation, Fig.8 presents a response for the force model in a deterministic approach.

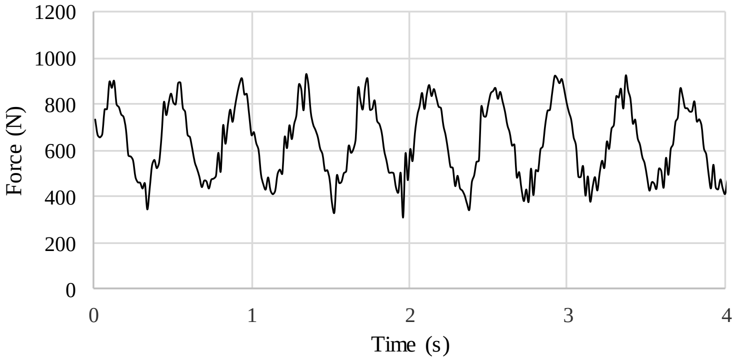


Figure 7. Force model's response with variability in its parameters

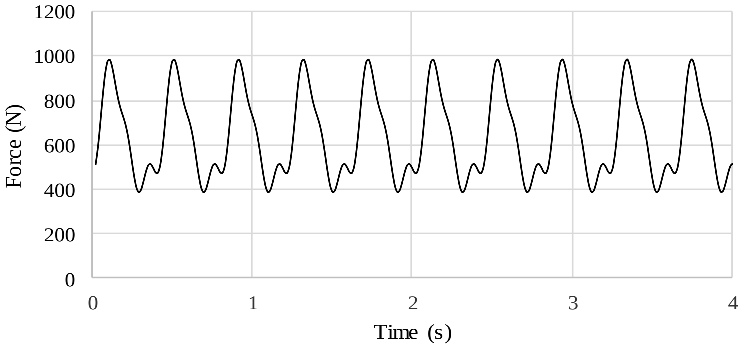


Figure 8. Force model's response with a deterministic approach

5 Simulation and results

Experimental data obtained by Rezende *et al.* [1] in experimental tests were used in the validation of the simulation results. The experimental signals from Fig.2-a and Fig.3-a were divided when a resonance, beat or acceleration reduction pattern was observed. As a result, the acquired signals were divided into three sections each, as presented in Fig.9 and Fig.10.

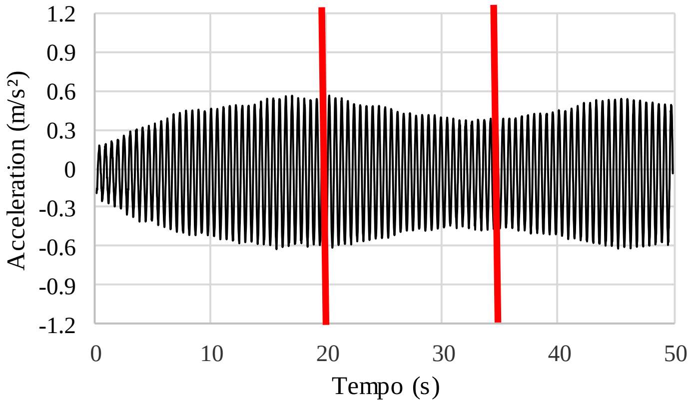


Figure 9. Split division proposed for the signal representing one pedestrian walking in 1.85Hz

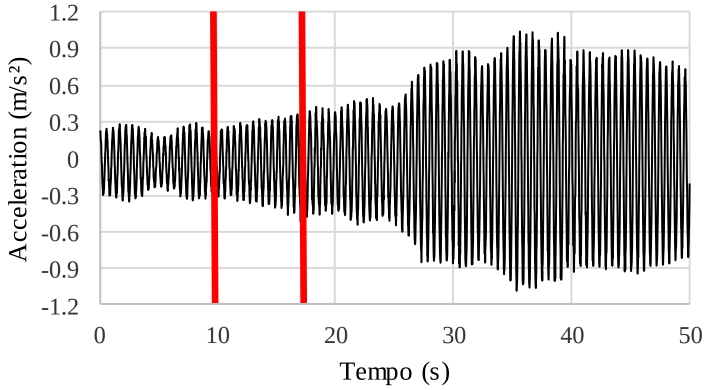


Figure 10. Split division proposed for the signal representing four pedestrians walking in 1.85Hz

Rezende *et al.* [2] conducted simulations with one and four people walking on the footbridge following a deterministic approach, assuming constant values of pacing frequency and pacing length for each signal part.

In the next section, the results obtained with the application of the probabilistic model in the simulations will be shown.

5.1 One person walking in 1.85Hz with stride variability

Figure 8 shows the signal relating to a person walking at 1.85Hz, with the proposed split of segments: 0-20s, 20-35s and 35-50s. In the 0-20s and 35-50s interval an increase in the acceleration amplitude is observed, which indicates a pattern in resonance. In the 20-35s, a decrease of the acceleration occurs, probably caused by a pedestrian pacing mismatch. One must consider the fact that even with the help of the metronome, people usually cannot walk perfectly at an intended frequency. In addition, the transit on the footbridge was not impeded during the test. This situation can generate different patterns in the signal due to the influence of passengers in normal use.

Table 3 shows the characteristics attributed to the parameters for each section, a step frequency and standard deviation value was chosen, based on the proposed signal division. For the first and third stretches, the signal showed the presence of resonance. Therefore, there were assigned step frequencies values closer to the natural frequency of the walkway, combined with low standard deviation, in order to represent a good synchronization with the footbridge natural frequency. For the second section, it was necessary to use a lower frequency value, along with a larger standard deviation, to highlight the gait mismatch in relation to the footbridge natural frequency.

Table 3. Characteristics of the parameters used in the simulation

Strech	Average Step Frequency [Hz]	Frequency Standard Deviation [Hz]	Average Step Length [m]	Step Length Standard Deviation [m]	Pedestrian mass [kg]
0-20s	1.851	0.001	0.71	0.071	65
20-35s	1.757	0.0025	0.71	0.071	65
35-50s	1.85	0.001	0.71	0.071	65

Figure 11 presents the deterministic theoretical results obtained by Rezende *et al.* [2], while Fig.12 presents the results obtained by the probabilistic model. There is no significant difference between the force and biodynamic models in both results. This occurs because the mass of the structure is much greater than the mass of a single person, so the human-structure interaction is practically negligible. Comparing the two results, there are few differences, but the behavior of the signals with the variability shows improvements through the generation of more regular oscillations (mainly in 20-35s signal part) and better approximations with the experimental response.

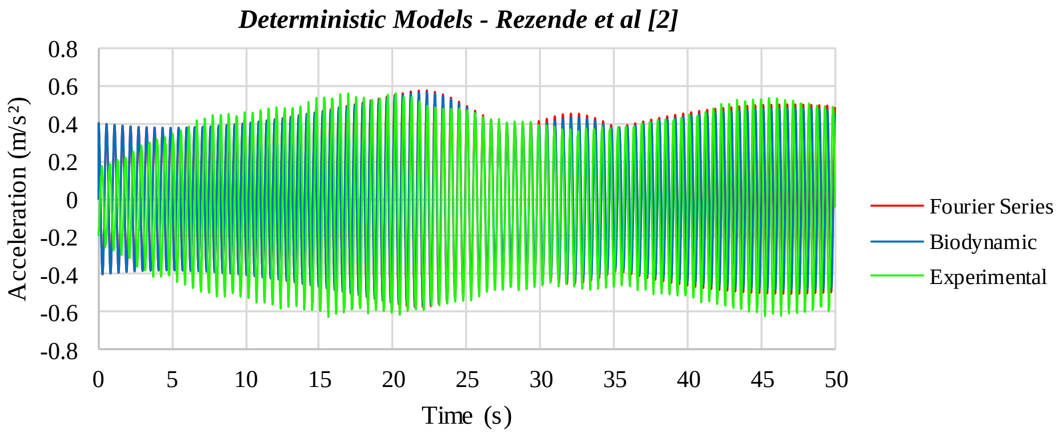


Figure 11. Experimental signal and deterministic signals for a person walking in 1.85Hz

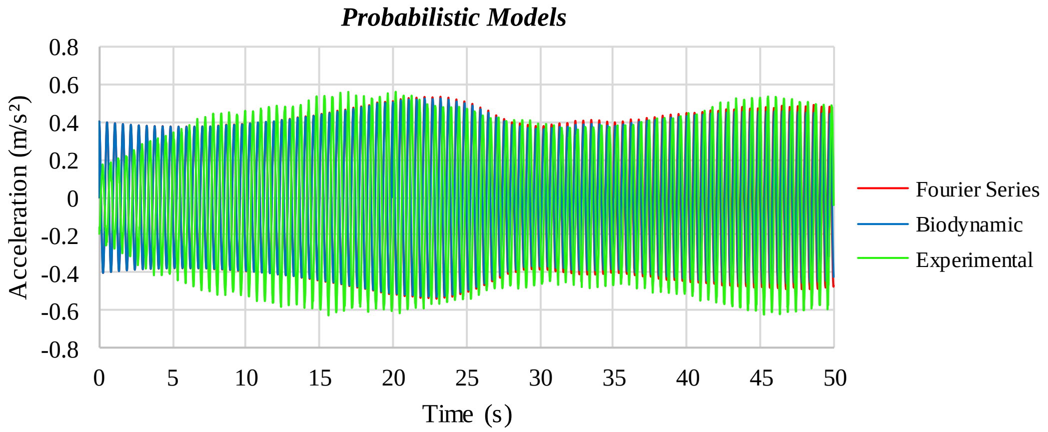


Figure 12. Experimental signal and probabilistic signals for a person walking in 1.85Hz

5.2 Four persons walking in 1.85Hz with stride variability

Similarly, to the previous section, three parts with different patterns were identified in the experimental signal of Fig.9. Between 0 and 10s, a beat situation is observed on the signal, which indicates that the four people were not in perfect synchronized movement. Then, between 10-17s, the signal begins to acquire resonance patterns. Finally, between 17-50s, the signal presents its greatest amplitudes, evidencing a more perfect resonance in this section. The four pedestrians sought to walk at 1.85Hz and were arranged in two pairs, one in front of the other and distant 1 meter as shown in Fig.13.

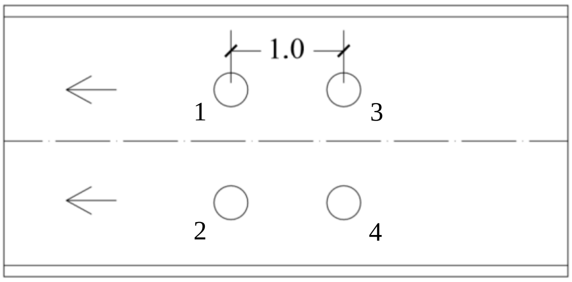


Figure 13. Sketch of the pedestrians' layout on the footbridge. Unit in meters.

In this case, the characteristics assigned to the frequencies are found in Table 4 and Table 5 presents the standard deviations for each stretch and these were chosen based on the proposed signal divisions.

Considering the simulations were conducted in a two-dimensional model, the non-plane layout created by the four pedestrians and the structure symmetry axis, has to be converted into a plane layout. Therefore, it was assumed that the front people (Pedestrian 1 and Pedestrian 2) would be simulated by a single biodynamic model, centered at the structure symmetry axis, with equivalent mass and stiffness calculated as a parallel spring system. The same process was repeated with the pedestrians from behind (Pedestrian 3 and Pedestrian 4).

The frequency values attributed to the pedestrians 3 and 4 were smaller than those of the pedestrians 1 and 2, regarding the care they must had had during the experiment in not walking faster than the front pedestrians. This assumption was adopted during the first and second signal division. However, in the third division, all individuals were assigned equal frequency values closer to 1.85 Hz, in order to generate higher acceleration amplitudes.

The length of the step used was 0.71m for the pedestrians 1 and 2 and 0.67m for the pedestrians 3 and 4 The standard deviation for this parameter was 0.071 for the whole group.

The standard deviations chosen for the probabilistic variables had more influence in this analysis results. It can be observed from the experimental signal of Fig.9 that the human step was being continuously adjusted with the structural natural frequency during the test, converging to the resonance at the end of the signal. Therefore, a smaller standard deviation was chosen in each division, denoting the decrease of the group mismatch in relation to the resonance frequency.

Table 4. Assigned values for average frequencies and step length for each stretch and group of pedestrians

	Pedestrians 1 and 2		Pedestrians 3 and 4	
Stretch	Average Frequency [Hz]	Average Step Length [m]	Average frequency [Hz]	Average Step Length [m]
0-10s	1.75	0.71	1.65	0.67
10-17s	1.845	0.71	1.735	0.67
17-50s	1.865	0.71	1.865	0.67

Table 5. Assigned standard deviation for the step frequency and length according to the stretch and mass for each pedestrian

Stretch	Frequency Standard deviation [Hz]	Step Length Standard deviation [m]	Pedestrian mass [kg]
0-10s	0.008	0.071	65
10-17s	0.005	0.071	65
17-50s	0.0035	0.071	65

Figure 14 shows the deterministic signs obtained by Rezende *et al.* [2] for the deterministic models of force and biodynamics, while Fig.15 presents the signals of the probabilistic models, where both are superimposed on the experimental signal.

Again, a good correlation with the experimental signals is observed, mainly by the biodynamic model, because it considers the human-structure relationship resulting a reduce in the signal’s amplitude. In addition, the probabilistic model obtained a regular signal and lower acceleration amplitude in comparation to the deterministic model, being even closer from the experimental amplitudes. It was able to notice de efficiency of the statistical methods for the dynamic analyzes related to the pedestrian’s walking movements.

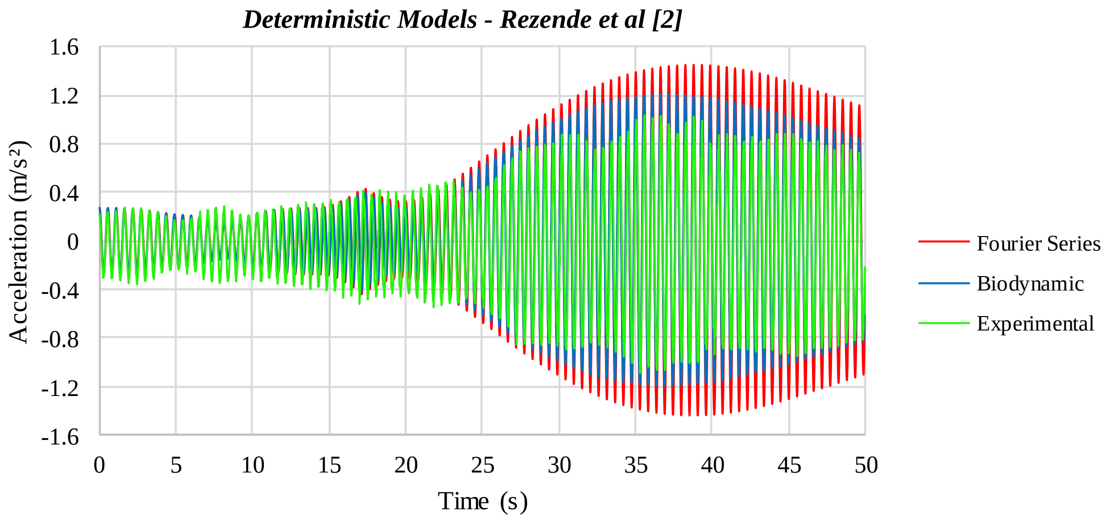


Figure 14. Experimental and deterministic signals for 4 pedestrians walking in 1.85Hz (Rezende *et al.* [2])

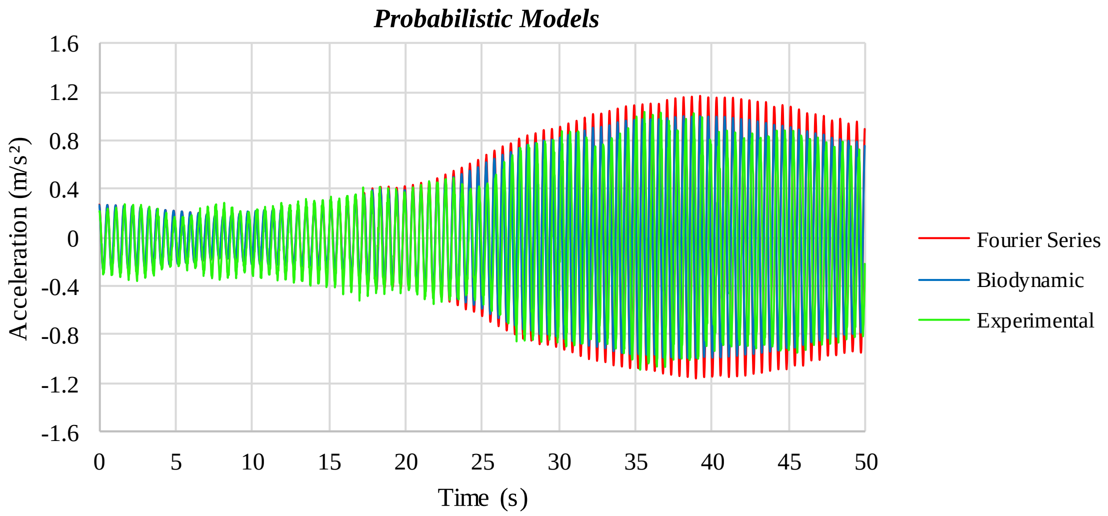


Figure 15. Probabilistic and experimental signals for 4 pedestrians walking in 1.85Hz

6 Conclusion

The simulations made for this paper allowed the evaluation of the effectiveness of the variability attribution in walking simulation parameters, representing satisfactorily the effect of pedestrian walking on the dynamic behavior of footbridges.

The probabilistic results had better correlation with the experimental signals than the deterministic analysis done by Rezende et al. [2], especially in moments of high resonance with the structure.

The biodynamic model obtained better correlations with the experimental results, especially for the case of four people walking at 1.85Hz. However, for the signal of one person walking at 1.85 Hz, the difference between the biodynamic and force models was negligible.

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