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# TOPOLOGY OPTIMIZATION OF SURFACE STRUCTURES SUPPORTED BY PILE GROUPS

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**Abstract.** This work investigates an aspect of topology optimization strategies in dynamic soil-structure interaction problems. The aim is to understand the influence of the presence of embedded pile groups in the optimal shape of a piled structure. The surface structure is modeled by classical three-dimensional finite elements. The embedded pile group that supports the structure is modeled via the impedance matrix method. The soil is an isotropic, viscoelastic, layered medium, and the piles are elastic isotropic bodies with fully bonded contact with the soil throughout their interfaces. Coupling between the structure and the pile foundation is obtained by establishing direct kinematic compatibility and equilibrium criteria at discrete nodes of the structure that connect to pile heads. The system is considered to be under arbitrary static external loads. Shape optimization of the surface structure is obtained with the bidirectional evolutionary structural optimization method (BESO), in which elements of the mesh of the structure are included or removed in order to achieve a certain optimal topology. Objective functions in this work is the stiffness of the structure. The results compare the difference between the optimized shapes in the case in which the structure rests on rigid supports, and the case in which energy dissipation to the soil through the piles is considered.

**Keywords:** Topology Optimization; soil-structure interaction

## 1 Introduction

Piled structures are widely used in many fields of engineering. From nuclear power plants to concert halls and commercial buildings, there are many examples of structures that rely on piles for structural support. In many cases, the vibration and displacement requirements are very restrict, increasing the importance of a good understanding of the dynamical response of these structures. The first step for this analysis is to model the interaction between the pile group and soil . A notable model of this interaction was developed by Kaynia and Kausel [1], which enables obtaining pile group displacement, rotation, torsion, and stiffness for a wide range of load, pile and soil properties. This formulation is used in the present work to obtain the stiffness matrix for the pile group that supports the structure.
After modeling the structure supported by the pile group using classical finite element method (FEM), a coupled stiffness matrix for the structure coupled with the pile group is then obtained by establishing kinematic compatibility in the nodes of the structure that are connected to each pile head. This procedure will be explained in more details in the formulation section.
One possible analysis that can be done using this coupled stiffness matrix scheme is the Structural Optimization of the piled structure. Structural optimization is the group of techniques used to find the best structure that yields requirements of stress, displacement, cost, manufacturing or others. Topology optimization is one approach with the aim to find the best spatial arrangement for a continuous structure (Huang and Xie [2]). After the initial work of Bendsøe and Kikuchi [3] using numerical methods, a wide range of methods were developed, most of them relying on performing a finite element analysis and optimizing the continuum in a mesh of discrete elements. One of these methods for discrete elements is BESO algorithm proposed by Huang and Xie [4] which consists of the systematic removal of inefficient elements of the structure while adding elements where they are needed.
The aim of this work is to perform an optimization procedure of a piled structure using the BESO method in order to understand the influence of the pile group on the optimized shape of the structure.

## 2 Problem Statement

The problem studied in this work is the optimization of an arbitrarily shaped structure, subject to static loads. The structure is modeled using the classical FEM approach. The structure is connected to a group of unconnected piles, embedded in soil. The soil is modeled as a uniform viscoelastic stratum, resting on a rigid bedrock.

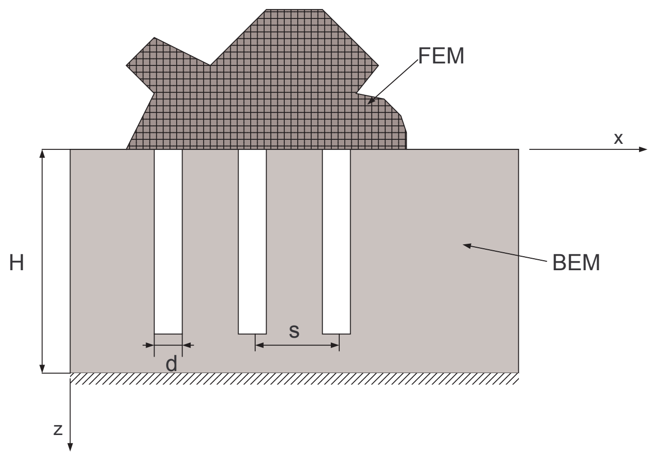


Figure 1. Physical and natural domains of the FEM structure.

The material and geometrical properties of the pile group are described in Table 1 and Table 2:

Table 1. Material properties and dimension of the embedded piles

| Nomenclature | Description |
|---|---|
| $E_p$ | Elastic modulus |
| $\rho_p$ | Mass density |
| $\nu_p$ | Poisson Ratio |
| $L$ | Length |
| $d$ | Diameter |
| $s$ | Center-to-center distance between adjacent piles |

Table 2. Material properties and dimension of the soil

| Nomenclature | Description |
|---|---|
| $E_s$ | Elastic modulus |
| $\rho_s$ | Mass density |
| $\nu_s$ | Poisson Ratio |
| $\beta_s$ | Material damping ratio |
| $H$ | depth |

The objective function for the optimization is to minimize the strain energy (mean compliance) $C$ for this structure, subjected to the load described above, under the volume constraint $V^*$.

$$\text{Minimize C} = \frac{1}{2}\mathbf{f}^{\mathbf{T}}\mathbf{u} \tag{1}$$

$$\text{Subject to: } V^* = \sum_{i=1}^{N} V_i x_i \tag{2}$$

$$x_i = 0 \text{ or } 1 \tag{3}$$

$\mathbf{f}$ is the vector of applied loads, $\mathbf{u}$ is the vector of nodal displacements, $V_i$ is the volume of a single element, $V^*$ is the prescribed volume of the whole structure and $x_i$ is the design variable that indicates presence (if equal to 1) or absence of an element.

## 3 Formulation

### 3.1 Formulation of the Piles

The formulation used for the piles group was derived by Kaynia and Kausel [1]. This formulation presents a solution for a pile group, connected to a rigid plate, embedded in a viscoelastic, layered soil media, supported by a half-space or a rigid bed rock. This formulation enables one to simulate several loading cases such as rocking, axial, bending and seismic excitation. It also enables one to simulate a group of piles, of different dimensions and material properties, whether they are connected or not to each other. In this work, the main interest from the implementation of this pile model is $K_e$, the stiffness matrix of the unconnected pile group. Matrix $K_e$ will be incorporated into the stiffness matrix of the structure to represent the response of a piled structure.

$$\mathbf{K_e} = \mathbf{K_p} + \mathbf{\Psi^T}(\mathbf{F_s} + \mathbf{F_p})\mathbf{\Psi}. \tag{4}$$

Equation 4 gives the formulation of matrix $K_e$, where $K_e$ is the dynamic stiffness of the pile group, $K_p$ is the stiffness matrix (each of which is modeled as one-dimensional finite beam element), $\Psi$ is the shape matrix of the displacements at each pile node, $F_s$ is the flexibility matrix of the soil medium and $F_p$ is the flexibility matrix of the nodes of a fixed-end pile. The definition of each term of this equation is outside the scope of this work but can be found in Kaynia and Kausel [1]

### 3.2 Formulation of the Structure

The structure that is coupled with the pile group is modeled using the classical finite element method. The structure is discretized using hexahedral 8-nodes elements, with mass density $\rho$, and defined by coordinates ($x_i$,$y_i$,$z_i$) in the physical domain and ($\xi_i$,$\eta_i$,$\zeta_i$) in the natural domain.

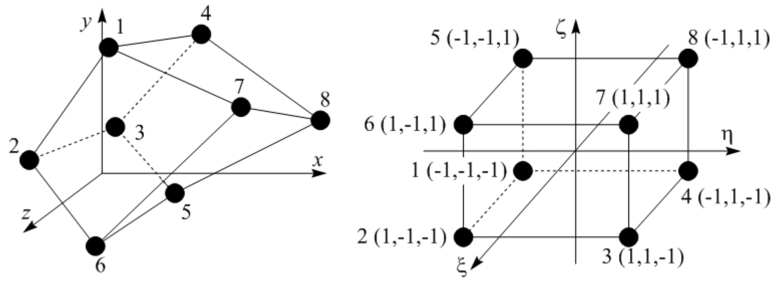


Figure 2. Physical and natural domains of the FEM structure.

The elemental stiffness is given by:

$$\mathbf{k_e} = \int_{V_e} \mathbf{B^T D B} dV = \int_{-1}^{1}\int_{-1}^{1}\int_{-1}^{1} \mathbf{B^T D B} det(J) d\xi d\eta d\zeta \tag{5}$$

in which $V_e$ is the volume of the element in the physical domain, D is the constitutive matrix of the element, N and B are a vector of shape functions and a matrix of its derivatives and J is the Jacobian responsible for transforming between physical and natural domain coordinates.
The global stiffness matrix of the structure $\mathbf{K_f}$ is then obtained by using the classical assembly scheme of the finite element method. A detailed deduction for each of these terms can be found in Cook et al. [5] and most finite element method textbooks.

### 3.3 Pile-Structure Coupling Scheme

The coupling between the structure and pile models described above is obtained by establishing kinematic compatibility and equilibrium in the nodes where the mesh of the structure connects with the pile heads of the pile group. In order to ensure this coupling, one must generate a mesh where there is a node that corresponds to the location of each pile head in the pile group.
The relation between the nodal displacements $\mathbf{u}$ and nodal forces $\mathbf{f}$ on the N nodes of the interface is described by:

$$\mathbf{f} = \mathbf{Ku} \,. \tag{6}$$

$$\mathbf{f} = \begin{bmatrix} f_x^1 & f_y^1 & f_z^1 & f_x^2 & f_y^2 & f_z^2 & \cdots & f_x^N & f_y^N & f_z^N \end{bmatrix}. \tag{7}$$

$$\mathbf{u} = \begin{bmatrix} u_x^1 & u_y^1 & u_z^1 & u_x^2 & u_y^2 & u_z^2 & \cdots & u_x^N & u_y^N & u_z^N \end{bmatrix}. \tag{8}$$

$$\mathbf{K} = \begin{bmatrix} k_f^{1,1} & \cdots & k_f^{1,n} & \cdots & k_f^{1,m} & \cdots & k_f^{1,N} \\ \vdots & \ddots & \vdots & & \vdots & & \vdots \\ k_f^{n,1} & \cdots & k_f^{n,n} + k_p^{i,i} & \cdots & k_f^{n,m} + k_p^{i,j} & \cdots & k_f^{n,N} \\ \vdots & & \vdots & \ddots & \vdots & & \vdots \\ k_f^{m,1} & \cdots & k_f^{m,n} + k_p^{j,i} & \cdots & k_f^{m,m} + k_p^{j,j} & \cdots & k_f^{m,N} \\ \vdots & & \vdots & & \vdots & \ddots & \vdots \\ k_f^{N,1} & \cdots & k_f^{N,n} & \cdots & k_f^{N,m} & \cdots & k_f^{N,N} \end{bmatrix}. \tag{9}$$

Each term $k$ of the final matrix $\mathbf{K}$ is a 3x3 stiffness matrix in the x, y and z directions. The sub-indices $f$ and $p$ refer to the stiffness of the structure and the pile, whereas the super-indices $n$ and $m$ indicate the nodes of the structure connected to the piles $i$ and $j$. An detailed example and deduction for these equations can be found in Tavares and Labaki [6].

### 3.4 Topology Optmization Method

The BESO procedure solves the problem of minimizing the strain energy (mean compliance) of the structure by evaluating the value of the sensitivity for each element. The sensitivity is defined by the value of the change of the strain energy when one element is added or removed, given by:

$$\alpha_i^e = \Delta C = \frac{1}{2}\mathbf{u_i^T K_i u_i} \tag{10}$$

The topology optimization method used for this work is the BESO procedure described by Huang and Xie [2] which has the following steps:

1. Discretization of the design domain and assignment of initial properties values 1 or 0 to the elements to build the initial design of the structure.
2. Perform finite element analysis and calculate the elemental sensitivity number.
3. Average the sensitivity number with its history information and then save the resulted sensitivity for the next iteration.
4. Determine the target volume for the next iteration.
5. Add and delete elements.
6. Repeat steps 2 to 5 until the volume constraint is satisfied and the convergence criteria achieved.

The variables necessary for the BESO optimization are described in the table below. For more details on the filtering scheme, stabilization of the solution and the criteria of convergence, one can refer to Huang and Xie [2].

Table 3. BESO parameters

| Nomenclature | Description |
|---|---|
| $ER$ | Evolutionary volume ratio |
| $AR$ | Volume addition ration |
| $r_{min}$ | filter radius scale |
| $\tau$ | convergence tolerance |
| $V^*$ | Target volume fraction |

## 4 Numerical Results

This section is divided in five subsections. In the first two sections the structure, load and variables of the problem are presented. In the following two sections we present the results for the optimization of a structure subjected to a horizontal and a vertical load. The final section is a discussion of the results obtained.

### 4.1 Definition of the Structure

The present method was used to study the influence of the piles on a prismatic structure supported by four piles (Fig. 4). The load is uniformly distributed on a small area on the top surface of the structure (Fig. 3).

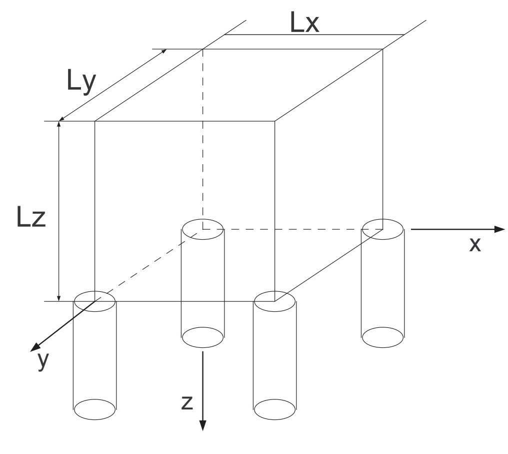


Figure 3. Modeled structure

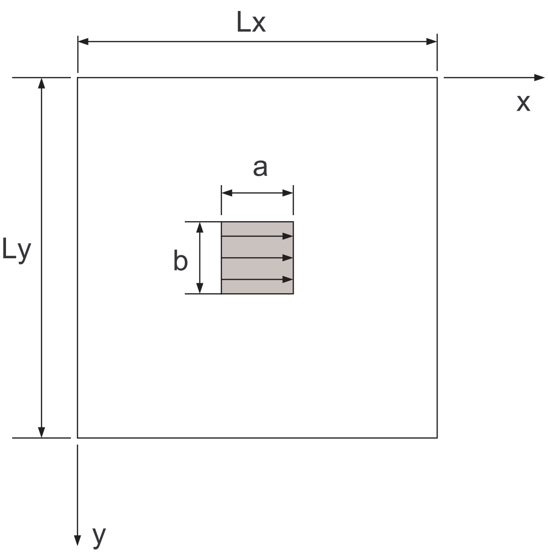


Figure 4. Load application

### 4.2 Input Variables for the Simulation

The soil medium is modeled with depth $H/d = 75$ (see fig. 1) and material damping $\beta_s = 0.05$. The piles have the following material and geometrical properties $E_p/E_s = 1$, $\rho_p/\rho_s = 1$, $\nu_p/\nu_s = 1$, $L/d = 37.5$, $s/d = 5$ and $d/a = 1$, and the structure is modeled with $b/a = 1$, $Lx/a = Ly/a = 5$, $Lz/a = 20$, $E_{st}/E_s = 1$, $\rho_{st}/\rho_s = 1$ and $\nu_{st}/\nu_s = 1$.

Table 4. Variables for BESO optimization

| Nomenclature | Value |
|---|---|
| $ER$ | 0.01 |
| $AR$ | 0.5 |
| $r_{min}$ | 1.5*a |
| $\tau$ | 0.0001 |
| $V^*$ | 0.3 |

The mesh was discretized in 27000 elements, having 30 divisions in the x, y and z direction.

### 4.3 Results - Horizontal load

Figure 5 shows the convergence history of the compliance and volume of the structure with the increasing number of optimization iterations for the case in which the structure is on rigid supports, and the load is applied in the x-direction. That is, full longitudinal displacement restriction is prescribed in the four bottom corners of the structure in all directions. The resulting optimized topology for this case is shown in Fig. 6. The corresponding results for the case in which the rigid supports are replaced by embedded piles are shown in Figs. 7 and 8. The normalized Compliance $C^*$ in these results is given by $C^* = C_i/C_1$, where $C_i$ is the compliance of the structure in the ith iteration and $C_1$ is the compliance of the structure in the first iteration.

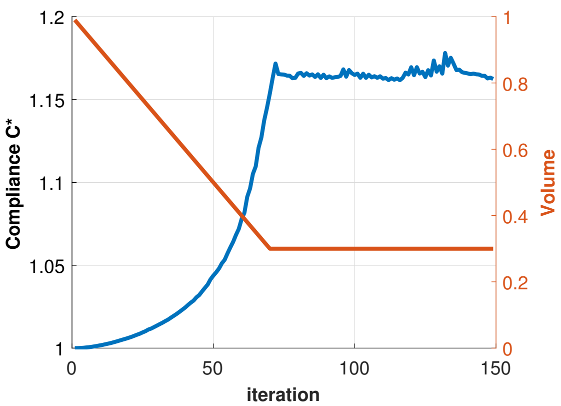


Figure 5. Mean Compliance x Volume, Clamped Structure

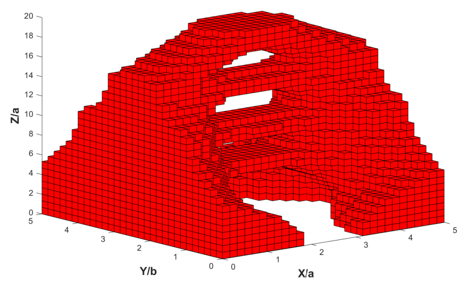


Figure 6. Final Topology, Clamped Structure

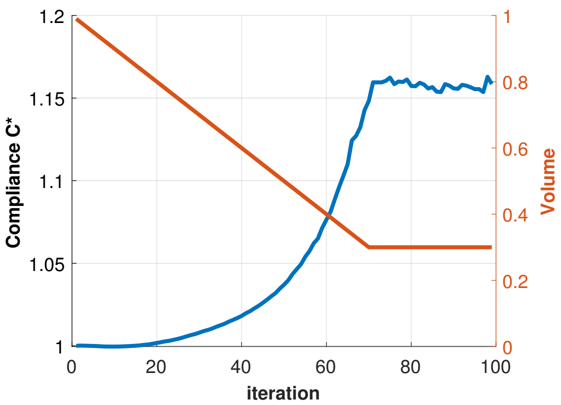


Figure 7. Mean Compliance x Volume, Piled Structure

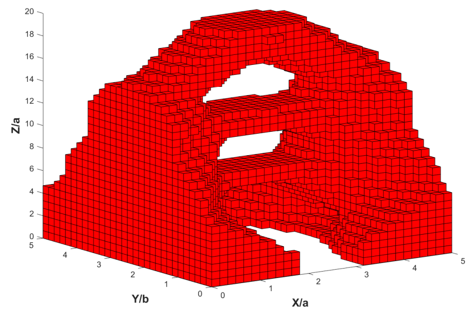


Figure 8. Final Topology, Piled Structure

## 4.4 Results - Vertical load

Figure 9 shows the convergence history of the compliance and volume of the structure with the increasing number of optimization iterations for the case in which the structure is on rigid supports. In this case, the load is applied in the z-direction. The resulting optimized topology for this case is shown in Fig. 10. The corresponding results for the case in which the rigid supports are replaced by embedded piles are shown in Figs. 11 and 12.

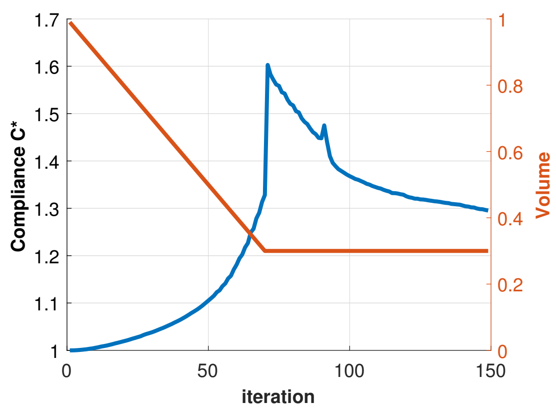


Figure 9. Mean Compliance x Volume, Clamped Structure

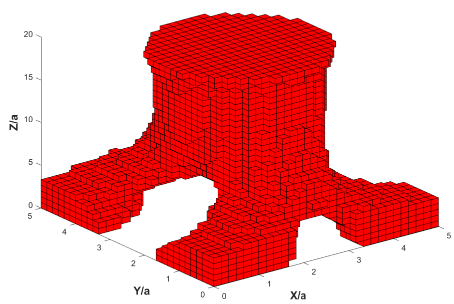


Figure 10. Final Topology, Clamped Structure

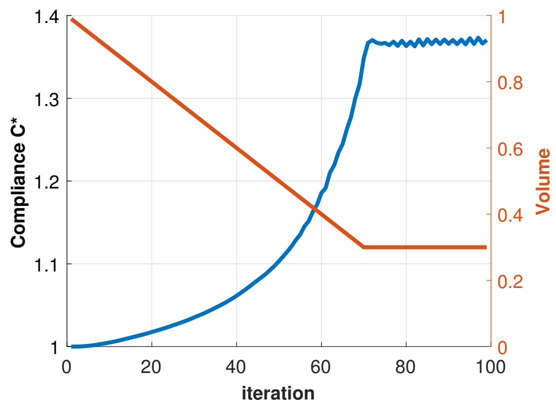


Figure 11. Mean Compliance x Volume, Piled Structure

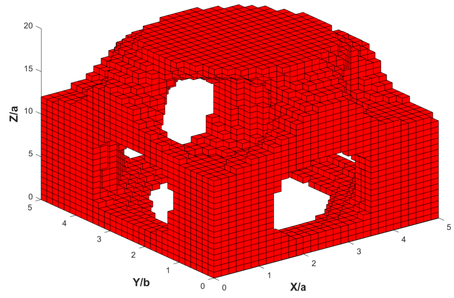


Figure 12. Final Topology, Piled Structure

### 4.5 Discussion

The results in this section show that consideration of a flexible pile group support for the structure strongly affects its final optimized shape. This can be seen both quantitatively in the achievable compliance, and qualitatively in the shape of the structure. The addition of the pile group led to a less stiff structure specially in the vertical load case, in which case a much different shape was also achieved.

## 5 Conclusion

This article presented an analysis of the influence of the presence of embedded piles in the optimal shape of a piled structure under static loads. The piles were modeled according to the influence matrix method, the soil was modeled as a homogeneous, isotropic layer supported by a rigid base, and the structure was modeled using classical finite elements. Coupling between the piles and the structure was obtained through direct continuity and equilibrium conditions at their shared nodes. Topology optimization of the structure was performed through BESO, in which the compliance of the structure was the objective function, under volume constraints. The results showed that both the achievable compliance and the final optimized shape depend on whether the structure is supported by piles or by rigid supports. This analysis indicate that energy transfer to the soil cannot be disregarded in the topology optimization of soil-structure interaction problems.

## Acknowledgements

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## References

[1] Kaynia, A. M. & Kausel, E., 1991. Dynamic of piles and pile groups in layered soil media. *Soil Dynamics & Earthquake Engineering*, vol. 10, pp. 386–401.

[2] Huang, X. & Xie, Y. M., 2010. *Evolutionary topology optimization of continuum structures: methods and applications*. John Wiley & Sons, Ltd.

[3] Bendsøe, M. D. & Kikuchi, N., 1988. Generating optimal topologies in structural design using a homogenization method. *Comput. Meth. Appl. Mech. Engng.*, vol. 71, pp. 197–224.

[4] Huang, X. & Xie, Y. M., 2007. Convergent and mesh-independent solutions for bi-directional evolutionary structural optimization method. *Finite Elements in Analysis and Design*, vol. 43, pp. 1039–1049.

[5] Cook, D., Malkus, D. S., Plesha, M. E., & Witt, R. J., 2001. *Concepts and Applications of Finite Element Analysis*. John Wiley & Sons, Ltd.

[6] Tavares, E. S. & Labaki, J., 2019. Fem-bem analysis of arbitrarily-shaped structures supported by pile groups. *7th International Symposium on Solid Mechanics, MECSOL2019*, vol. .