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Simultaneous Optimization of Fiber and Material Distribution for Composite Materials Based on Topology Optimization

André Luis Ferreira da Silva, Ruben Andres Salas, Emílio Carlos Nelli Silva
andre_fersi@usp.br
Department of Mechatronics System Engineering, University of Sao Paulo Avenida Professor Mello de Morais, 2231, 05508-030 São Paulo, SP, Brazil

Abstract. New additive manufacturing technologies are been developed to obtain particular composite structures. In the case of fiber-reinforced composite structures, fiber angle influences material properties. This work presents a method based on topology optimization, which combines two models to design structures made of fiber-reinforced composite, considering simultaneously the distribution of material and the fiber orientation.

Keywords: Topology Optimization, Fiber Orientation, Composite structures, SIMP

1 Introduction

Fiber-reinforced materials have been used in several engineering applications due to the advantages provided by their properties. These properties can be modified according to the form which the fibers are oriented in the composite. By using new additive manufacturing techniques, it is possible to customize the orientation of fiber according to the desired application. However, it is important to ensure fiber continuity to avoid stress concentrations. Moreover, it is relevant to determine the distribution of the material to ensure there is no material in unnecessary regions, which in some cases represent great energy saving and prevent large amounts of pollutants from being released into the atmosphere. There are several methods for optimizing the fiber orientation, however, in most cases, there are issues when the fiber angle is considered directly as a design variable, as multiple local minima. To circumvent these issues, some works propose methods where candidate angles are chosen *a priori*, which limit the solution space. Thus, in this work is used a method named SPIMFO (*Self-Penalizable Interpolation Model for Fiber Orientation*) which define the sine and cosine functions as convergent Taylor series, avoiding the problems as multiple local minima, no limiting the solution space. The SPIMFO method is applied simultaneously with SIMP (*Solid Isotropic Microstructure with Penalization*) model to determine optimized topologies considering at the same time the material distribution and the optimized fiber angles. A filter-based in a modified Helmholtz equation is used to ensure fiber continuity. The performance of the method is illustrated by numerical examples. The modified Helmholtz filter proves to be efficient to both design variables, ensure the fiber continuity and prevent the checkerboard and the mesh dependence problems.

2 Theoretical Formulation

The constitutive equation for an orthotropic material in a linear range, when plane stress is considering, can be represented by Eq. (1) [1]

$$\boldsymbol{\sigma} = \mathbf{Q} \ \boldsymbol{\varepsilon} \tag{1}$$

where $\boldsymbol{\sigma}$ is the stress tensor and ε is the strain tensor, both in Voigth notation, and $\mathbf{Q}$ is the constitutive matrix for a ortotropic material. Matrix $\mathbf{Q}$ can be define in terms of compliance matrix $\mathbf{S}$ according to Eqs. (2) and (3) [1]

$$\mathbf{S} = \begin{bmatrix} \frac{1}{E_1} & -\frac{\nu_{12}}{E_1} & 0 \\ -\frac{\nu_{21}}{E_2} & \frac{1}{E_2} & 0 \\ 0 & 0 & \frac{1}{G_{12}} \end{bmatrix} \tag{2}$$

$$\mathbf{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{3a}$$

$$Q_{11} = \frac{S_{22}}{S_{11}S_{22} - S_{12}^2} \tag{3b}$$

$$Q_{12} = -\frac{S_{12}}{S_{11}S_{22} - S_{12}^2} \tag{3c}$$

$$Q_{22} = -\frac{S_{11}}{S_{11}S_{22} - S_{12}^2} \tag{3d}$$

$$Q_{66} = \frac{1}{S_{66}} \tag{3e}$$

where E_i are Young's modulus, ν_{ij} are the Poisson ratio and G_{12} is the shear modulus in plane 1-2 [1].

It is necessary to consider the fiber angle in the constitutive equation, and this can be achieved by using a change of coordinates as shown in Eq. (4)

$$\begin{bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{bmatrix} = \mathbf{T}^{-1}\,\mathbf{Q}\,\mathbf{R}\,\mathbf{T}\,\mathbf{R}^{-1} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} = \bar{\mathbf{Q}} \begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{bmatrix} \tag{4}$$

where $\bar{\mathbf{Q}}$ is the transformed constitutive matrix, $\mathbf{T}$ is the transformation matrix and $\mathbf{R}$ is the Reuter matrix (see Eqs. (5) and (6))[1]

$$\mathbf{T} = \begin{bmatrix} c^2 & s^2 & 2\,s\,c \\ s^2 & c^2 & -2\,s\,c \\ -sc & sc & c^2 - s^2 \end{bmatrix} \qquad c = cos(\theta), \quad s = sin(\theta) \tag{5}$$

$$\mathbf{R} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix} \tag{6}$$

3 Topology Optimization Formulation

The models SIMP [2] and SPIMFO [3] are used to determine the optimized distribution of material and fiber angle respectively. In the SIMP model, the material distribution is defined through a pseudo-density design variable named $\bar{\rho}$, which is penalized by a penalty parameter p. The purpose of the penalty is to drive the values of $\bar{\rho}$ to 1 or 0, which represent the domain regions where there are material and void respectively. Eq. (7) presents the formulation of SIMP model [4].

$$\bar{\mathbf{Q}}(\mathbf{x}) = \bar{\rho}(\mathbf{x})^p\ \bar{\mathbf{Q}}^0(\mathbf{x}), \quad p > 1 \tag{7a}$$

$$\int_{\mathbf{V}} \bar{\rho}\ d\mathbf{V} \leq V; \quad 0 < \bar{\rho}_{min} \leq 0 \leq \bar{\rho} \leq 1, \quad \mathbf{x} \in \mathbf{V} \tag{7b}$$

where $\bar{\mathbf{Q}}^0(\mathbf{x})$ represents the constitutive matrix of a particular material, V is the volume constraint and *bfV* represents the domain.

The optimization of fiber orientation by using topology optimization usually presents local minima problem when fiber angle is considered directly as a design variable [5]. There are methods which circumvent this issue establishing candidate angles *a priori* (see works [5–8]). However, the model SPIMFO, proposed by Salas et al. [3], can avoid the local minima problem without using candidate angles. The main feature of this model is the approximation of the sine and cosine functions by Taylor series, as shown in Eq. (8)

$$s = sin(\theta) = \sum_{p_f=0}^{p_f^{max}} \frac{(-1)^{p_f}}{(2\,p_f+1)!} \left(\pi\hat{\theta}\right)^{2p_f+1} \tag{8a}$$

$$c = cos(\theta) = \sum_{p_f=0}^{p_f^{max}} \frac{(-1)^{p_f}}{(2\,p_f)!} \left(\pi\hat{\theta}\right)^{2p_f} \tag{8b}$$

where p_f is the penalty parameter and $\hat{\theta}$ is a pseudo-orientation variable which varies from -1 to 1. Few terms are used in the series defined by Eq. (8) in the first iterations, and as the optimization process progresses, more terms are added. This approach permits circumvent the local minima problem.

The well-known problem of compliance minimization, shown in Eq. (9), is solved in this work.

$$\begin{aligned} min \quad & \int_V \mathbf{b} \cdot \mathbf{u} \, dV + \int_{\Gamma_N} \mathbf{t} \cdot \mathbf{u} \, dS \\ s.t. \quad & \int_V \boldsymbol{\sigma} : \nabla \mathbf{v} \, dV = \int_V \mathbf{b} \cdot \mathbf{v} \, dV \int_{\Gamma_N} \mathbf{t} \cdot \mathbf{v} \, dS \\ & \int_V \bar{\rho}(\mathbf{x}) \, dV - \mathcal{V} \leq 0 \\ & \hat{\theta}_{min} \leq \hat{\theta} \leq \hat{\theta}_{max} \end{aligned} \tag{9}$$

where $\mathbf{b}$ is the body forces, $\mathbf{t}$ is the traction forces, $\mathbf{u}$ are the displacements, $\mathbf{v}$ are test functions and $\mathcal{V}$ is the maximum volume fraction.

4 Numerical Examples

Fig. 1 shows the domain, material properties, element and parameters used in numerical examples. The cantilever represented by Fig. 1 is made of glass/epoxy and is clamped on the left side. A force $F = 0.1 \ N/mm$ is applied on the right side. The initial guesses for the pseudo-orientation $\hat{\theta}$ and for pseudo-density $\hat{\rho}$ are $0°$ and 0.6 respectively. A filter based on a modified Helmholtz equation, proposed by Lazarov and Sigmund [9] is used to deal with the checkerboard and mesh dependence problems in SIMP and to ensure the fiber continuity in SPIMFO. Two different continuation schemes are applied in SIMP penalization parameter. For the first result (see 2(a)), p is equal 1 at the beginning of optimization and its value increases by one every 15 iterations until it reaches 20. In the second result, the penalization parameter starts at 10 and its value increases every 30 iterations until it reaches 20. The results for the different continuation schemes for SIMP model are shown in Fig. 2. Fig. **??** shows the convergence of the objective function over iterations.

Material Properties
$E_1 = 181 \ GPa$
$E_2 = 10{,}3 \ GPa$
$G_{12} = 7{,}71 \ GPa$
$\nu = 0{,}28$

Filter	
$r_\theta \, [mm]$	$r_{\bar{\rho}} \, [mm]$
7,5	5

SPIMFO Parameters			
$\hat{\theta}_c^{min}$	$\hat{\theta}_c^{max}$	p_f^{ini}	p_f^{max}
- 89/180	90/180	2	17

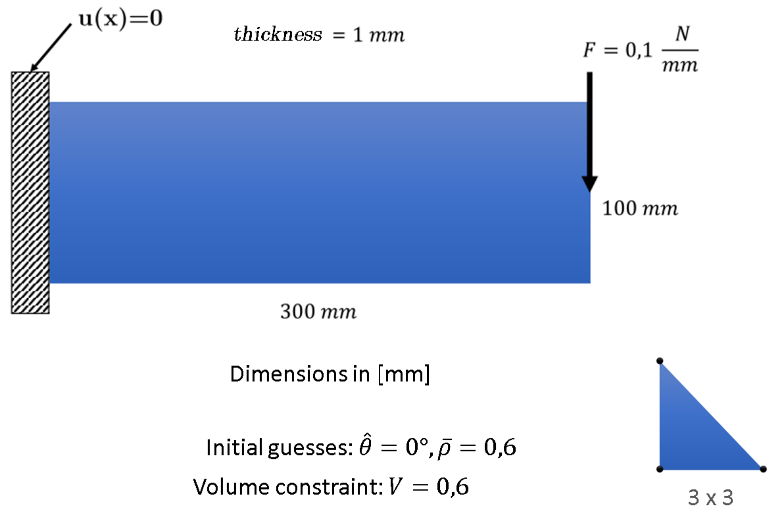
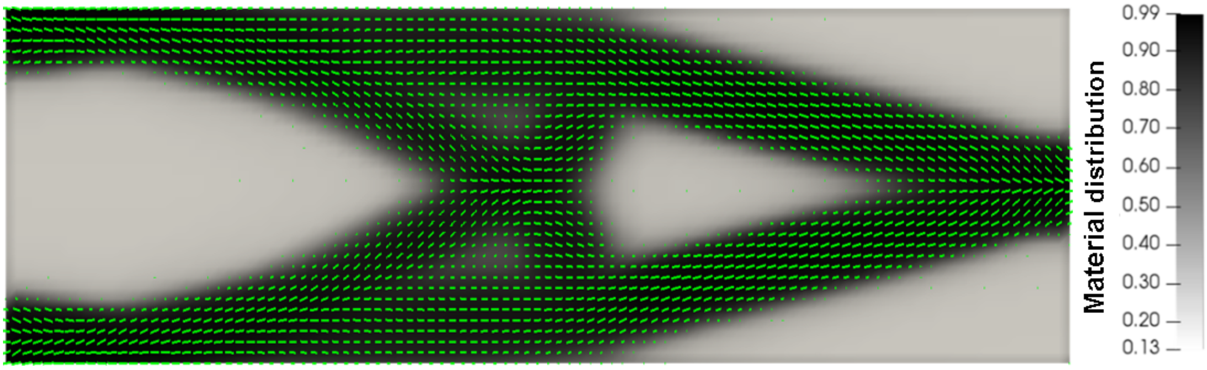
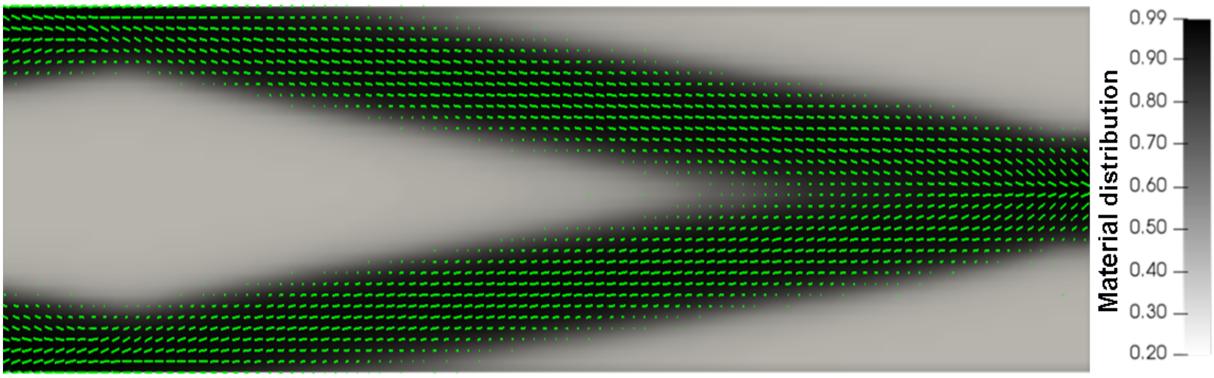


Figure 1. Domain, material properties, element and parameters for numerical examples.



(a) Result for $p_{init} = 1$ and $p_{max} = 20$.



(b) Result for $p_{init} = 10$ and $p_{max} = 20$.

Figure 2. Results for different continuation schemes for SIMP model.

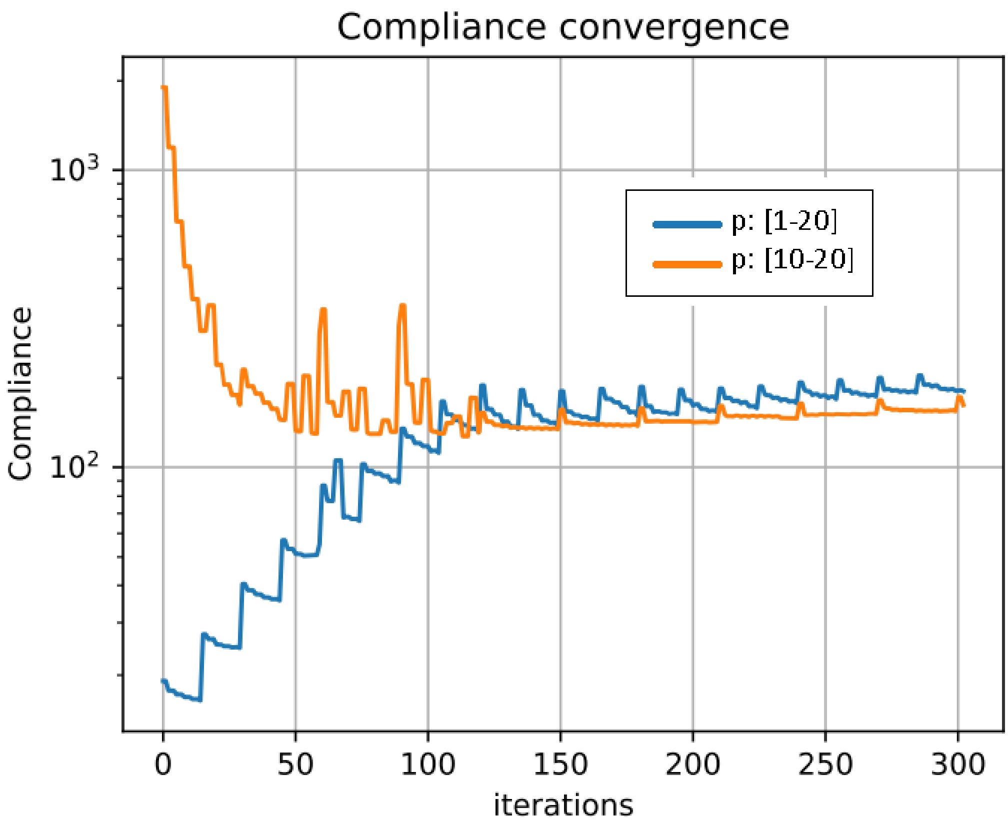


Figure 3. Convergence curve of compliance values.

In result represented by Figs. 2(a) and 2(b), black and gray colours represent the presence of material and void respectively, and green lines represent the fibers. The material distribution observed in Fig. 2(a) is similar to results presented in literature where only isotropic material are considered (see [4]). It can be observed that fiber orientation tend to follows the path of material distribution, even with the presence of two members in the center of the structure. Fig. 2(b) shows a structure with two branches which emerge from the left side and meet where force is applied. However, even though it is a different distribution of material, the fibers also follow the path of material distribution. For both results, there is not abrupt changes in fiber direction, which means that the continuity of the fibers has been achieved.

5 Conclusion

Optimized composite structures were obtained by using topology optimization combining the models SIMP, for the material distribution, and SPIMFO for fiber orientation. In results, there is no presence of checkerboard and there is continuity in fiber, this can be attributed to Helmholtz filter. The result obtained for continuation scheme where $p_{init} = 10$ and $p_{max} = 20$ (see Fig. 2(b)) is similar to results present in literature where only distribution material is considered (see [4]). Fiber angle tends to follow the path formed by material distribution as can be verified in Fig. 2. Fig. 3 shows that the compliance value increase when the value of penalty factor p change during the iterations.

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