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NONLINEAR ANALYSIS OF VISCOELASTIC RECTANGULAR PLATES

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Abstract. *In this work, the non linear vibrations of thin, viscoelastic and isotropic clamped rectangular plates, subjected to concentrated harmonic load are studied. The Von Karman non linear strains relations are used and to describe the clamped boundary conditions, rotational springs are considered. The viscoelastic material is described as the Kelvin-Voigt model and the Raylrigh Ritz method is used to obtain a system of non linear dynamic equilibrium equations with 39 degrees of freedom which is solved, in turn, by the Runge-Kutta tnethod. A parametric detailed analysis is performed to study the influence of axial load, viscosity parameter and geometry on the non linear response of the plate. The frequency-amplitude relation and resonance curves were plotted. The frequency-amplitude curves showed that as the viscosity parameter is increased, the maximum amplitudes and the degree of nonlinearity are reduced, it was also observed that when both the dimentions of the plate and external load are increased, the frequency-amplitude relations are quasi linear, The resonance curves showed that the plate has a typical hardening behavior with two coexisting atractors with high sensitivity to initial conditions.*

Keywords: Viscoelastic plates, Kelvin-Voigt model, Nonlinear vibrations.

1 Introduction

Plates are plane structural elements with large applications in several engineering áreas and depending on its geometry and imposed external loads, it is necessary to study the non linear dynamic response of the plate. In literature, it is possible to find several works related to the nonlinear behavior of viscoelastic plates, a detailed bibliographic revision can be obtained in Amabili [1]. A lrge number of works related to the non linear vibrations of plates can be found in literature but, when non linear damping is considered, a reduced number of work is found.

Viscoelastic material have several applications in engineering such as metals in high temperatures, elastomeric, biological and composite materials. In general, viscoelastic materials have in its constitutive relation an elastic and a damping part and it is necessary to consider a mechanical model that considers correctly the strain-stress relation. The most common are the Maxwell, Kelvin-Voigt, Linear Solid (Zener) and Boltzmann models (Flügge [2], Amabili [3]). The stress-strain relations of a viscoelastic material are obtained by fluence and relaxation tests, which are weel described by the Zener and Boltzman models. The Kelvin-Voigt model has a limitation in the relaxation test but captures correctly the fluence phenomena and it is large applied in vibrations of viscoelastic materials. (Balasubramanian et al [4]).

Xia and Lukasiewicz [5, 6] studied the dynamic response of a laminated plate, where the external layers were considered as isotropic and elastic and the internal layer as viscoelastic described by the Kelvin-Voigt model. Rossihkin and Shitikova [7] studied the free non linear vibrations of a viscoelastic plate considering the Riem ann-Liouville fractional derivative. The modal interaction was analyzed when subjected to different internal resonances and, it was observed, that depending on the degree of the fractional derivative several vibrational regimes can co-exist. Bilasse, Azrar and Daya [8] developed a numerical method to study the non linear vibrations of sandwich viscoelastic plates. The computational procedure was based on the finite element method coupled with ah harmonic equilibrium of a complex mode of the Galerkin method. Balkan e Mecitoğlu [9] studied numerical and experimentally the dynamics behavior of sandwiche viscoelastic plates subjected to non uniform blasts. The external layers were considered as isotropic and elastic and the internal layer as viscoelastic described by the Kelvin-Voigt model. It was observed that, for and efficient vibration control with small amplitudes, it is more convenient to increase the thickness of the internal layer. Amabili [10] analyzed the non linear vibrations of viscoelastic rectangular plates considering the Kelvin-Voigt model and comparing with a viscous damping model. It was observed that the frequency response of the plate is different for both models considered. Balasubramanian et al [4] studied the non linear oscillations of viscoelastic plates and comparing some results with a finite element model using ABAQUS. It was observed that the Kelvin-Voigt model has limitations in describing large amplitude oscillations of the plate and, to fit numerical and experimental results it was necessary to change the viscoelastic parameter of the model.

In this work, the non linear vibrations of a clamped rectangular plate subjected to a concentrated harmonic load are studied. To describe the clamped boundary conditions, rotational springs are considered in all boundaries and the Kelvin-Voigt model is applied to descried the viscoelastic material. The Rayleigh-Ritz method is applied together with the Hamilton principle to obtain a set of 39 non linear equations of dynamic equilibrium. Both, the resonance curves and frequency amplitude relation are obtained to observe the non linear behvior of the plate when varying the viscoelastic parameter, the geometry of the plate and the external load value.

2 Mathematical formulation

In this work a perfect, flexivel, rectangular, clamped, isotropic, viscoelastic plate subjected to an harmonic concentrated load is studied. The plate has coordinates (*O; x; y; z*) and displacement fields *u*, *v* e *w* with length *a*, with *b* and thickness *h* as observed in Fig. 1. In the following section, all fomulation is based in the work of Balasubramanian et al [4].

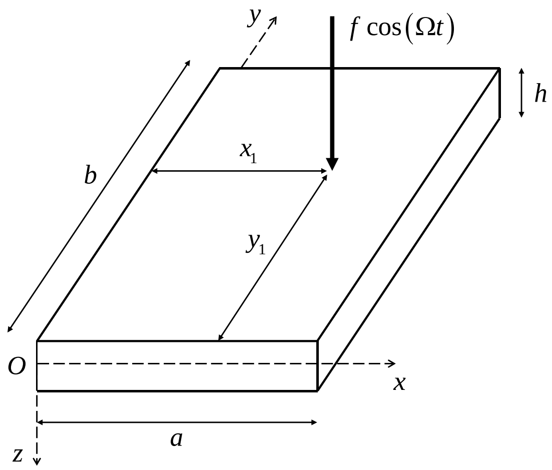

Figure 1. Rectangular viscoelastic plate

The von Kármán non linear strain – displacement relations were considered where, ε_x, ε_y e γ_{xy} are strain components in any point of the plate which are related to the strain of the middle surface given by ε_{x0}, ε_{y0} e $\gamma_{xy,0}$ and the changes of curvature of the middle surface k_x, k_y e k_{xy} given by Eq.1.

$$\varepsilon_x = \varepsilon_{x,0} + zk_x, \quad \varepsilon_y = \varepsilon_{y,0} + zk_y, \quad \gamma_{xy} = \gamma_{xy,0} + zk_{xy}$$

$$\varepsilon_{x,0} = \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2, \quad \varepsilon_{y,0} = \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2, \quad \gamma_{xy,0} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial x}\frac{\partial w}{\partial y}. \tag{1}$$

$$k_x = -\frac{\partial^2 w}{\partial x^2}, \quad k_y = -\frac{\partial^2 w}{\partial y^2}, \quad k_{xy} = -2\frac{\partial^2 w}{\partial x \partial y}$$

The viscoelastic material is described by the Kelvin-Voigt model, the stress-strain relation is described by:

$$\sigma = E(\varepsilon + \eta\dot{\varepsilon}). \tag{2}$$

Then, the constitutive equation of the plate is given by:

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} + \eta \begin{bmatrix} \frac{E}{1-\nu^2} & \frac{\nu E}{1-\nu^2} & 0 \\ \frac{\nu E}{1-\nu^2} & \frac{E}{1-\nu^2} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix} \begin{bmatrix} \dot{\varepsilon}_x \\ \dot{\varepsilon}_y \\ \dot{\gamma}_{xy} \end{bmatrix}. \tag{3}$$

where E is the Young modulus and η is the viscoelasticity parameter in seconds (s)

In this work, rotatory inertia and shear deformation are not considered, then the strain energy of the plate can be writen as:

$$U_P = \frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^a\int_0^b (\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \tau_{xy}\gamma_{xy})\,dxdydz. \tag{4}$$

Substituting Eq. (1) and Eq. (3) in Eq. (4) it is obtained the strain energy which contains elastic (U_E) and viscoelastic (U_V) terms which are given by:

$$U_E = \frac{Eh}{2(1-\nu^2)}\int_0^a\int_0^b\left(\varepsilon_{x,0}{}^2+\varepsilon_{y,0}{}^2+2\nu\varepsilon_{x,0}\varepsilon_{y,0}+\frac{1-\nu}{2}\gamma_{xy,0}{}^2\right)dxdy + \frac{Eh^3}{2(12(1-\nu^2))}\int_0^a\int_0^b\left(k_x{}^2+k_y{}^2+2\nu k_x k_y+\frac{1-\nu}{2}k_{xy}{}^2\right)dxdy \quad (5)$$

$$U_V = \eta\frac{Eh}{(1-\nu^2)}\int_0^a\int_0^b\left(\varepsilon_{x,0}\dot{\varepsilon}_{x,0}+\varepsilon_{y,0}\dot{\varepsilon}_{y,0}+\nu\varepsilon_{x,0}\dot{\varepsilon}_{y,0}+\nu\varepsilon_{y,0}\dot{\varepsilon}_{x,0}+\frac{1-\nu}{2}\gamma_{xy,0}\dot{\gamma}_{xy,0}\right)dxdy + \eta\frac{Eh^3}{12(1-\nu^2)}\int_0^a\int_0^b\left(k_x\dot{k}_x+k_y\dot{k}_y+\nu k_x\dot{k}_y+\nu k_y\dot{k}_x+\frac{1-\nu}{2}k_{xy}\dot{k}_{xy}\right)dxdy \quad (6)$$

finally:

$$U_P = U_E + U_V. \quad (7)$$

As observed in Eqs. (5) and (6) the viscoelastic terms (U_V) can be obtained by deriving in time the elastic terms and multiplied by the viscoelastic parameter (η), as described by:

$$U_V = \eta\left(\frac{dU_E}{dt}\right). \quad (8)$$

Rotational springs are applied in all boundaries to describe the clamped effect then, the strain energy due to the rotational springs is given by:

$$U_M = \frac{1}{2}\int_0^b k_r\left(\left(\left(\frac{\partial w}{\partial x}\right)_{x=0}\right)^2+\left(\left(\frac{\partial w}{\partial x}\right)_{x=a}\right)^2\right)dy + \frac{1}{2}\int_0^a k_r\left(\left(\left(\frac{\partial w}{\partial y}\right)_{x=0}\right)^2+\left(\left(\frac{\partial w}{\partial y}\right)_{x=b}\right)^2\right)dx. \quad (9)$$

where: k_r is the stiffness of the springs. The boundary conditions of the plate are:

$$\begin{aligned} &u=v=w=0, \quad M_x = \pm k_r\frac{\partial w}{\partial x}, \text{ at } x=0,\ a \\ &u=v=w=0, \quad M_y = \pm k_r\frac{\partial w}{\partial y}, \text{ at } y=0,\ b \end{aligned} \quad (10)$$

The viscoelastic plate was also considered as pre-tensioned by in plane loads in both directions as seen in Fig. 2, then strain energy due to in-plane load is given by:

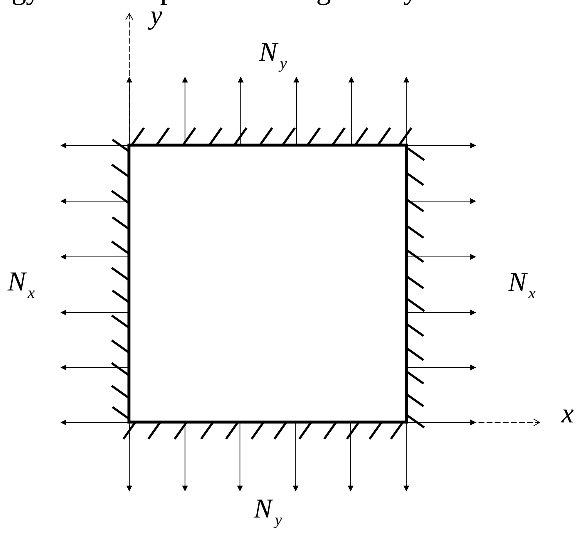

Figure. 2. Clamped viscoelastic plate with in-plane loads

$$U_C = \frac{1}{2}\int_0^a\int_0^b \left(N_x \left(\frac{\partial w}{\partial x}\right)^2 + N_y \left(\frac{\partial w}{\partial y}\right)^2 \right) dydx. \tag{11}$$

Then, the total strain energy of the plate is given by:

$$U = U_P + U_M + U_C. \tag{12}$$

The work done by tem concentrated external harmonic load, acting at $x = x_1$ and $y = y_1$, can be written as:

$$W = f\cos(\Omega t)\int_0^b\int_0^a w\delta(x - x_1)\delta(y - y_1)dxdy. \tag{13}$$

where f, Ω, t are, respectively, load intensity, frequency of excitation and time; w is the transversal field displacement and δ is the Dirac delta.

Finally, the kinetic energy of the plate is given by:

$$T = \frac{1}{2}\rho h\int_0^b\int_0^a \left(\dot{u}^2 + \dot{v}^2 + \dot{w}^2\right)dxdy. \tag{14}$$

The expansions for the axial and transversal field displacements, considering simple-supported boundary conditions are given in Eq. 9. These field displacements does not satisfy forced boundary conditions but, assuming rotational springs it is possible to consider clamped boundaries for the plate.

$$\begin{aligned} u(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} u_{m,n}(t)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \\ v(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} v_{m,n}(t)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \\ w(x,y,t) &= \sum_{m=1}^{M}\sum_{n=1}^{N} w_{m,n}(t)\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right) \end{aligned} . \tag{15}$$

where m and n are the halwave number in x and y directions, respectively; M and N are total number of terms used in each expansion and $u_{m,n}(t)$, $v_{m,n}(t)$ and $w_{m,n}(t)$ are the unknown amplitudes of each displacement. Then, the vector of generalized coordinates can be written as:

$$\mathbf{q} = \left[u_{m,n}(t),\ v_{m,n}(t),\ w_{m,n}(t)\right]^T. \tag{16}$$

the dimension of $\mathbf{q}$ is related to N_q, which representes de humber of degrees of freedom of the plate.

The Rayleigh-Ritz method is applied together with the Hamilton principle to obtain a set of nonlinear differential equatins of dynamics equilibrium which are in turn solved by the Runge-Kutta method. The modified Hamilton principle can be written as (Amabili [1]):

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}_j}\right) - \frac{\partial T}{\partial q_j} + \frac{\partial U}{\partial q_j} = \frac{\partial W}{\partial q_j}, \text{ for } 1 \le j \le N_\xi. \tag{17}$$

Finally, the nonlinear system of equations can be described as:

$$\mathbf{M}\ddot{\mathbf{q}} + (\mathbf{G} + \mathbf{G_2} + \mathbf{G_3})\dot{\mathbf{q}} + (\mathbf{K} + \mathbf{K_2} + \mathbf{K_3})\mathbf{q} = \mathbf{F}\cos(\Omega t). \tag{18}$$

where $\mathbf{M}$ is the diagonal mass matrix with dimensions $N_q \times N_q$; $\mathbf{G}$, $\mathbf{G}_2$ and $\mathbf{G}_3$ are, respectively, the viscoelastic matrix of linear, quadratic and cubic terms; $\mathbf{K}$, $\mathbf{K}_2$ and $\mathbf{K}_3$ are, respectively, the elastic matrix of linear, quadratic and cubic terms; $\mathbf{F}$ is the load vector with represents the projection of the concentrated harmonic load in the generalized coordinates and $\ddot{\mathbf{q}}$ $\dot{\mathbf{q}}$ $\mathbf{q}$ are the acceleration, velocity and displacement vectors, respectively.

3 Numerical results

Consider a viscoelastic plate with the following physical and geometrical properties h = 1,5 *mm*, $a = b$ = 26 *cm*, E = 5,62 *MPa*, ρ = 1430 *kg/m³* e ν = 0,5 (Balasubramanian et al [4]). The static traction in-plane loads are N_x = 100 *N/m* and N_y = 90 *N/m*. The concentated harmonic load is located at x_1 = 0,11 *m* and y_1 = 0,19 *m* four levels loads were considered as 0,01 *N*, 0,04 *N*, 0,07 *N*, 0,10 *N*, and also, four levels of the viscoelastic parameters (η): 0,0050 *s*, 0,0020 *s*, 0,0018 *s*, 0,0012 *s*. To garante the clamped boundary condition, the rotational springs have the following stiffness k_r = 1000 *N/rad*. It was considered a model with 39 degrees of freedom as given in Table 1.

Table 1. Generalized coordinates utilized in the expansions of the displacements

Displacement	Generalized coordinates
u	$u_{2,1}$, $u_{4,1}$, $u_{6,1}$, $u_{8,1}$, $u_{2,3}$, $u_{4,3}$, $u_{6,3}$, $u_{8,3}$, $u_{2,5}$, $u_{4,5}$, $u_{6,5}$, $u_{2,7}$, $u_{4,7}$
v	$v_{1,2}$, $v_{3,2}$, $v_{5,2}$, $v_{7,2}$, $v_{1,4}$, $v_{3,4}$, $v_{5,4}$, $v_{7,4}$, $v_{1,6}$, $v_{3,6}$, $v_{5,6}$, $v_{1,8}$, $v_{3,8}$
w	$w_{1,1}$, $w_{1,3}$, $w_{1,5}$, $w_{1,7}$, $w_{3,1}$, $w_{3,3}$, $w_{3,5}$, $w_{3,7}$, $w_{5,1}$, $w_{5,3}$, $w_{5,5}$, $w_{7,1}$, $w_{7,3}$

First, the natural frequency of the plate was obtained, as can be seen in Table 2, there agreement with reference Balasubramanian et al [4].

Table 2. Comparison of natural frequencies values

Natural Frequency,	Balasubramanian et al [4]	Present work	(%)
$\omega_{1,1}$ (Hz)	20,90	21,02	0,57

The frequency-amplitude curves are observed in Fig. 3, as can be seen, the plate displays hadening behavior for all four cases of viscoelasic η. It is also possible to observe that, the hardening effect is stronger for small values of the viscoelastic parameters and, if this value is increased, the hardening effect is reduced.

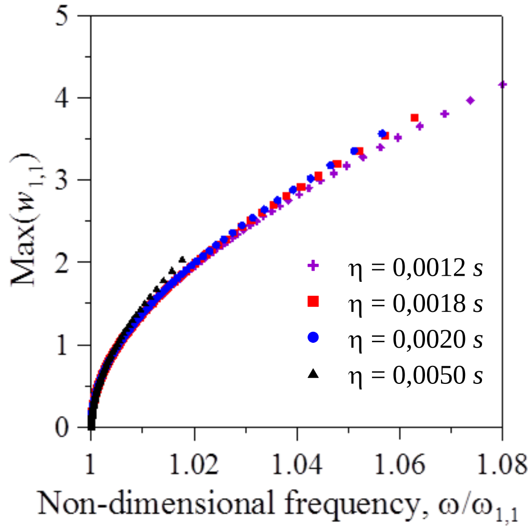

Figure 3. Frequency-amplitude curves for different values of η

After this initial analysis, now considering the external harmonic load, the resonance curves were obtained for increased values of the frequency of the external load and considering several values of the viscoelastic parameter. Figure 4 displays the resonance curves and were plotted using the continuation method (Del Prado [11]), in these figures, continuous lines represent stable vibrations and dotted lines represent unstable vibrations. Figure 4a,display the obtained values and Fig. 4b display the obtained values by Balasubramanian et al [4], both figures show very good agreement.

As can be observed, for small values of amplitude of the external load, the resonance curves display linear behavior with small amplitude oscillations and a mean peak at $\omega/\omega_{1,1}$ = 1,0. If the value of the amplitude of excitation is increased, the plate starts to display hardening behavior with large amplitude oscillations with jumps between small and large amplitude oscillations with stable and unstable non linear paths.

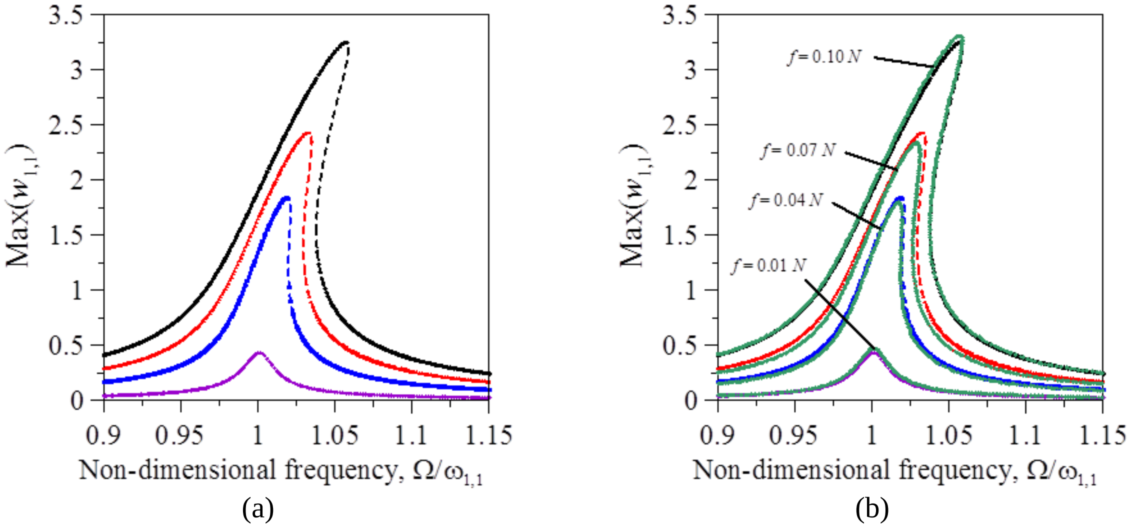

Figure 4. Resonance curves of viscoelastic plate (a) obtained in this study (♦ f = 0.01 N; ■ f = 0.04 N; ▲ f = 0.07 N; ● f = 0.10 N) and (b) comparison between curves obtained by Balasubramanian et al [4]: (✚)

3.1 Parametric Analysis

In this section, changes on the geometry and on the values of the in-plane loads were introduced. This was done looking to understand the influence of both the geometry and axial load on the non linear response of the plate.

Table 3 displays the values of the geometry adopted in analysis as well as the obtained natural frequencies of the plate.

Table 3. Plate geometry variations

Plate variation	Plate dimensions (*m*)		Natural frequency (*Hz*)	Load position at *y* coordinate (*y1*)
	a	*b*		
1.5*a*	0.26	0.39	17.99	0.285
2.0*a*	0.26	0.52	16.82	0.380

Now, Table 4 displays the values of the reduced in-plane loads as well as the natural frequencies for each case, for this, the vales of the plate geometry were adopted as shown in Table 3.

Table 4. In-plane loads variation

Plate variation	In-plane loads (*N/m*)		Natural frequency (*Hz*)
	N_x	N_y	
75%	75	67.5	18.29
50%	50	45	15.05

Now, the frequency-amplitude relations for each plate case were obtained and are displayed in Fig. 5. As can be observed, all curves show hardening behavior and due to the low degree of nonlinearity, the curve related to η = 0.0050 *s* was no plotted. In Fig. 5(a), Fig. 5(b) and Fig. 5(c), black dots represent the original plate geometry, red dots represent the plate for *b* = 1,5*a* and blue dots represent the plate for *b* = 2,0*a*. In Fig. 5(d), Fig. 5(e) and Fig. 5(f), black dots represent the original in-plane load of the plate, red dots represent the plate with 50% of the original load and blue dots represent the plate with 75% of the original in-plane load. As can be observed, depending on the adopted values of geometry of in-plane load, the nonlinearity degree is affected. The non-linearity is higher for a square plate as well as for a plate with low values of in-plane loads.

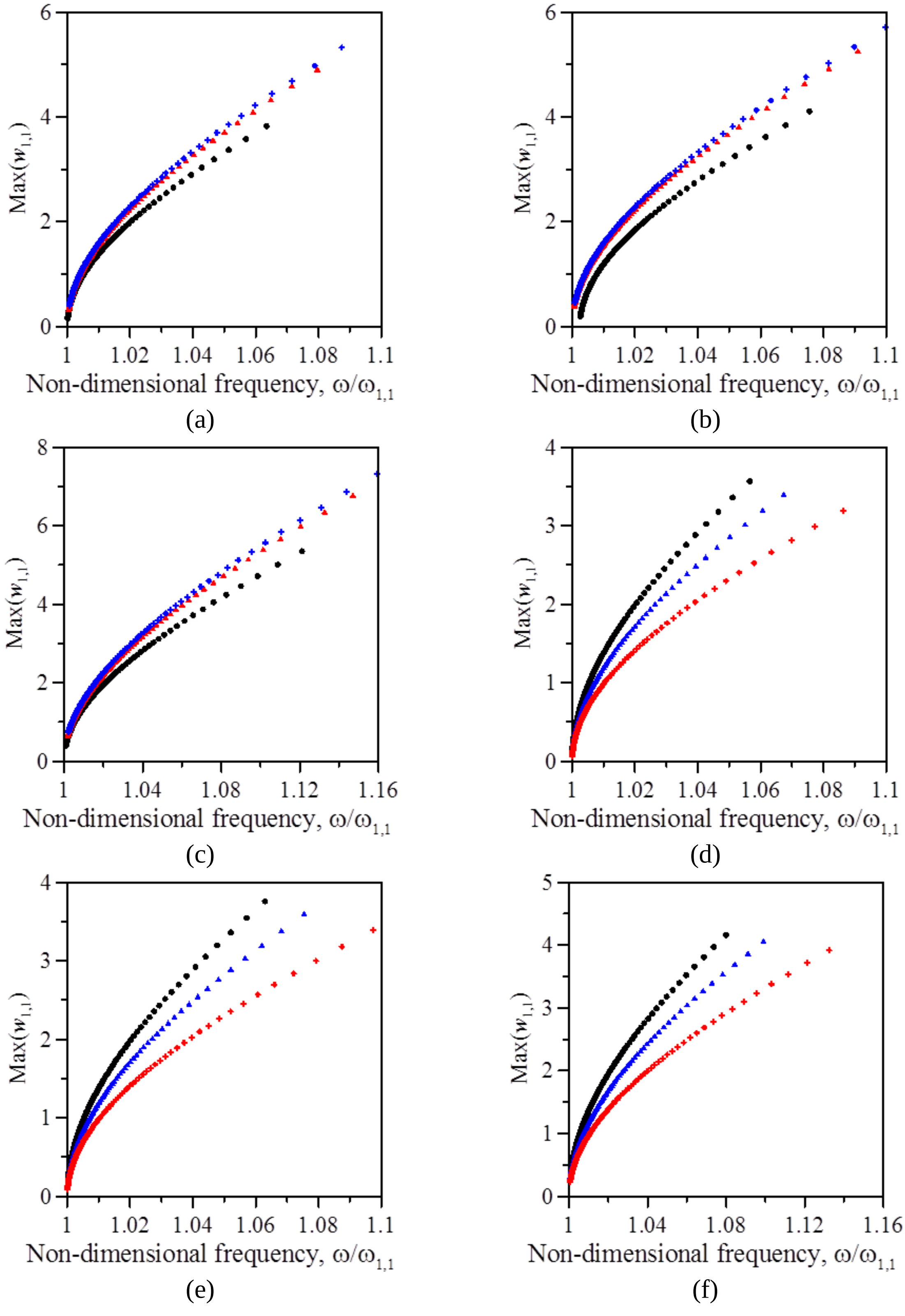

Figure 5. Amplitude-frequency curves of the viscoelastic plate for the (a) geometry variation: η = 0.0020 *s*; (b) geometry variation: η = 0.0018 *s*; (c) geometry variation: η = 0.0012 *s*; (d) in-plane loads variation: η = 0.0020 *s*; (e) in-plane loads variation: η = 0.0018 *s*; (f) in-plane loads variation: η = 0.0012 *s*;

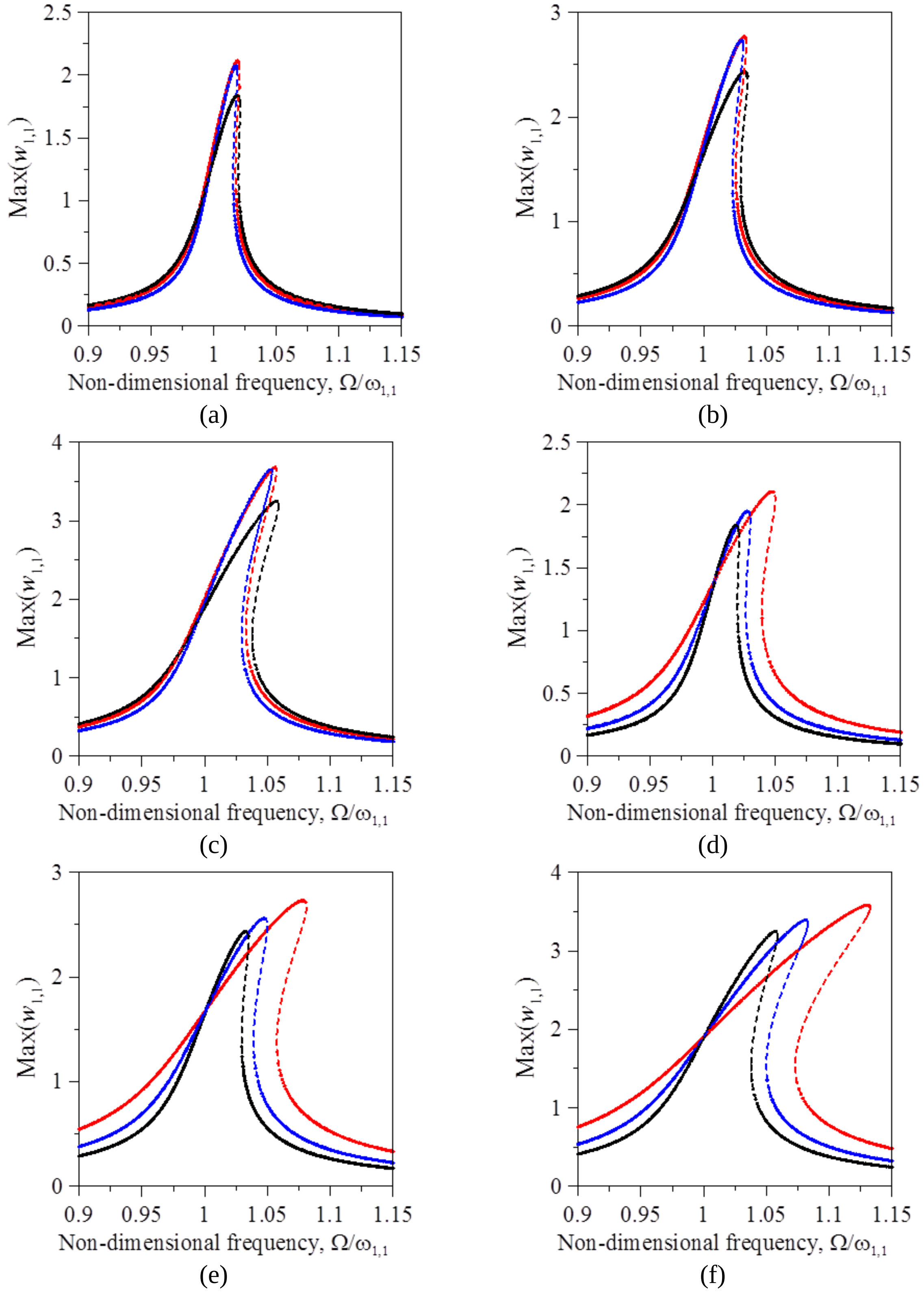

Figure 6. Resonance curves of the viscoelastic plate for the (a) geometry variation: *f* = 0.04 *N*; (b) geometry variation: *f* = 0.07 *N*; (c) geometry variation: *f* = 0.10 *N*; (d) in-plane loads variation: *f* = 0.04 *N*; (e) in-plane loads variation: *f* = 0.07 *s*; (f) in-plane loads variation: *f* = 0.10 *N*;

Now the concentrated external harmonic load will be considered. Figure 6 displays the obtained resonance curves and colors follow the same criteria adopted previously in Fig. 5. Due to the small degree of nonlinearity, the resonance curve for f = 0,01 N was no displayed. As can be observed, all curves show hardening behavior with jumps from large to small amplitude oscillations where stable and unstable solutions co-exist.

It is also possible to observe that, that higher nonlinearity is seen in the plate with lower values of the in-plane loads as well as with smaller geometric dimensions. Figure 6(b), Fig. 6(c) and Fig. 6(d) the phase portraits and Poincare mapping corresponding to gray lines of Fig. 6(a) between $\Omega/\omega_{1,1}$ = 1.05, $\Omega/\omega_{1,1}$ = 1.075, $\Omega/\omega_{1,1}$ = 1.1), are displayed. It is possible to verify the coexistence of attractors of large and small amplitude vibrations and all atractors have 1T periodicity. Also, in Fig. 6(a) all curves in intersect at $\Omega/\omega_{1,1}$ = 1.0, which means that, independing on the parameters, the plate will display the same amplitu oscillations for all cases.

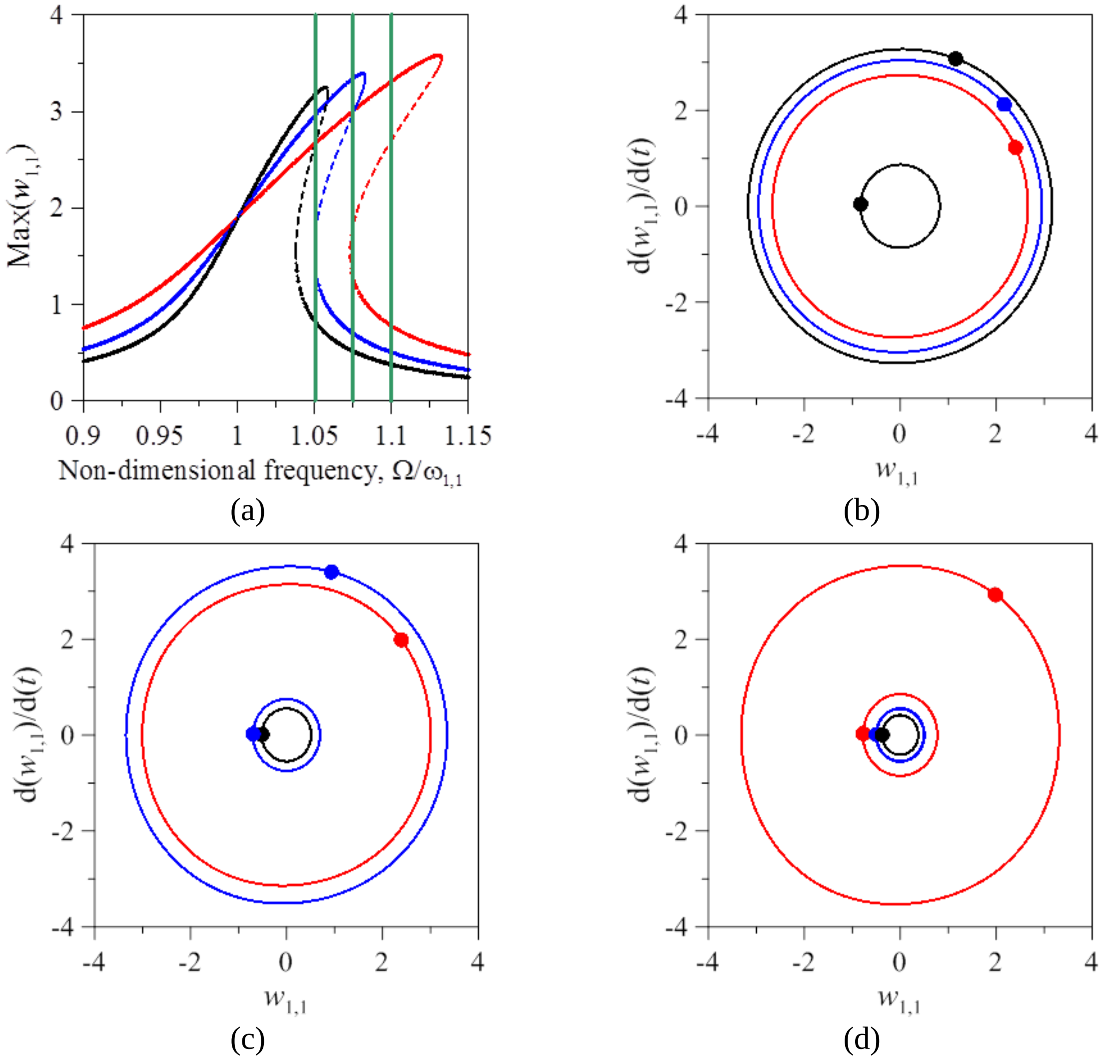

Figure 6. (a) Resonance curves for in-plane loads variation and *f* = 0.10 *N* with the three frequency points analyzed; (b) Poincaré maps for: $\Omega/\omega_{1,1}$ = 1.05; (c) Poincaré maps for: $\Omega/\omega_{1,1}$ = 1.075; (d) Poincaré maps for: $\Omega/\omega_{1,1}$ = 1.10.

4 Concluding remarks

In this work, the nonlinear vibrations of a tensioned clamped viscoelastic plate subjeted to concentrated harmonic external axial load was studied and the Kelvin-Voigt model was applied to ddes cribe the stress-strain relation. A model with 39 degrees of freedom was obtained using the Rayleigh-Ritz method and the Hamilton principle.

To study the degree of nonlinearity, the frequency-amplitude relations were obtained showing hardening behavior and for incremental values of the viscoelastic parameter (η) the plate shows linear behavior.

The resonance curves of the plate were obtained showing hardening behavior but, the degree of the nonlinearity, depends on the geometric and physical characteristics as well as on the value of the internal in-plane loads. Also, is was showed the phase portraits and Poincaré mapping of the plate in regions where there is coexistence of large and small amplitude vibrations. All responses displayed 1T stables oscillations.

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