XL CILAMCE
IBERO-LATIN AMERICAN CONGRESS ON COMPUTATIONAL METHODS IN ENGINEERING
NOVEMBER 11-14, 2019
Praiamar Natal Hotel & Convention
Natal, RN-BRAZIL

BOUNDARY ELEMENT TOPOLOGY OPTIMIZATION OF ANISOTROPIC MEDIA

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Abstract. The objective of this work is to present the implementation of a topological sensitivity based material removal procedure in a standard Boundary Element Method formulation. The approach used selects the areas showing the lowest sensitivities, where material is removed by opening a small cavity. As the iterative process evolves, the original domain has volume progressively removed, until a desired material volume is achieved. Because the Boundary Element Method does not employ domain meshes in linear cases, the resulting topologies are completely devoid of intermediary material densities. Additionally, with Bézier Curves, a method of smoothing lines was implemented during the topological optimization scheme, aiming to reduce some usual problems in the optimization process, such as the irregularity of the boundaries, eliminating the need for post-processing and reducing the computational cost of the process. Anisotropic elastic cases are presented and compared with their isotropic counterpart optimums.

Keywords: Topology Optimization, Boundary Element Method, Bézier Curves.

1 Introduction

Topology optimization is an engineering tool widely used in the design phase to define the topology of a component without compromising its ability to withstand the required service conditions. This process aims to improve the distribution of material within a given controlled location, considering a cost function and the boundary conditions applied (Bensøe and Sigmund [1]).

The Boundary Element Method (BEM) is an alternative to other numerical methods, such as the Finite Element Method (FEM), especially in cases where greater precision is required, to evaluate stresses and strains in the topology optimization routine. Among them, it can be highlighted the smaller mesh size, which reduces the computational processing time, as well as the greater capacity of the method to capture singularities and discontinuities, such as stress concentration problems or infinite domain (Brebbia and Dominguez [2]). The BEM transforms partial differential equations that govern the domain of the problem into integral equations involving boundary values, resulting in the reduction of the dimension of the problem in a unit. Thus, only the contour (or surface) needs to be discretized, facilitating the generation of meshes and favoring processes that require constant changes in the geometry under analysis, as in the optimization problems, as shown in Fig. 1.

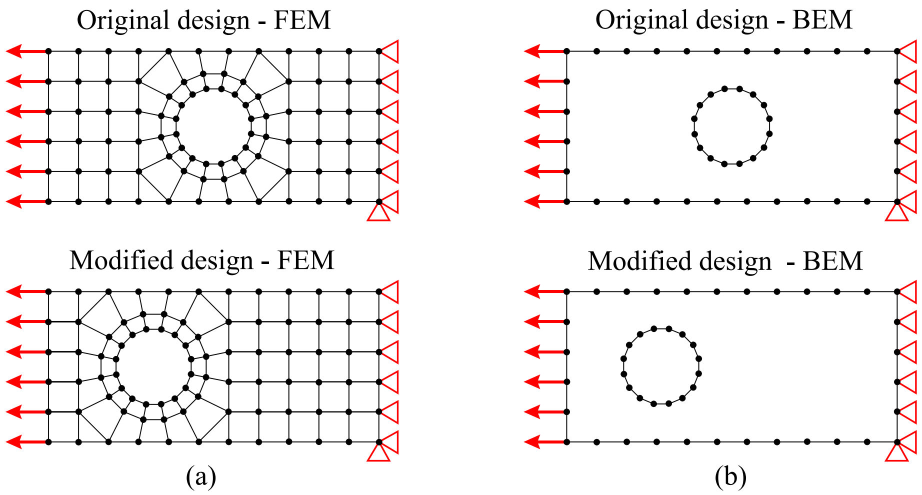

Figure 1. Examples of meshes of the (a) FEM and (b) BEM.

In this work, the anisotropic fundamental solution proposed by Cruze and Swedlow [3] for plane problems is used to calculate the displacements and surface tractions of the boundary in the boundary integral equation.

The implementation of a contour smoothing method during an optimization routine tries to reduce some common problems in the process, such as a contour irregularity, due to the way in which the material is removed (Anflor and Marczak [4]). In addition, it is expected to eliminate the need for post-processing, reducing the computational cost of the process.

2 Review of Topological Derivative on 2D elasticity theory

The topological derivative is a function used to evaluate the sensitivity of the topology of a domain Ω to be modified, through a cost function Ψ (Marczak [5]). It allows evaluating the sensitivity of the problem when a small hole is generated in a given position of the domain. Mathematically, it can be expressed as (Feijoo *et al.* [6]):

$$D_T{}^*(\hat{x}) = \lim_{\epsilon \to 0} \frac{\Psi(\Omega_\epsilon) - \Psi(\Omega)}{f(\epsilon)} \tag{1}$$

where $D_T{}^*(\hat{x})$ is the topological derivative value at $\hat{x}, \epsilon$ is the hole radius, $\Psi(\Omega_\varepsilon)$ and $\Psi(\Omega)$ are the values of the cost function in the original domain and in the modified domain Ω_ε and $f(\varepsilon)$ is a regularizing function of the analyzed problem.

According to Feijoo *et al.* [6] and Novotny *et al.* [7], since the domains Ω_ε and Ω are in different topological spaces, it is not possible to establish a one-to-one mapping between them, making difficult or even impossible to calculate the topological derivative. To solve this problem, the authors introduce the idea that the creation of a hole can be realized causing a small perturbation $\partial\epsilon$ in a hole whose radius tends to zero, already existing in the domain Ω_ε. In this way, a homeomorphism can be established between the domains and the topological derivative can be redefined as:

$$D_T(\hat{x}) = \lim_{\substack{\epsilon\to 0 \\ \partial\epsilon\to 0}} \frac{\Psi(\Omega_{\epsilon+\partial\epsilon}) - \Psi(\Omega_\epsilon)}{f(\epsilon+\partial\epsilon) - f(\epsilon)}. \tag{2}$$

Since it only provides the sensitivity of the problem to the increase of the hole and not its creation, even considering that the expansion of a hole of radius ϵ when $\epsilon \to 0$ is equivalent to creating it, the equivalence of the two equations can be mathematically proven, as demonstrated by Novotny *et al.*[7] and Feijoo *et al.* [6].

The use of Total Potential Energy as a cost function is common in topological optimization, since the minimization of the internal energy, besides simplifying the calculations, results in components of high stiffness, that is, of high strength. For an anisotropic material, Giusti *et al.* [8] has shown the capability of the topological derivative as a sensitivity tool for optimization problems and the expression for the topological derivative can be easily calculated for any point of the structure, and it is given by:

$$D_T(\hat{x}) = \mathbb{P}\sigma(\hat{x}) \cdot \nabla^s u(\hat{x}) \tag{3}$$

where $\mathbb{P}$ is a polarization tensor given as a function of the material properties $\sigma(\hat{x})$ are the stresses and $\nabla^s u(\hat{x})$ are the strains at point $(\hat{x})$ (Oliver *et al.* [9]). The values of the topological derivative can be easily found in optimization problems by the Boundary Element Method, since the determination of the stress and strains of the domain is a post-processing step of the method (Marczak [10]).

3 Bézier Curves

The Bézier Curve is a polynomial curve widely used in several computer programs that work with animation and manipulation of images. The first published work dates back to 1962 by the French engineer Pierre Étienne Bézier, who was responsible for several contributions to Computer-Aided Design and Computer-Aided Manufacturing technologies (Farouki [11]). The interpolation function $P(t)$ of the Bézier Curve is defined by *n + 1* control points p_i by the Bernstein polynomials $B_{n,i}(t)$ as:

$$P(t) = \sum_{i=0}^{n} p_i B_{i,n}(t) = \sum_{i=0}^{n} p_i \frac{n!}{i!\,(n-i)!} t^i (1-t)^{n-i}. \tag{4}$$

The interpolation function can be expressed in its parametric form splitting the values of $x(t)$ and $y(t)$ as:

$$x(t) = \sum_{i=0}^{n} x_i B_{i,n}(t). \tag{5}$$

$$y(t) = \sum_{i=0}^{n} y_i B_{i,n}(t). \tag{6}$$

The parameter t varies from 0 to 1, since the basis functions of the Bernstein Polynomials form a partition of unity. Thus, Bernstein polynomials provide a percentage value of how each one of the control points contributes to the Bézier Curve shape (Farouki [11]).

4 Methodology

The smoothing routine is designed to ensure that the correct contour parts are modified either for the use of the Bezier Curves or any other smoothing algorithm. Because it is not a post-processing step, the routine must be able to identify the segments that have been modified from those that are the original contour, as well as the portions that have boundary conditions. The optimization scheme follows the idea proposed by Marczak [5] and is divided into six steps, shown in Fig. 2, and listed below:

1. The BEM problem is solved, and the DT is evaluated on the domain;
2. The points with the lowest values of DT are selected;
3. Holes are created by punching out disks of material centered on the selected points;
4. The contour is divided and the contours to be smoothed are selected;
5. Smoothing routine is applied;
6. Check stopping criteria, rebuild the mesh, and return to step 1, if necessary.

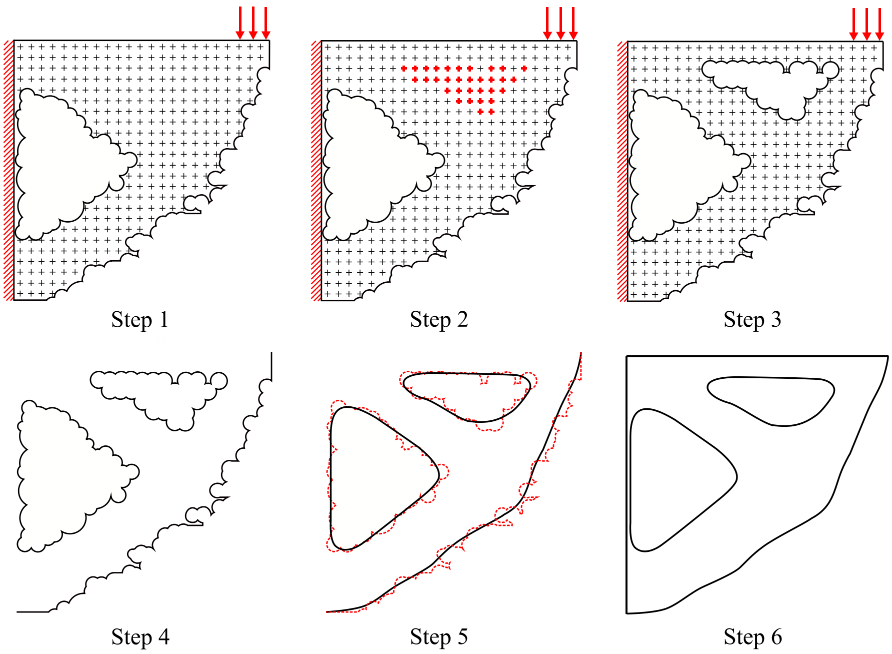

Figure 2. Topology optimization scheme.

5 Results

The results obtained with the implementation of the algorithm are demonstrated in this section.

5.1 *Benchmark 1* – Orthotropic Rectangular Cantilever

In this rectangular domain orthotropic problem, a load is applied at the center of the right edge, while the left edge is clamped. The radius of the holes used to remove material is 3% of the smallest edge of the domain and 40 holes are created at each iteration. The internal grid has 4851 points and Bézier smoothing is applied every 5 iterations. The process stopped when $A_f = 0.45A_0$. The evolution history of the optimization process with contour smoothing is demonstrated in Fig. 3. The constitutive tensor of the material is given by:

$$\mathbb{C} = \begin{bmatrix} 0.3400 & 0.1689 & 0 \\ 0.1689 & 0.3400 & 0 \\ 0 & 0 & 0.1401 \end{bmatrix}. \quad (7)$$

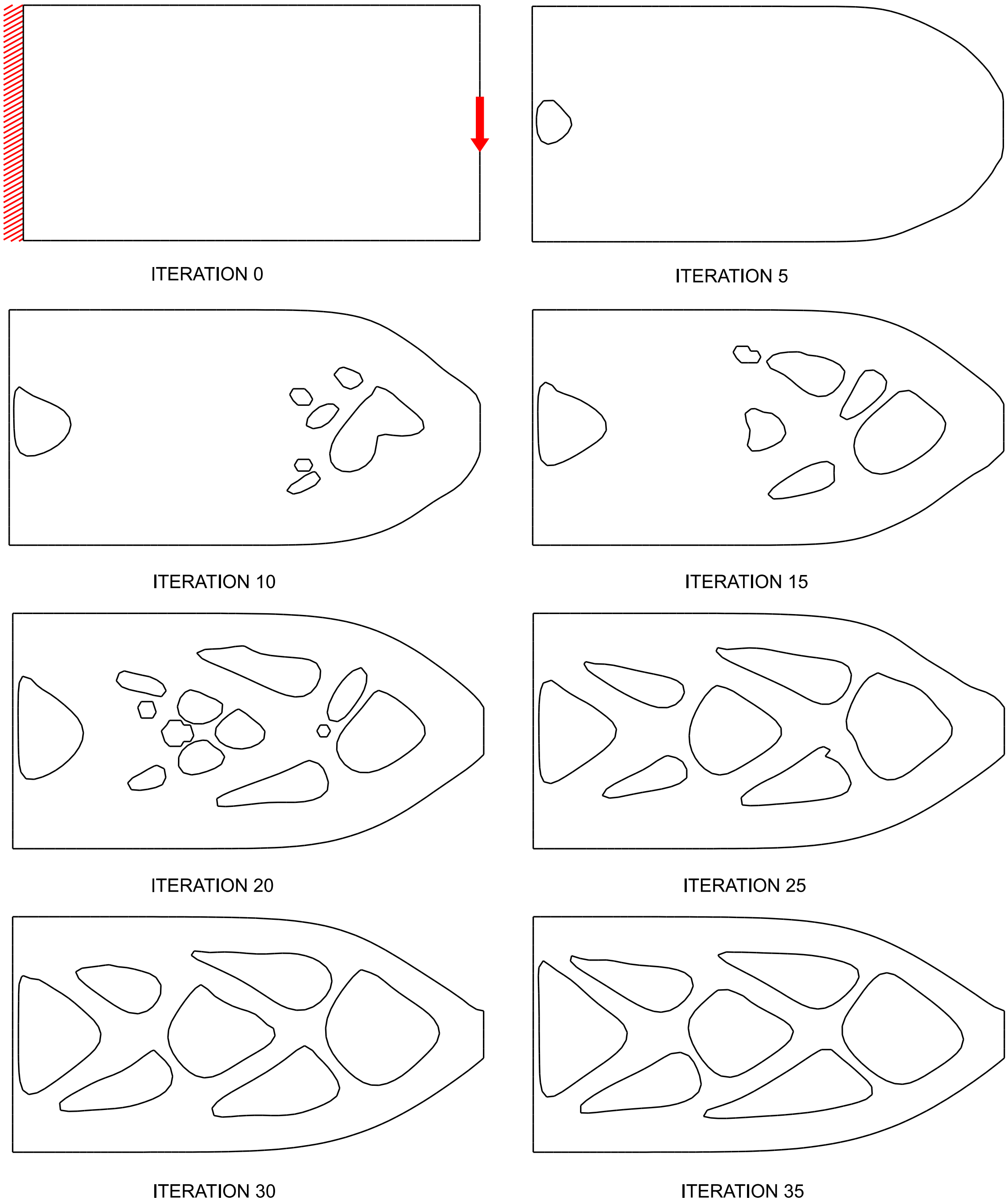

Figure 3. Optimization history of *benchmark 1* with the contour smoothing process.

The results obtained using the smoothing technique presented a more refined pattern of reinforcements in the geometry, compatible with the results obtained by Giusti *et al.*[8] using different optimization methods. The results can be improved changing material removal rate and the point density used in the internal grid during the optimization, as demonstrated by Marczak [5]. However, decreasing the rate of material removal and increasing the internal points density increase the computational cost of the process.

It is important to highlight the difficulty of maintaining symmetry during the topological optimization process in originally symmetrical problems. This difficulty is related to the numerical validation of the sensitivity of the cost function, which is subjected to rounding and truncation problems, as well as the material removal strategy, which does not occur symmetrically (Marczak [10]).

5.2 *Benchmark 2* – Anisotropic Rectangular Cantilever

In this anisotropic problem, the loading conditions are the same as in *Benchmark 1*. The radius of the holes is 3% of the smallest edge. In the beginning of the process 30 holes are created at each iteration and smoothing is applied every 5 iterations. After iteration 25, the material removal rate is decreased to 10 holes and Bézier smoothing is applied every 10 iterations. The internal grid has 4851 points and the process stopped when $A_f = 0.5A_0$. The evolution history of the optimization process is shown in Fig. 4. The constitutive tensor is:

$$\mathbb{C} = \begin{bmatrix} 0.2782 & 0.1250 & 0.0728 \\ 0.1250 & 0.2184 & 0.0120 \\ 0.0728 & 0.0120 & 0.1401 \end{bmatrix}. \tag{8}$$

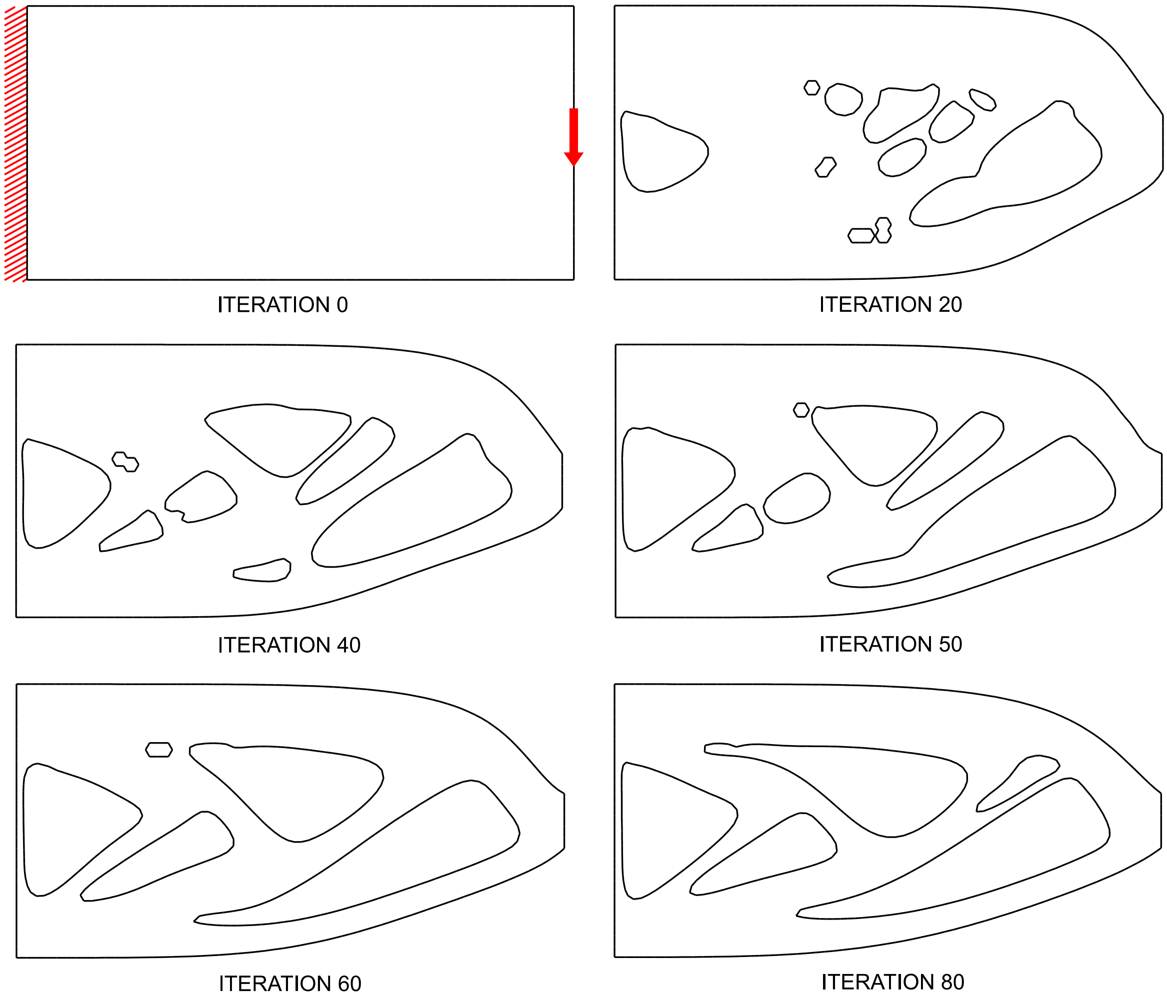

Figure 4. Optimization history of *Benchmark 2*.

5.3 *Benchmark 3* – Orthotropic Rectangular Cantilever

In this rectangular domain orthotropic problem, a load is applied at the right of the top edge, while the left edge is clamped. The radius of the holes used to remove material is 3% of the smallest edge of the domain and the internal grid has 4851 points. In the beginning of the process 30 holes are created at each iteration and Bézier smoothing is applied every 5 iterations. After iteration 30, the material removal rate is decreased to 10 holes each iteration and Bézier smoothing is applied every 7 iterations. The evolution history of the optimization process with contour smoothing is shown in Fig. 5. The process stopped when $A_f = 0.5A_0$. The constitutive tensor of the material is given by:

$$\mathbb{C} = \begin{bmatrix} 21.1302 & 3.3145 & 0 \\ 3.3145 & 9.7199 & 0 \\ 0 & 0 & 5.000 \end{bmatrix}. \tag{9}$$

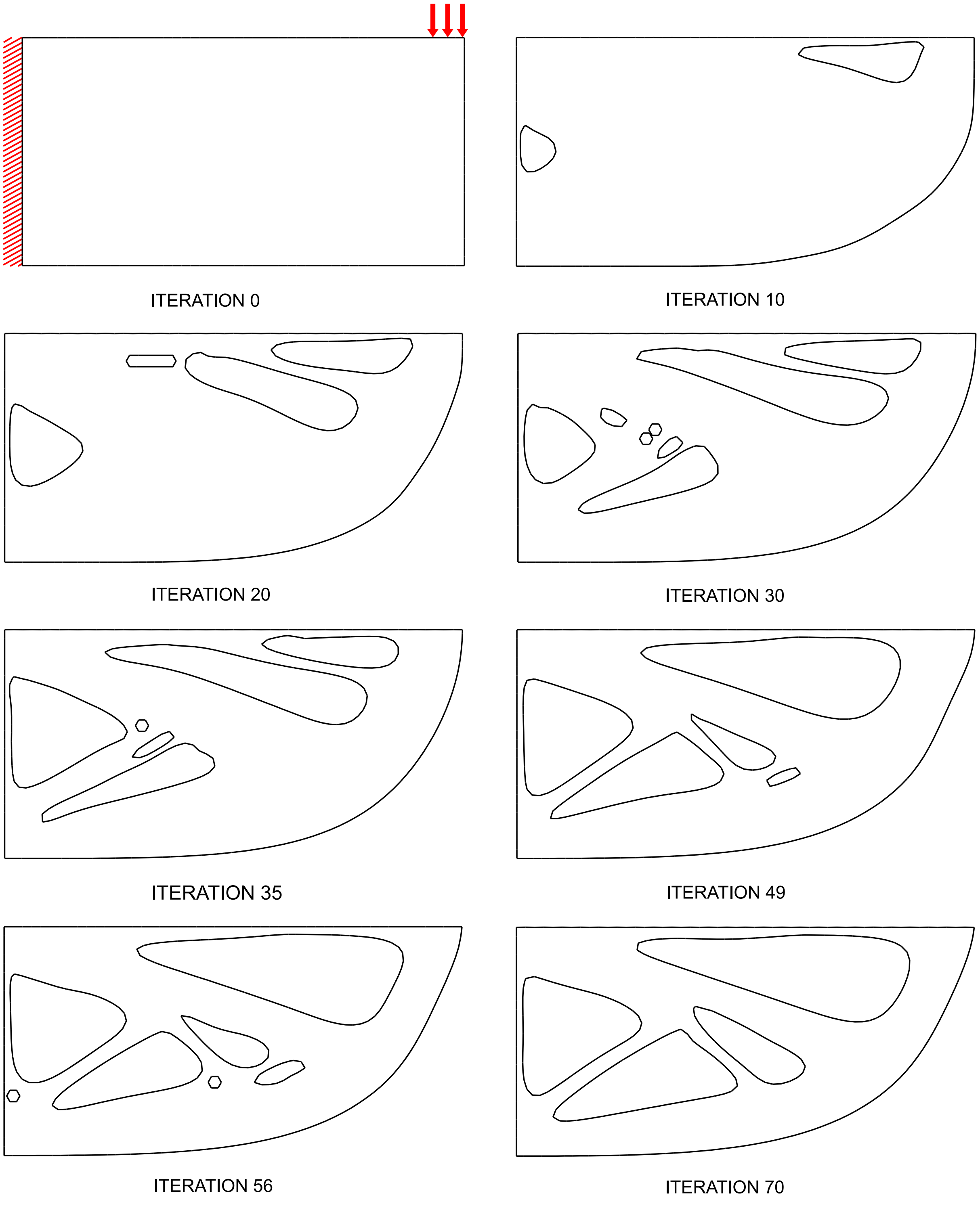

Figure 5. Optimization history of *Benchmark 3*.

5.4 *Benchmark 4* – Orthotropic Rectangular Cantilever

In this problem, the loading conditions are the same as in *Benchmark 3*. The radius of the holes used to remove material is 3% of the smallest edge of the domain and the internal grid has 4851 points. In the beginning of the process 30 holes are created at each iteration and after 10 iterations, the material removal rate is decreased to 20 holes each iteration. Bézier smoothing is applied every 5 iterations and the process stopped when $A_f = 0.5A_0$. The evolution history of the optimization process with contour smoothing is demonstrated in Fig. 6. The constitutive tensor of the material is the same as in Benchmark 3, but the material axes are rotated 30 degrees counter clockwise with the horizontal axis.

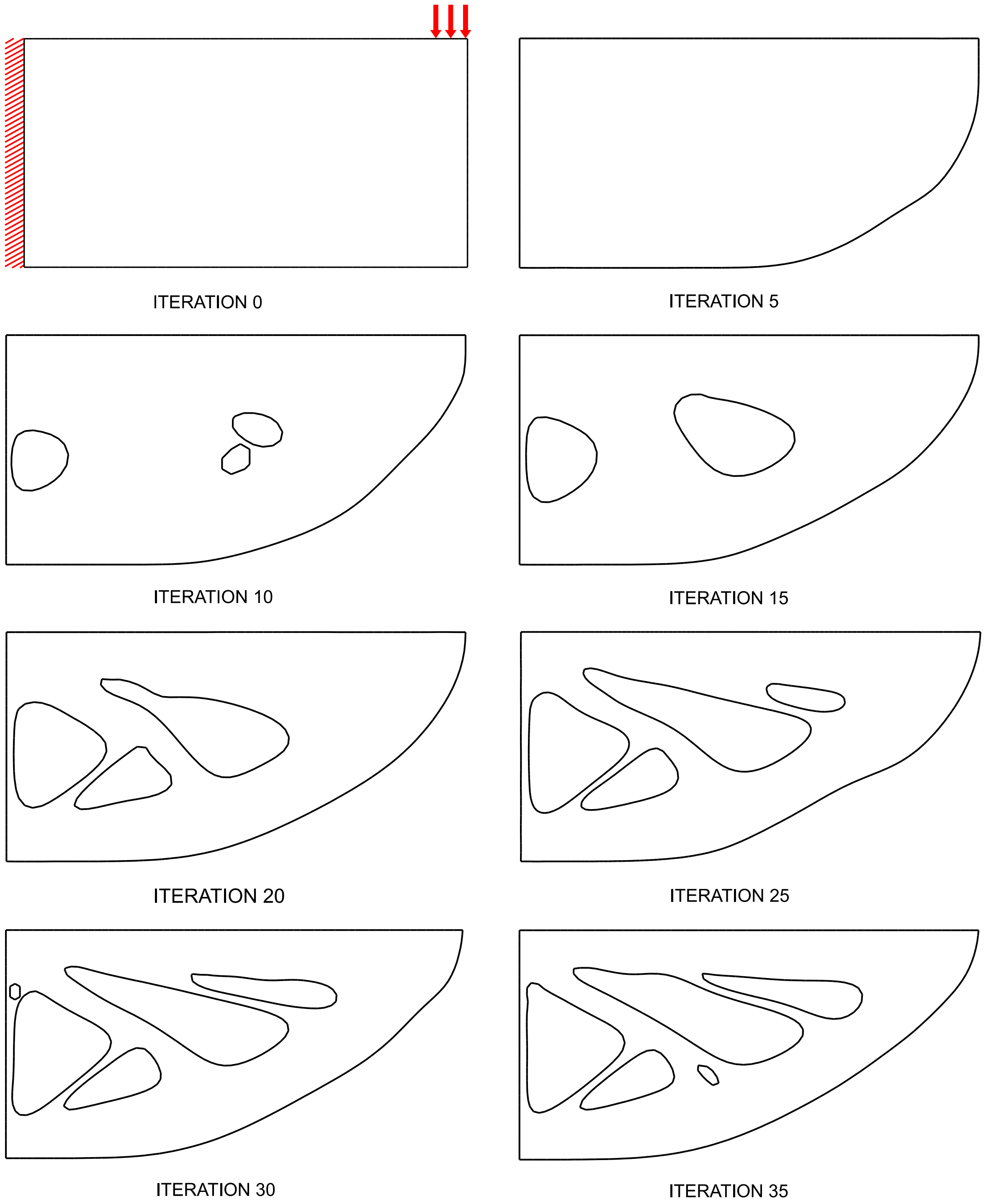

Figure 6. Optimization history of *Benchmark 4.*

A phenomenon observed in both cases, which can negatively influence the results of the stress analysis of the pieces in the post-processing of the topological optimization process, are the sharp corners in the internal contours of the optimized structures. In both examples, this phenomenon occurs due to the organization of the contour points before the application of the smoothing function and the size of the elements used. For closed contours, such as the inner contours of the pieces, it is necessary to choose appropriately which points will be the first and last points of the curve to be smoothed. This choice must be made to ensure that these points are not close to corners, thereby ensuring that the line connecting the first two points has little angular variation with respect to the line connecting the last two points, thus eliminating the possibility of appearance of sharp corners.

An instantaneous increase in the area is observed when the smoothing function is applied. The topological optimization strategy of material removal by drilling holes does not allow material to be added after the hole is created and may result incorrect geometry changes, since the validation of the sensitivities of the topological derivative is subject to rounding errors (Marczak [10]). Thus, it can be said that the phenomenon of area increase, resulting of the Bézier Curve application, influences the result and is beneficial to the optimization process, providing better results than in the non-smoothing process with the same settings.

6 Conclusions

The present work demonstrate the results obtained in a topology optimization scheme for anisotropic plane structures using the topological derivative, the boundary element method and a smoothing routine.

The results demonstrate a great potential of the anisotropic topological derivative and smoothing algorithms within the topological optimization process, since they provide visible improvements in the results of the topologies, besides the reduction of the computational cost of the whole process. The smoothing algorithm made it easier to obtain optimized topologies with the BEM much closer to the classical results obtained through other methods, such as the Finite Element Method. In addition, the BEM is established as a method for topological optimization without the disadvantages associated with the existence of intermediate densities.

The use of Bézier Curve as a smoothing method presented good results and good stability during the execution of the program. Some aspects of the algorithm, such as the correct organization of the points to be smoothed in closed contours, can be modified and tested in future works, avoiding the formation of sharp corners and, consequently, the need for post-processing after the optimization.

The presented results proved that the formulation generates optimal topologies and is an interesting field of investigation for integral equation methods.

Acknowledgements

F. L. Lorenzi acknowledges CAPES/Brazil and R. J. Marczak acknowledges CNPq through project 310649/2017-0 for the financial support during the development of this research.

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