A TOPOLOGY OPTIMIZATION APPROACH FOR THE 2D ACOUSTIC INVERSE PROBLEM IN TIME DOMAIN

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Abstract. Acoustic Tomography (AT) is a method widely used in the oil and gas industry for the purpose of finding and/or monitoring sub-sea bed hydrocarbon reservoirs. This approach is based on propagation and measurement of acoustic waves in order to acquire information about unknown media. As a result, physical properties of the medium can be identified by solving an inverse problem, which is usually formulated as the minimization of the difference between the response measured by the receivers, and a simulated response obtained by a numerical model. An efficient method to solve inverse problems and to find the distribution of material parameters is the Topology Optimization Method (TOM). This work proposes a topology optimization approach for the acoustic inverse problem in the time-domain using finite element method. A material interpolation scheme based on the Solid Isotropic Material with Penalization (SIMP) is employed to represent the velocity model for the acoustic problem. The sensitivity analysis is carry out using an automated differentiation tool and the optimization problem is solved by a Limited-Memory BFGS algorithm. Two examples considering different velocity models are analyzed and a perturbation is imposed on the measured responses in order to avoid inverse crime.

Keywords: Topology optimization, SIMP method, Acoustic inverse problem, Finite element method.

1 Introduction

Acoustics is a field of knowledge concerning the energy generation, propagation and reception by vibrational waves in a material medium. Among the applications of acoustics can be mentioned sonars for detection and location in offshore exploration and submarines, sounders for depth measurement or bathymetry and design of theaters or concert halls [1].

In some problems, both physics governing the waves behavior and the properties of the medium are known. Thus, the variable of interest is the acoustic field being this problem called the "forward problem". However, in many practical applications, the physics governing the problem is known, the properties of the medium are an unknown and this problem is named the "inverse problem" [2].

In the inverse problem, the response is estimated, compared to the observed measurements and modified until reaching a certain acceptance level, being possible to consider the noise, the losses and the media imperfections [3]. Some causes of the acoustic signals scattering are ambient noise (wind, sea waves or seismic activity), shipping, fishing activity or reverberation in acoustic transducers (return of part of the transmitted energy) because of the imperfections in the volume and boundaries of the medium [1].

There are different approaches to solve an inverse problem. An efficient approach is the use of the Topology Optimization Method (TOM), which is based on an iterative material distribution in a fixed domain so that a cost function reaches an extreme value. The TOM has been applied to inverse problem such as dielectric properties identification [4], thermal cartography by using optimality criteria [5] and damping acoustic wave propagation [6].

This present work adresses to study the acoustic inverse problem using a topology optimization approach. This paper is organized as follows. In Section 2, the theorical formulation about acoustic wave propagation is given. In Section 3, the topology optimization method is formulated and the material model is presented. In Section 4, the implementation details of the problem are shown. In section 5 some numerical examples and results are presented and at the end the the conclusions are discussed in the last section.

2 Theorical Formulation

The acoustic wave equation in the time domain is given by:

$$\frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \nabla^2 u = q(x,t) \tag{1}$$

where $u(x,t)$ is the pressure field, c is the wave propagation velocity in the medium and $q(x,t)$ is the excitation.

If the equation 1 is valid, then multiplying by an arbitrary test function and integrating over the whole domain is also valid, with this

$$\int_\Omega \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} v \mathrm{d}x - \int_\Omega \nabla^2 u v \mathrm{d}x = \int_\Omega q(x,t) v \mathrm{d}x \tag{2}$$

integrating the term containing the laplacian of u:

$$-\int_\Omega \nabla^2 u v \mathrm{d}x = \int_\Omega \nabla u \cdot \nabla v \mathrm{d}x - \int_{\partial\Omega} v \nabla u \cdot n \mathrm{d}s \tag{3}$$

The weak form of the equation is given by (4)

$$\int_\Omega \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} v \mathrm{dx} + \int_\Omega \nabla u \cdot \nabla v \mathrm{dx} - \int_{\partial\Omega} v \nabla u \cdot n \mathrm{d}s = \int_\Omega q v \mathrm{dx}. \tag{4}$$

For the time integration, different methods can be used. In this work we will use the backward difference finite method. The main idea is to replace the derivatives in the differential equation by finite differences that approximate them. Substituting the second derivative of the pressure field over time by the backward difference finite method:

$$\frac{d^2u(x,t)}{dt^2} \simeq \frac{u^{k+1} - 2u^k + u^{k-1}}{\Delta t^2} \tag{5}$$

Rearranging the equations

$$\int_\Omega \frac{1}{c^2}\frac{u^{k+1} - 2u^k + u^{k-1}}{\Delta t^2} v\mathrm{dx} + \int_\Omega \nabla u \cdot \nabla v\mathrm{dx} - \int_{\partial\Omega} v\nabla u \cdot n ds = \int_\Omega qv\mathrm{dx} \quad . \tag{6}$$

Equation (6) is the weak form of the acoustic wave propagation equation, discretized with backward difference finite method in time.

3 Topology Optimization

The Topology Optimization Method(TOM) is a powerful optimization technique which determines a material distribution in a given design domain in order to extremize a cost function. The TOM usually combines the optimization algorithms and the finite element method to distribute material in a fixed design domain with the aim of extremizing a cost function.

In this work, the Solid Isotropic Material with Penalization(SIMP) [7] is used as a material model, then the wave velocity can be written as:

$$c = c_1 + (c_2 - c_1)a^p \tag{7}$$

where a $\in$ $(0, 1)$ is the pseudo-density function, p is the penalization and c_1 and c_2 are the material properties. The value of the c goes to the first material first material c_1 when the pseudo-density function is zero and goes to the second material c_2 when a equal to one.

The SIMP is plotted for different penalization exponents in Figure 1 . As we increase the value of the penalization, the value of the functional is forced to go to the extremities.

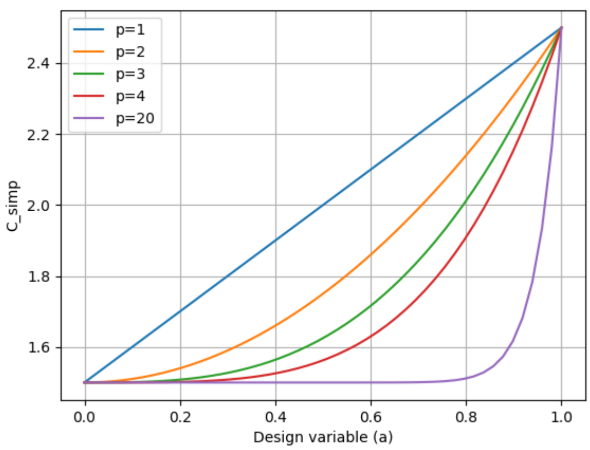

Figure 1. The SIMP for different penalizations.

The optimization problem is formulated as below:

$$\begin{aligned}
\min_{a} : \quad & \mathcal{J} = \int_0^T \int_\Omega (u - u_{obs})^2 \; dxdt \\
\text{s.t.} : \quad & \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2} - \nabla^2 u = q(x,t) \\
: \quad & \nabla u \cdot n = 0 \\
: \quad & 0 \leq a_i \leq 1 \\
: \quad & i = 1, 2, \ldots, N_{desvar}
\end{aligned} \tag{8}$$

where the functional objective of the problem is to minimize the sum of the squares of the differences between the signal obtained by the forward problem and a reference signal that describes the medium behavior under a certain acoustic excitation. The constraints are the finite element acoustic wave propagation in transient regime and the box constraint of the design variable. A Neumann boundary condition is considered for the case, which get a zero value over the boundary.

4 Numerical Implementation

In this section, some numerical aspects and the details of the implementation are discussed. Primarily, the initial domain is discretized by finite elements and designs variables are defined with a uniform values guess and the finite element method problem is solved with pressure field as the response. Then, the function objective and the constraints are evaluated, the sensitivity analysis is done by calculating the adjoints equations. A interior point based algorithm is used for the optimization, after each optimization iterations the design variable and the design domain are updated. Therefore, the process is repeated until a convergence criteria is achieved.

A flowchart of the implemented algorithm for solving the acoustic inverse problem using topology optimization is presented in Figure 2.

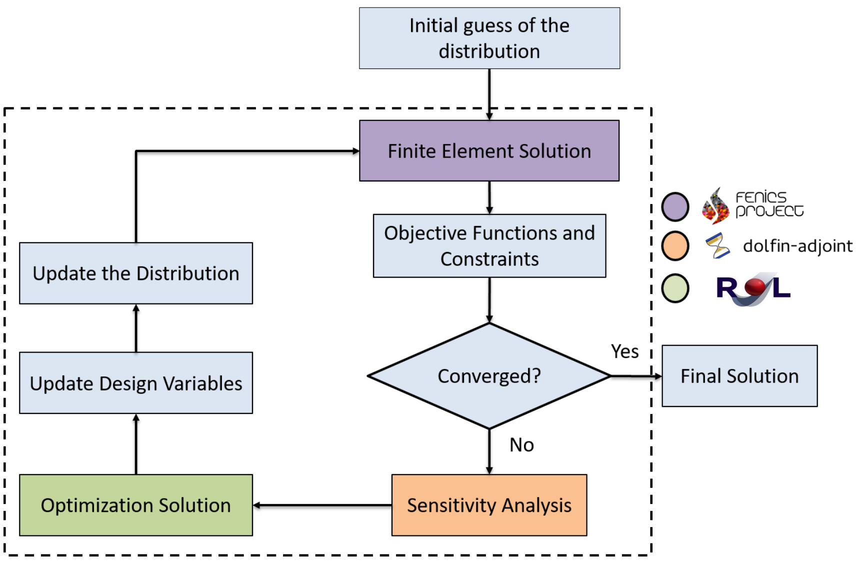

Figure 2. The flowchart of the optimization.

All the problem is implemented with the aid of python and its libraries. The finite element solution is obtained using Fenics, the adjoint calculation is done by the dolfin ajdoint. For the optimization the Rapid Optimization Library (ROL) with a Limited-Memory BFGS algorithm is used.

5 Numerical Results

In this section, the numerical results of the inverse problem using TOM are analyzed by means of two examples.

A sinusoidal function is used as the excitation source $q(x,t)$, which is applied during an initial interval:

$$q(x,t) = \begin{cases} sin(2\pi f t), & \text{if } t < t_d. \\ 0, & \text{otherwise.} \end{cases} \tag{9}$$

where t_d is predefined time interval for the excitation. The excitation source is located at $(x,y) = (500\text{m}, 750\text{m})$ and the receivers are distributed on the boundary of the domain. Figure 3 shows the source and receivers location. The design domain is discretized in 3200 elements, using a constant strain triangle (CST) with a symmetric orientation as presented in Figure 3.

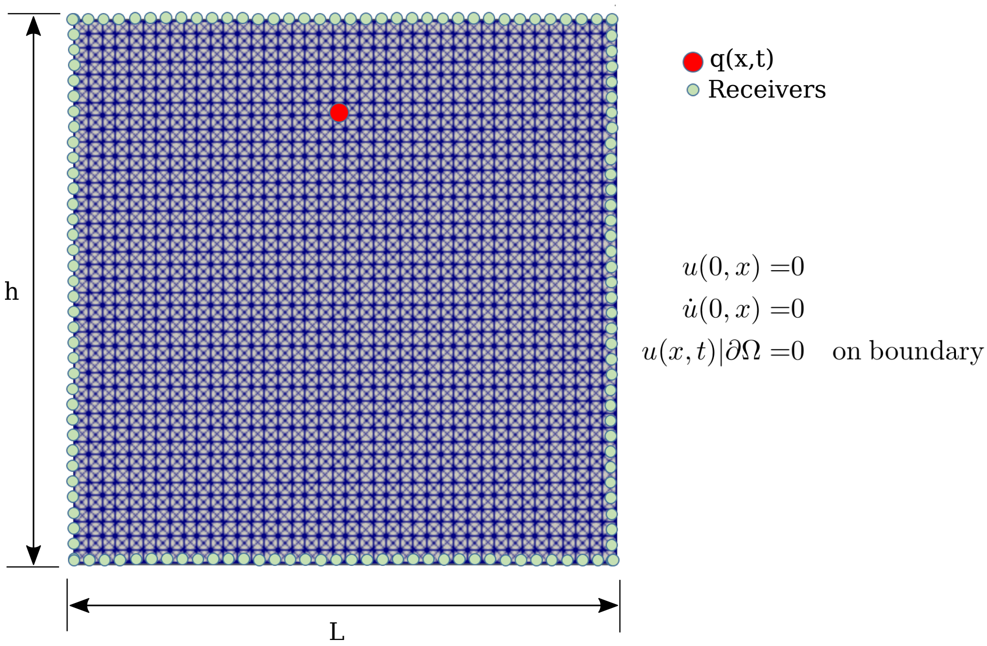

Figure 3. Mesh, boundary and initial conditions.

When experimental data are not available, simulated data from a forward model is necessary in order to solve the inverse problem. However, if the same mathematical model and numerical approximation are used to generate this simulated data and to solve the inverse problem, the so-called inverse crime is committed. In order to avoid this issue, different approaches can be used [8]. One way to avoid the inverse crime is introducing a random error in numerical response. Thus, a set of perturbed response has the form $u_{obs} \approx u(1 + r\sigma)$, where r is a random number and σ is the standard deviation that can be arbitrarily chosen. A Gaussian distribution with mean $\mu = 0$ and the standard deviation $\sigma = 0.01$ is used to generate a random error.

All the required material properties to solve this problem are presented at table 1.

Table 1. Material properties and input parameters.

Parameters	c_1(m/s)	c_2(m/s)	L(m)	h(m)	t_d(s)	f(Hz)	p
Value	1500	2500	1000	1000	5	10	1

5.1 Example A: Two-layer velocity model

The first example considers a velocity model for a two layer domain. This model essentially consists of different velocity for each layer, with a : in the top layer with $c_1 = 1500\mathrm{m/s}$ and $c_2 = 2500\mathrm{m/s}$ in the bottom layer. An initial guess is imposed for the design variable as an intermediate material, i.e., $a = 0.5$, which is equivalent to $c = 2000\mathrm{m/s}$. The optimization procedure will minimize the difference between the responses of these two models, in order to reconstruct the reference velocity model. Figure 4 illustrates the evolution of the optimization over the iterations without considering the perturbation.

The same procedure is carried out, however some random perturbation is added to the observed model. These results are presented in Figure 5.

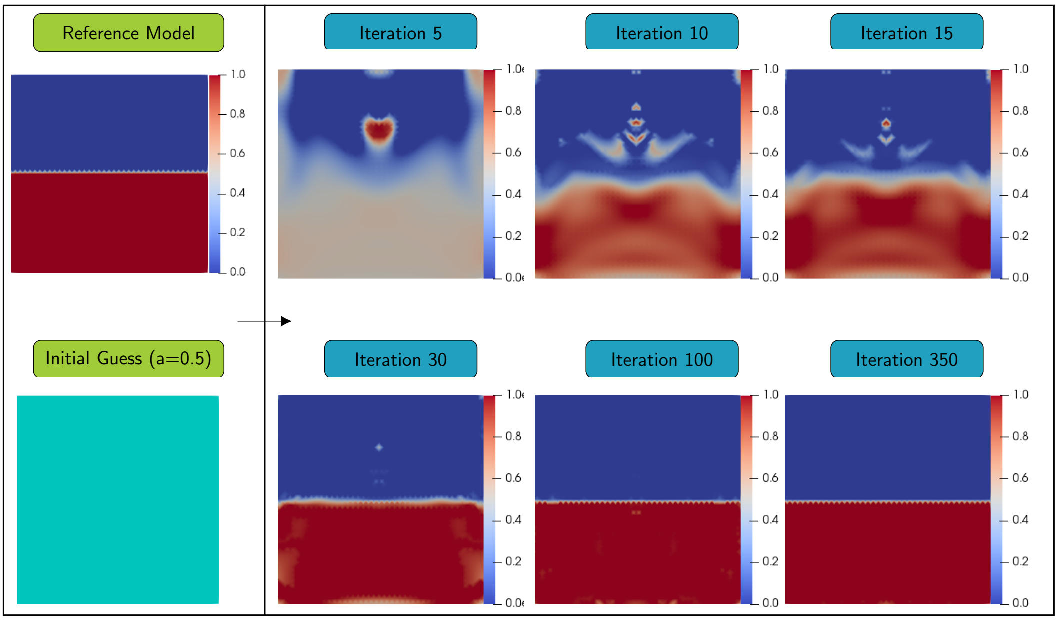

Figure 4. The evolution of the optimization for the example A configuration with inverse crime.

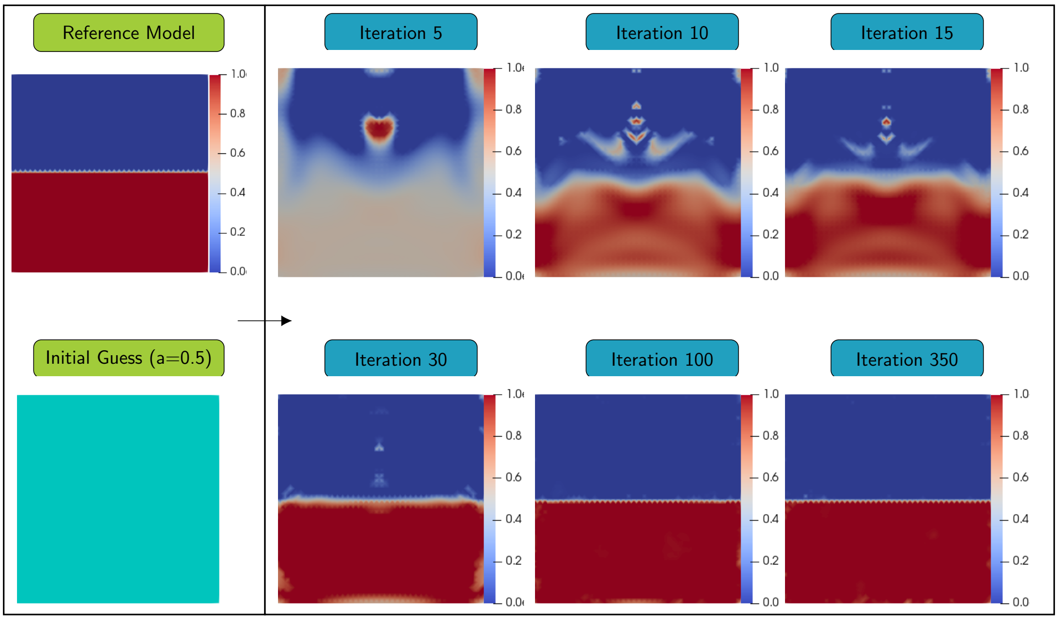

Figure 5. The evolution of the optimization for the example A configuration without inverse crime.

For a better comparison between the inversion procedure with/without inverse crime, both convergence curves are plotted in Figure 6. We can see that cost functional presents a clearly difference between the magnitudes due to the introduction of the random perturbation, which modifies the least-squares objective function.

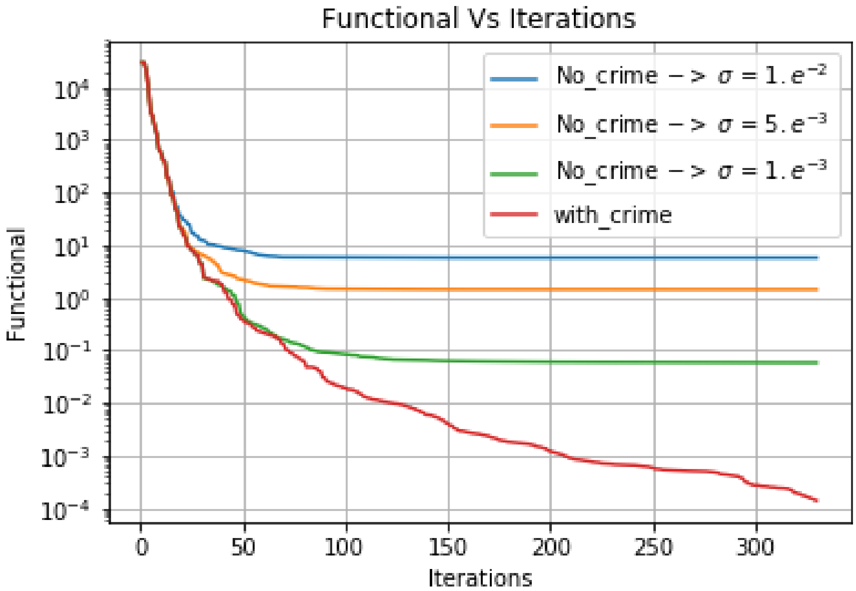

Figure 6. Convergence curve for the example A.

The optimization process is carried out using different values of standard deviation. It is noted that increasing the standard deviation, i.e, increasing the perturbation, the functional converges to a higher value. However, the optimization procedure was able to reconstruct the reference velocity model for all analyzed perturbations scenarios. The cost function in the iteration 300 is plotted over the standard deviations in Figure 7, along with their respective solutions.

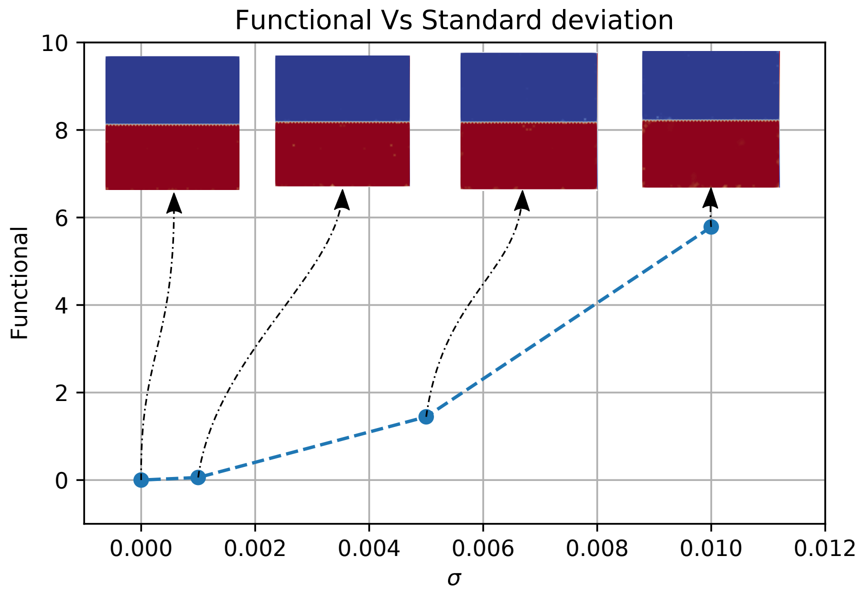

Figure 7. Convergence curve for the example A.

5.2 Example B: Velocity model with circular inclusion

In this case, we consider a circular inclusion in a domain, where the circle has a velocity of $c_1 = 1500\mathrm{m/s}$ and the remaining of the domain with $c_2 = 2500\mathrm{m/s}$. The same homogeneous initial guess is used for the design variable, i.e., $a = 0.5$, which is equivalent to $c = 2000\mathrm{m/s}$. The optimization will reconstruct the reference model by minimizing the cost functional. Figure 8 illustrates the optimization evolution over the iterations.

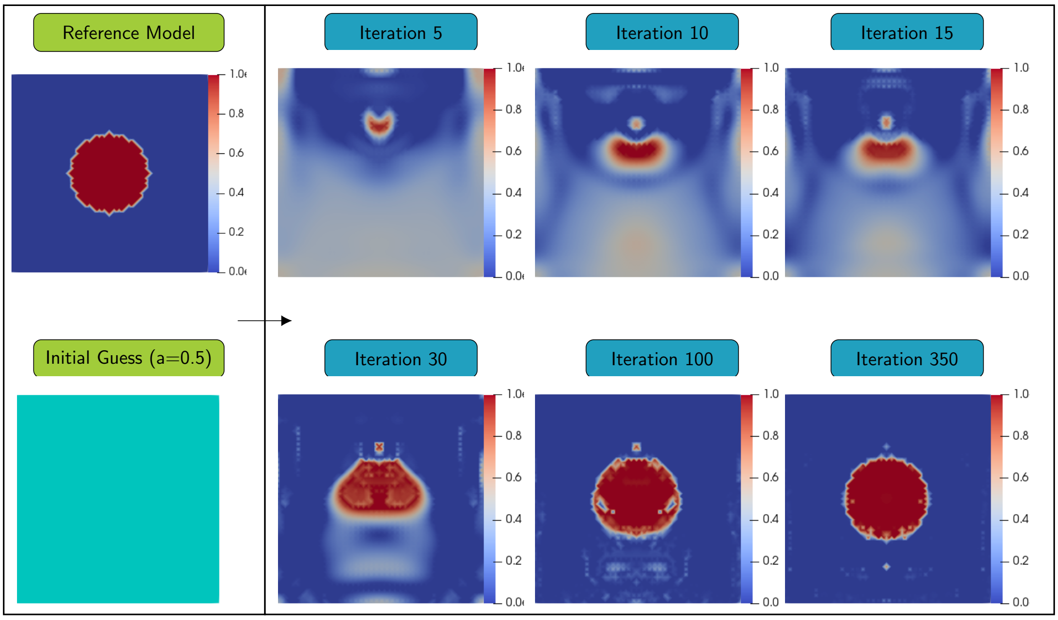

Figure 8. The evolution of the optimization for the example B with inverse crime.

The same procedure is carried out, however some random perturbation is added to the observed model. These results are presented in Figure 9.

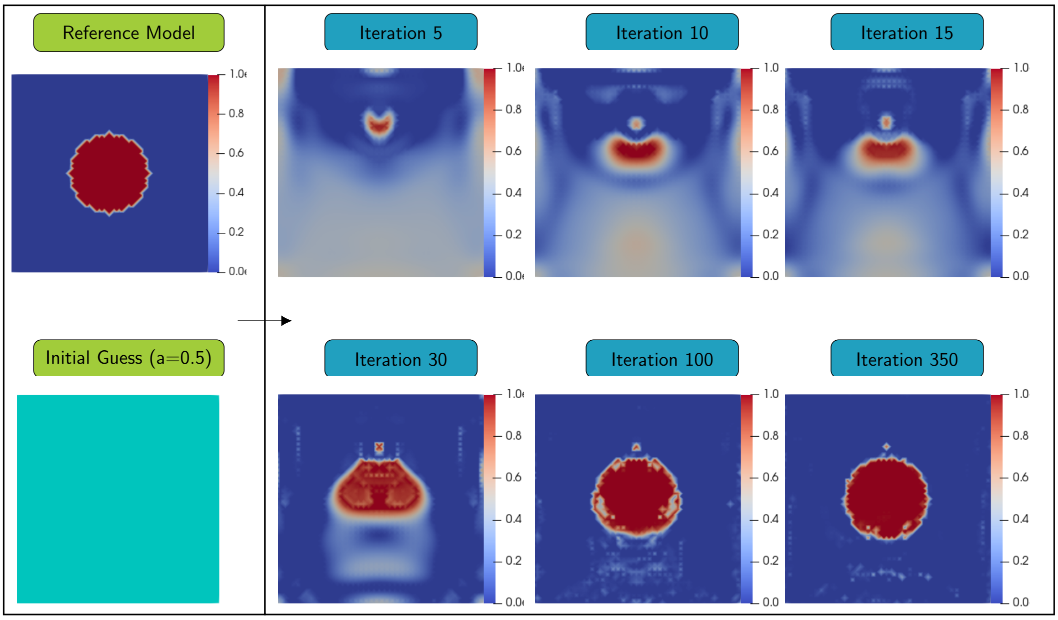

Figure 9. The evolution of the optimization for the example B without inverse crime.

The same behavior is observed for this example, as we can see in Figure 10 where the cost functional presents a difference between the magnitudes due to the introduction of the random perturbation. However, it should be noted that the proposed procedure was able to reconstruct the reference velocity model even with the perturbation that was added to the response.

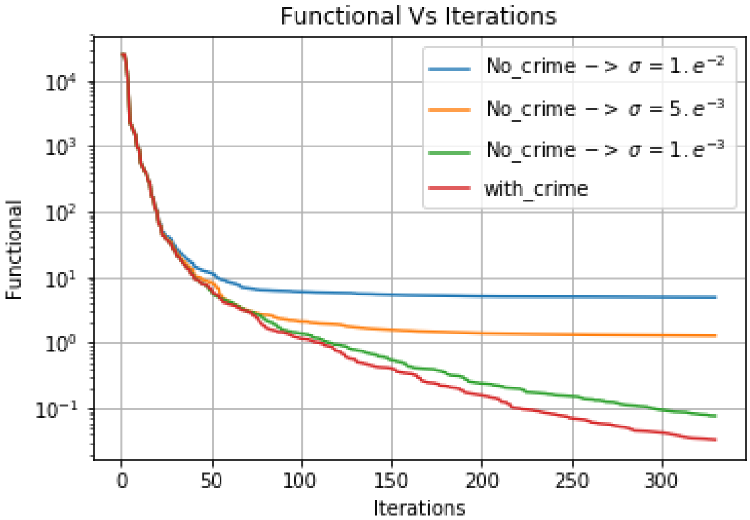

Figure 10. Convergence curve for the example B.

As observed for the previous example, we can also note that higher functional values are obtained when increasing the standard deviation of the perturbation. Again, the optimization procedure was able to reconstruct the reference velocity model for all analyzed perturbations scenarios, as can be seen in Figure 11 where the functional values are plotted along with their respective solutions in the iteration 300.

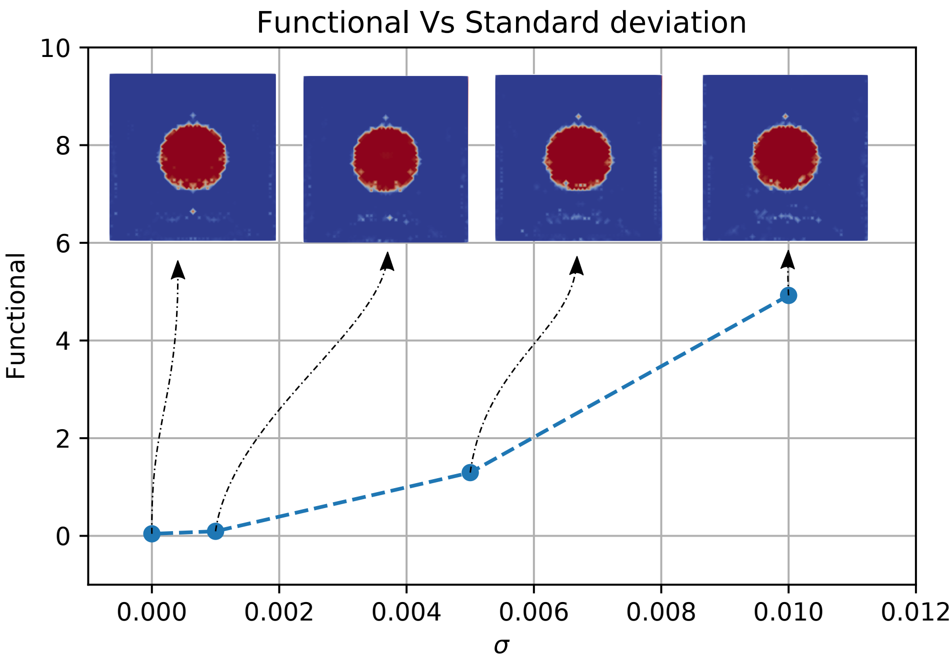

Figure 11. Convergence curve for the example B.

6 Conclusion

The topology optimization has been applied for the inverse acoustic problem and the SIMP method is used as a material model. From these preliminary results, these techniques can be seen that this approach using SIMP model is a promising way to solve acoustic inverse problems. A linear interpolation scheme, i.e., $p = 1$ was capable of reconstruct the velocity models without the use of filters or regularization terms, however it may be necessary for other velocity models.

In future works, we intend to evaluate different velocity models and interpolation schemes. Also, different solvers will be tested in order to reduce the computational cost of the optimization algorithm.

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