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NUMERICAL SOLUTION OF THERMAL STRESSES IN A COMPOSITE HOLLOW DISK UNDER STEEP TEMPERATURE

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Abstract. This work studies the effects of thermal stresses in composite disk during heating by imposing high heat fluxes at the inner face. The medium is formed by layers of steel, cement and rock. Thermal stresses are caused by distinct displacements in different layers. It is important to analyze the movements of contraction and expansion of these layers in addiction to verify the tensions caused by temperature difference. This configuration, or say, composite layers, is found in completed oil wells. These stresses can induce fissures and failures that can let a movement of fluids from oil reservoir through them and cause environmental problems in oceans and seas where oil is extracted. The numerical solution is based on finite element method (FEM) applied by the commercial code ANSYS® which was employed in the simulations. Materials of layers were assumed isotropic and no thermal resistance between them was considered. Transient heat transfer was analyzed. The idea of this work is to investigate the interaction between steel and cement because it configures a point of concern due to the fact that coefficients of thermal expansion from these materials are different and there is a large probability of unwanted fissures. Another point is to establish an optimal heat flux that does not allow a formation of fissures and empty spaces between the layers that allow a flow of fluids from the oil field through them, especially at the steel-cement interface.

Keywords: Thermal Stresses, Finite Element Method, Oil Wells, Composite Structures

1 Introduction

One of the concerns from oil and gas industry is about the procedures and techniques that can be applied in a process of plug and abandonment to prevent some environmental problems in the future. A good process of plug and abandonment aims to prevent leakages along the well to all fluids remain permanently confined as explained by Mainguy et al. [1].

The process of plug and abandonment, also called P&A, is divided in five stages: Planning; Regulatory Approval; Plugging Operation; Wellhead and Casing Removal; Reporting Requirements as described by Kaiser and Dodson [2].

Planning consists in review the wellbore design, geologic and reservoir conditions, investigate items related with healthy and safety issues, as well as regulatory requirements to evaluate de difficulties that may occur during the process. It is a part based on studies to evaluate and aim some potential problems and concerns which can occur during and after the P&A.

Regulatory approval is the official notification required to start the process of P&A. This part depends of norms and laws from government bodies where the wells are placed. In general gases, the regulatory approval contains a description of the work requirements, assessment of expected environmental impacts of the operation and procedures to minimize such impact.

Plugging operation is part of the process when well is filled with cement to lock the reservoir. Firstly, it is necessary to remove downhole equipment such as: packers; production tubing; gas lift mandrels and pumps. Secondly, the well must be cleaned to remove some impurities that can be a problem during the application of cement. Its injection to create the plug takes 8-12 hours until it hardens. The length of plug varies from 30-60 meters and its location is located near the middle of wellbore. Some wells need two or more cement plugs and the number depends of the conditions presented during the planning of the procedure.

After these processes to plug and abandon an oil well, it is important to point out the necessity to maintain equipment monitoring some failures that can induce a fluid movement to the seabed or a complete deterioration of plug. With these information, it is possible to assume that this process is one of the most expensive for oil companies and new studies are been developed to find cheap procedures with a high level of reliability, even better than currently used.

Vrålstad et al. [3] pointed out a review comparing different plugging materials to show the challenges of each them and establishing a comparison focused on cost-effective of each solution and how effective are them concerning leaks and failures mechanisms during and after P&A. Myrseth et al. [4] developed a study that discuss about the number of oil wells in Norway and the cost of P&A process accordingly with the government rules. They decided to develop an open-source database to be a support tool for the oil sector to reduce bottlenecks and analyzing of efficient gains from technology improvements. One of the process cited is based on exothermic reaction with thermite.

Thermite reaction is suitable source to several engineering applications and may be applied in advanced materials synthesis as shown by Durães et al. [5]. It is a reaction that raises temperatures over 3000 K and it is characterized by a combination of ferric oxide (Fe_2O_3) and aluminum (Al) powders. Their combination results in a large amount of heat with formation of aluminum oxide (Al_2O_3) and Iron (Fe). This reaction is started by some ignition, which can be from mechanical means such as friction or compression shear and it is important to this work as a reference of heat generation because it can produce a amount of energy necessary to create a plug at oil wells and guarantee their isolation through a long period of time. But the heat generation can affect mechanically the structures near where thermite reaction occurs because it can generate thermal stresses, and this can be a problem to ensure the integrity of plug.

Stresses caused by a temperature gradient is one of the important parts of structural analysis because a large number of projects are developed to operate in some circumstances where the effects of temperature fields cannot be neglected. With the development of new industries and modern processes, many machines and structures experience extreme thermal environments as mentioned by Noda [6]. Temperature gradients are responsible for the movement of particles from a heater font to a cooler part of the materials and these movements can cause inner tensions.

When a solid body is heated or cooled, the temperature distribution will depend on: size and shape, thermal conductivity of the material and rate of temperature change as mentioned by Callister et al. [7]. These factors are important to analyze the effect of temperature gradients over the material. If a body suffers an abrupt variation of the temperature, compressible stresses are observed due the expansion of a hotter region to the detriment of other one and these tensions are balanced by tensile interior stresses.

There are previous studies published about thermal stresses which helped the development of this study. Al Farran [8] developed a stress analysis due an asymmetric temperature distribution in a composite material. Yang et al. [9] studied a transient heat flux of a gun barrel and taking into account the thermal contact resistance between two materials. Jane and Lee [10] studied the thermoelastic response of an infinitely along annular multilayered cylinder. Lee et al. [11] develop a study based on one-dimensional multilayered hollow cylinder with two different boundary conditions at outer face. Ootao and Tanigawa [12] developed a work based on multilayered hollow circular disk with piecewise power low nonhomogeneity with properties expressed as power functions of the radial coordinate.

Because of the scenario involving a huge amount of heat, which culminates in a huge temperature field, oil and gas industry needs to perform thermal stresses studies at these new plugs formed by thermite reactions because there is formation of a heat-affected zone during this process, residual tensions can emerge and cause cracks at structure which allows flux of oil through them.

2 Methodology

2.1 Finite element method

Finite element method is a numerical tool to solve problems from some subjects such as: structural analysis, heat transfer, mass transport and fluid flow. Basically, it consists in reduce a large domain into small finite elements due process of discretization and establish interpolation of them with special functions that varies with geometry, derivative order or both them to solve problems numerically.

The main constituents of a finite element method to solve a boundary-value problem are: variational or weak statement and the approximate solution of equations through the use of finite element functions as mentioned by Hughes [13]

To illustrate the finite element method, also known FEM, the Eq. (1) is an example of one-dimension differential equation defined in a domain [0,1]→$\mathbf{R}$ as illustrated:

$$u_{,xx} + f = 0 \tag{1}$$

The Eq. (1) has two boundary conditions: $u(1) = g$ and $u_{,x}(0) = -h$ and, because of this, is called two-point boundary-value condition problem with g and h both constants. The term f is called smooth function because their first order derivatives are continuous into the domain of function. The formulation presented by the Eq. (1) is called strong form of the problem, or classical form. In a situation like this, the solution of Eq. (1) is:

$$u = g + (1 - x)h + \int_x^0 \left[\int_0^y f(z)dz \right] dy \tag{2}$$

Equation 2 is the exact solution of the problem proposed by Eq. (1). But the main objective is, at this moment, to obtain approximate solutions because it is impossible in some circumstances finding the exact solution of the problem. Next part is to turn Eq. (1) into its weak form.

To define weak form, it is important to characterize trial and weighted functions as described by Hughes [13]. The trials have square-integrable derivatives and take on the value $u(1) = g$. The weighted functions, or variations, are the counter-part of trials because they must satisfy $w(1) = 0$. These conditions exist because both weak and strong forms of equations must keep the same boundary conditions. Without these points, it is impossible to transform the equation and apply Galerkin's Approximation Method. The weak form is presented by Eq. (3)

$$\int_0^1 w_{,x} u_{,x} dx = \int_0^1 w f dx + w(0)h \tag{3}$$

Equation 3 is the same one described by Eq. (1) with important elements for the application of FEM. The $w's$ are virtual forces or temperatures induced in the system. Product wf refers to distributed force or temperature along a body length and the product $w(0)h$ is the application of a boundary condition, force or temperature, at this point of body.

With Eq. (3) it will be possible to start the FEM and applying Galerkin's Method. First step is defining some space finite-dimensional approximations for trial and weighted functions.

The approximate solution of the weak form presented by Eq. (3) is

$$\int_0^1 w^h{}_{,x} u^h{}_{,x} dx = \int_0^1 w^h f dx + w^h(0)h \tag{4}$$

Here are two groups of modified functions from subordinated conjunctions which allow to the same ones of functions used to create weak form of differential equation showed by Eq. (3): u^h and w^h. Both them still maintain the boundary conditions $u^h(1) = g$ and $w^h(1) = 0$, but the u^h has a special modification presented by Eq. (5)

$$u^h = v^h + g^h \tag{5}$$

The manipulation shown by Eq. (5) is necessary because Galerkin's Method would have a problem with the application of boundary conditions at point $x = 1$. The term v^h is from the weighted functions group and g^h is from the trials. This is important to point out because, with the previous information, $v^h(1) = 0$ and $g^h(1) = g$. The next stage is manipulate Eq. (4) with interpolation functions between the finite elements to determine the matrices with linear equations and solve them to find the temperatures or displacements in a discrete medium.

The explanation above is for a one-dimensional situation, but can be expanded for two and three dimensions using Stokes' Theorem to determine weak form of differential equations. Interpolation functions for elements can be obtained from isoparametric elements, especially bilinear quadrilateral element for bidimensional cases and trilinear hexahedral for tridimensional studies.

2.2 Numerical simulation

ANSYS® software where used for numerical simulations of the thermal fields and structural analysis.

The study is done based in a composite hollow disk with three layers: steel, cement and a rock structure. The thermal properties are in Table 1. No thermal resistance was considered and the layers are assumed isotropic. The properties were taken based on a temperature of 60 ^{o}C because this value is observed in the medium before exothermic reaction.

The reference geometry for this simulation is presented in Fig. 1. The inner layer is steel, the outer is rock structure and between them is the cement layer At this work, the emphasis is studying the effects of displacements and stresses at radial direction only. To obtain these results, thickness of disk was reduced and this action minimize the effects of tension at longitudinal direction

Table 1. Thermal properties of materials

Material	Density (kg/m^3)	Thermal Conductivity (W/m^{o}C)	Specific Heat Capacity (J/kg^{o}C)
Steel	7800	54.21	448
Cement	2010	0.53	736
Rock Structure	2630	3.34	953

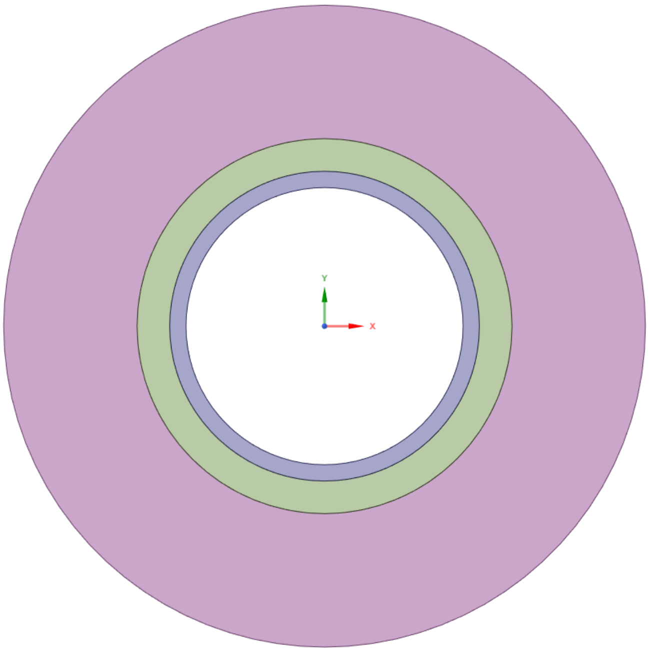

Figure 1. Visualization of the geometry - top view

The mesh of thermal field simulation is shown at Fig. 2. It is important to point out special refinements at contact interface between the layers to capture with more precision temperature at these regions. There is also a refinement at the inner face of the disk because of heat flux incidence. It is composed by quadratic elements.

Independence of mesh is shown by Table 2 taking maximum temperature of simulation as a parameter of comparison.

Table 2. Mesh validation - thermal field

Nodes	Temeprature inner face (^{o}C)	Error %
17000	783,37	-
25500	783,37	0
34000	783,37	0

Table 2 shows a interesting fact. There is no difference between temperatures even with mesh variation and when occurs a situation like this one is possible to conclude that the numerical simulation of thermal field is capable to represent results as good as an analytical formulation of the same problem.

Heat source was based on results from an exothermic reaction proposed by Abdelal et al. [14]. This model is focused on a thermite reaction to create a new type of plug for wells to replace the cement ones. Based on this model of reaction, it is possible to modeling this heat source and transform it into a heat

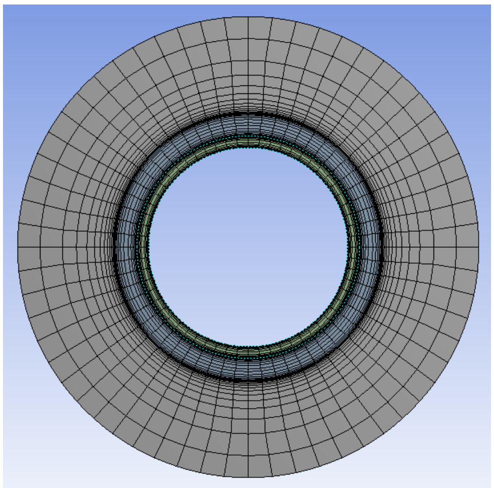

Figure 2. Mesh of thermal simulation - top view

generation at inner face of the disk along a period of time since the beginning of phenomena until the layers reach thermal equilibrium. Dimensionless heat generation in function of time is shown by Fig. 3

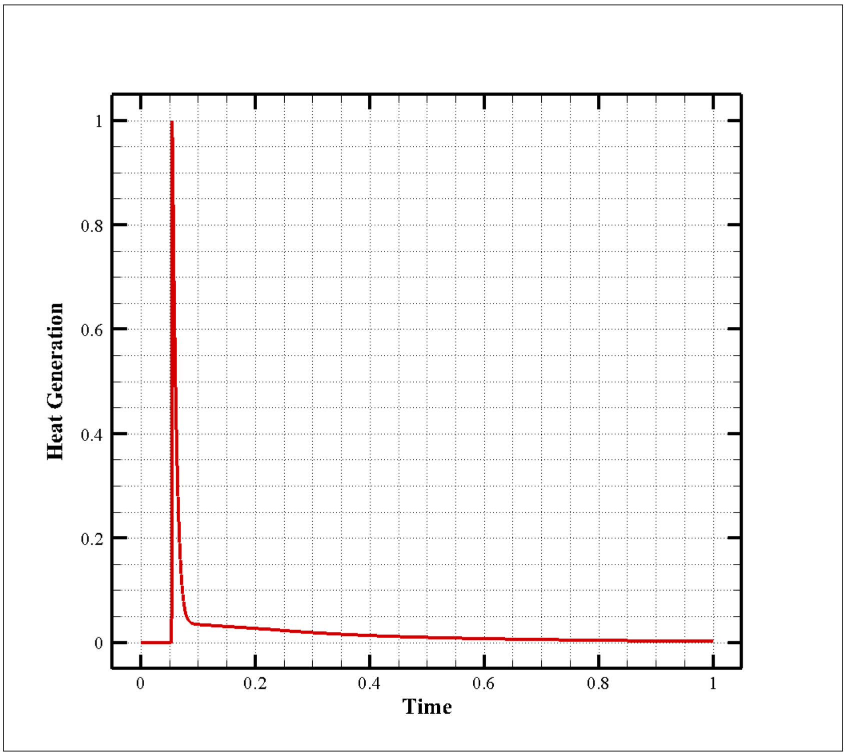

Figure 3. Dimensionless heat generation

Figure 3 represents a heat generation of $\dot{q}$ MW/m^3. The present work also works with generations of $1.5\dot{q}$ MW/m^3 and $2\dot{q}$ MW/m^3 to compare their effects on the structure. Both generations have the same behavior as observed in Fig. 3.

Structural analysis starts with additional properties such as the linear coefficient of thermal expansion, Young's modulus, Poisson's coefficient. These properties are placed in Table 3.

Mesh validation is presented by Table 4 by taking tension values at inner face of disk. It is important to point out the importance to do it on structural simulation even the mesh is same one used at thermal case because there is a difference between results as can be observed on it

A visualization of the mesh used at structural analysis is presented at Fig. 4. In a comparison with

Table 3. Structural properties

Materials	Linear Thermal Expansion ($^{o}C^{-1}$)	Young's modulus (GPa)	Poisson's Coefficient
Steel	1.2E-05	200	0.3
Cement	1.0E-05	30	0.18
Rock structure	7.5E-06	58.6	0.28

Table 4. Mesh validation - structural analysis

Nodes	Tension inner face (GPa)	Error %
17000	7.6687	-
25500	7.6608	0.103
34000	7.6918	0.301

mesh represented at Fig. 2, it is possible to realize both meshes are equal. This occurs because in thermal stresses analysis is important to capture not just the values of temperature, but it is necessary creating a nodal correspondence to capture the numerical solution from the temperature field in a specific node and, with this, calculate thermal stresses at the same point. Without this correlation, the results will be uncorrected with no physical validation.

Some conditions are needed to be imposed at this simulation to evaluate stresses and displacements at radial direction. Considering a part of a multilayer hollow disk with small thickness, upper and bottom faces are fixed to block movements at longitudinal direction and some rotation between the layers. If these actions are not implemented in the simulation, it will not represent precisely the real conditions observed in an oil well.

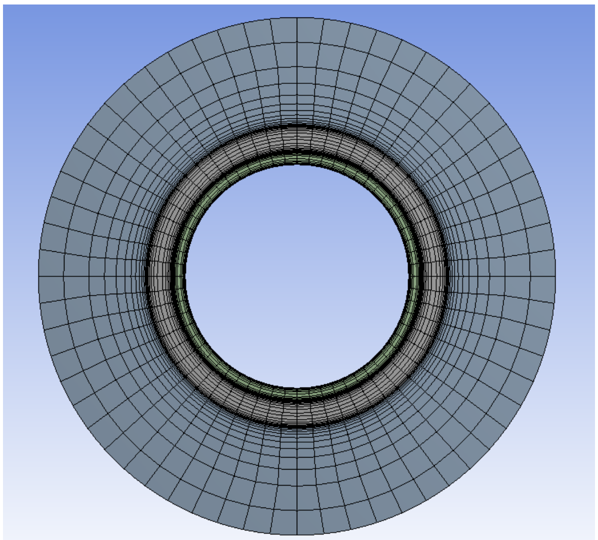

Figure 4. Mesh of structural simulation - top view

3 Results and discussion

Results of strain and stresses are plotted at Fig. 5 and Fig. 6. These results were taken at last step of the simulation to visualize what happened after the exothermic reaction of thermite.

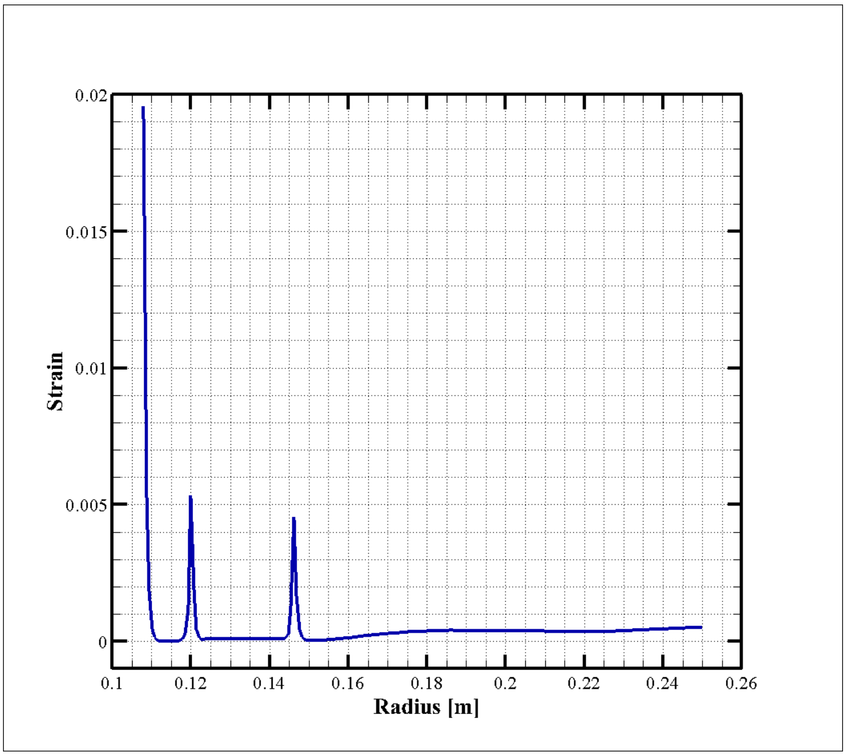

Figure 5. Variation of strain - $\dot{q}$ heat generation

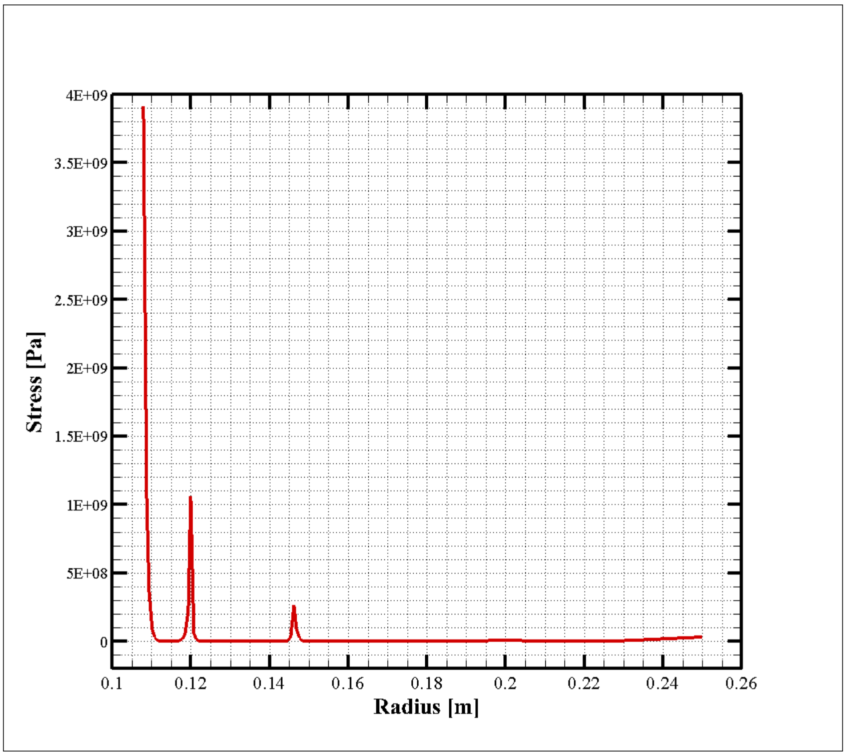

Figure 6. Variation of stress - $\dot{q}$ heat generation

It is interesting to point out some information from Fig. 5 and Fig. 6. First one is the fact that there is a huge deformation and stress at the inner face of the cylinder. This can be explained because it is the region where there is the first contact between heat generation and the multilayer disk, inducing high temperatures at steel layer.

Another consideration is about the contact region between steel-cement and cement-rock structure layer. Both have similar values of strain as shown by Fig. 5, being that steel-cement interface has a little bigger value than other one. When the comparison is made between stresses as shown by Fig. 6, there is difference. Steel-cement interface has more tensions between them compared with cement-rock structure interface. This fact can be explained by the Table 1 where is possible to check the thermal conductivity of these materials. There is a huge difference between them, which impacts the heat flux through them, concentrating heat at the steel layer, which increases the temperature of it. With this situation, steel layer tries to expand, but the cement layer acts as a block foreclosing the movement.

Stresses observed at cement-rock structure interface have the same origin of the steel-cement, but the values are smaller. Cement has a low value of thermal conductivity which affects the heat flux and, accordingly, the temperatures are lower compared with ones taken at steel-cement interface. With smaller temperatures, thermal stresses are lower at this interface.

A comparison between heat generations is presented by Fig. 7, Fig. 8, Fig. 9 and Fig. 10. First two figures are a comparative of strain and stresses observed with heat generations of $\dot{q}$ MW/m^3 and $1.5\dot{q}$ MW/m^3. Last ones compare $\dot{q}$ MW/m^3 and $2\dot{q}$ MW/m^3 heat generation results.

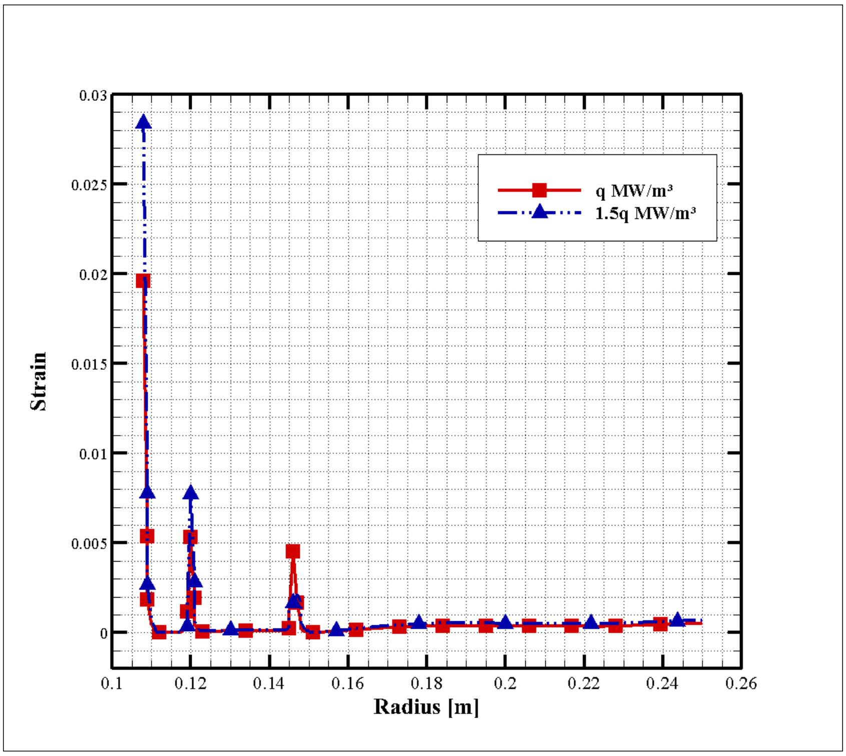

Figure 7. Strain comparison - $\dot{q}$ MW/m^3 and $1.5\dot{q}$ MW/m^3 heat generation

Firstly, observing Fig. 7 there are some interesting conclusions. At the steel-cement interface, displacement is bigger in a case with $1.5\dot{q}$ MW/m^3, but at the cement-rock structure interface occurs the opposite situation, with a bigger displacement with $\dot{q}$ MW/m^3 heat generation. A similar situation occurs with tension comparison as described by Fig. 8, with a increment of tensions at the steel-cement layer prompted by a high heat generation and almost no difference of tensions at the cement-rock layer. With this, is possible to realize a small effect of the heat generation increment at last layer because there is a concentration of thermal energy at steel caused by low conductivity of cement, which increases temperature and thermal tensions at this part. The insulating behavior of cement do not allow a efficient heat conduction throw the disk even with increment of heat generation.

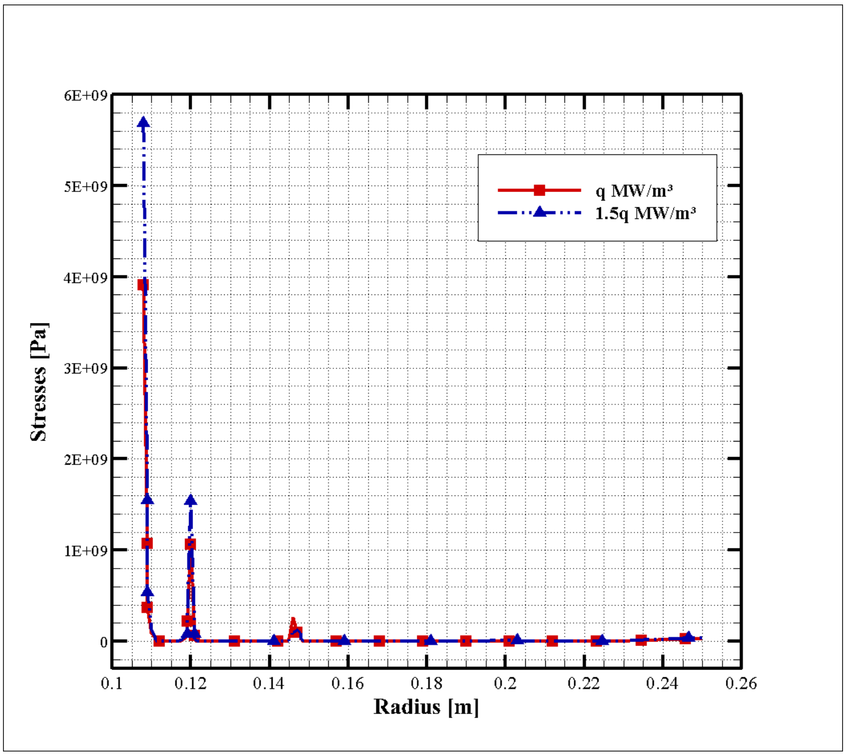

Figure 8. Stress comparison - $\dot{q}$ MW/m^3 and $1.5\dot{q}$ MW/m^3 heat generation

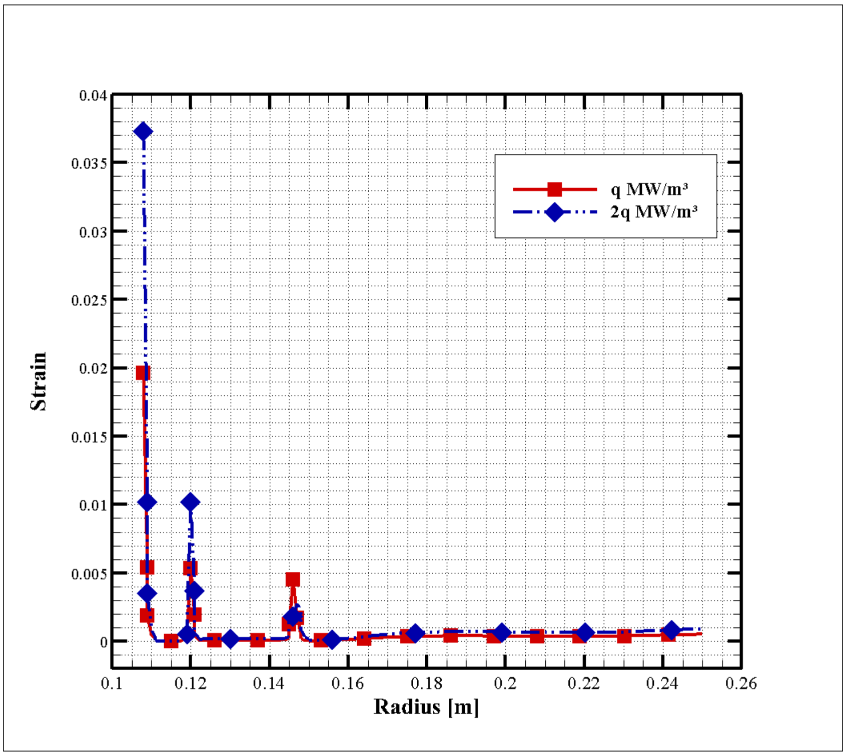

Figure 9. Strain comparison - $\dot{q}$ MW/m^3 and $2\dot{q}$ MW/m^3 heat generation

Checking Fig. 9 and Fig. 10 it is visualized the same phenomena. Increment of strain and stresses at inner and steel-cement layer, but no change at cement-rock layer due the low thermal conductivity of cement. There is no effective effect caused by variation of huge heat generation applied at these simulation conditions.

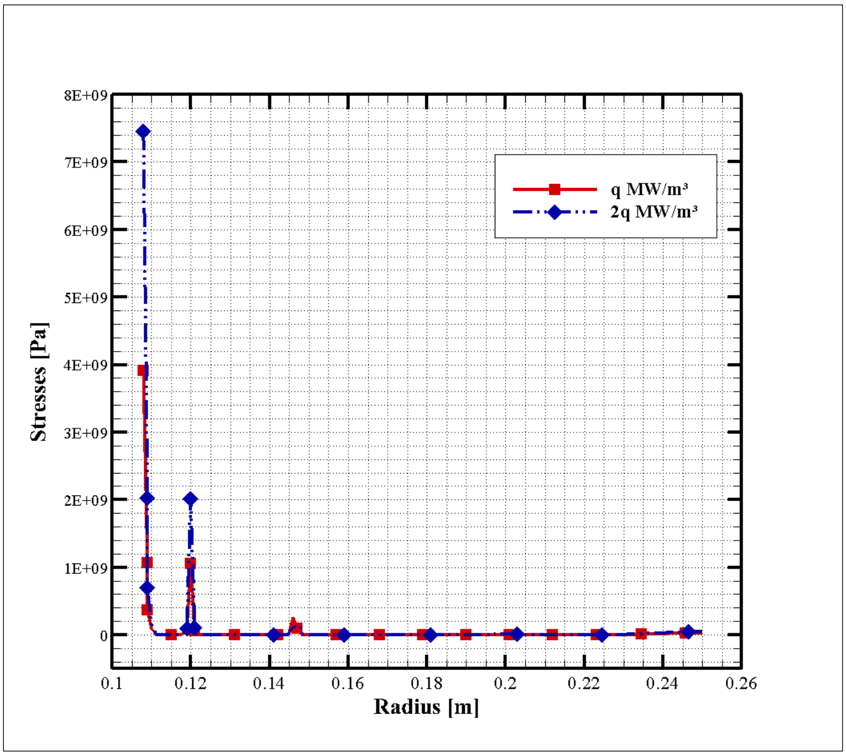

Figure 10. Stress comparison - $\dot{q}$ MW/m^3 and $2\dot{q}$ MW/m^3 heat generation

4 Conclusions

Results of Fig. 6 show a huge tension at the inner layer which value is near 400 MPa at the end of simulation. Based on information provided by Callister et al. [7] and Beer and Johnston Jr [15] it may be pointed out some concerns about cracks that can occur due creep because steel layer is under a huge temperature field, for a long period of time, caused by thermite reaction and the thermal field can induce stresses at material. A situation of steel under huge temperatures and under long time load application due thermal stresses can cause permanent deformation or rupture of structure below ultimate tensile strength. This situation is potentiality higher when there is a increment of heat generation as showed by Fig. 8 and Fig. 10.

Another concern is because of crystal structures from materials. Mechanical properties of steel are directly correlated with microstructure and they can be changed due variation of temperature. Abrupt increment followed by quick cooling can induce internal tensions at microstructure that can reduce ductility and further fractures. A situation of huge temperature during a long time at steel alloys is able to cause permanent changes at them which cannot return to their state before thermite reaction.

Steel-cement interface has a considerable tension between layers reaching almost 110 MPa at the end of analysis as presented by Fig. 6. As proposed by Odelson et al. [16], cement is very sensitive with variation of temperature and can lose strength in a situation of heat increment because also occurs an alteration of microstructure due escaping water. High temperatures at cement reduce Young's modulus and, accordingly, compressible ultimate strength, facilitating fractures and cracks due the stresses. Even low thermal conductibility of cement do not allow a great increment of its temperature compared to steel, its contact with hot medium is enough to induce this situation. And, as showed by Fig. 8 and Fig. 10, reduction of cement strength is bigger with increment of heat generation.

Cement-rock structure interface also needs a special attention. Even the fact cement blocks heat flux due low conductivity, heat increment is checked and thermal stresses can be induced. Figure 6 shows a tension of almost 22 MPa. It is a small value compared with others, but rock structures has problems when temperatures increase because there is an alteration of proprieties that reduces strength and can cause fractures. Jaeger et al. [17] describes how heat affects rock structures reducing dramatically strength. A interesting point in this situation is the fact that strain at these structures increases with reduction of ultimate tensile strength and these results of stresses and strain has a relationship with confining pressure of these rocks, which is a point of investigation.

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