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ROBUST MULTIOBJECTIVE OPTIMIZATION OF REINFORCED CONCRETE FRAMES USING RELIABILITY CONSTRAINTS

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Abstract. In most structural engineering designs deterministic models are used with requirements defined by standards, specifically, designs of reinforced concrete structures in Brazil are based on the NBR 6118:2014. This standard recommends the use of safety coefficients, without the consideration of the uncertainties associated to concrete structures designs. On the other hand, the Reliability-Based Robust Design Optimization (RBRDO), that will be studied in this paper, is characterized by optimization problems where the design uncertainties are treated statistically, allowing to measure the level of structural reliability and performance variability. In this type of problem, some constraint associated with the probability of failure is present in its formulation. The method to obtain the failure probability in this work will be FORM (First Order Reliability Method). Robust optimization problems aim to obtain a robust design, which in addition to a good performance, a low sensitivity to uncertainties of the problem are intended. The robustness measures employed in this work are mean and the standard deviation of functions of interest. This leads to a Robust Multiobjective Optimization (RMO) problem, due to multiple objectives, which has several optimum solutions called Pareto points. The main aim of the present research is to develop a computational tool to efficiently obtain robust optimum pareto designs of reinforced concrete framed structures under uncertainties. Such optimum pareto points will be found through Weighted Sums (WS), Min-Max e Normal Boundary Intersection (NBI) methods that were implement in the Python language. In addition, it will be used pre-existing in-house finite element libraries, which will be the method used for structural analysis. The reliability and optimization public libraries, also in the same language, will be considered. The applications in this work, is two 2D frames of reinforced concrete with one and three floors, with respectively 3 and 30 bars.

Keywords: Robust optimization, Reinforced concrete frames, Reliability

1 Introduction

Currently, in structural engineering designs deterministic models are used with requirements defined by standards, which uses semi-probabilistic methods, applying safety factors that increase the solicitations and the minors for the resistances. Such approach is also commonly employed when optimization procedures are used to obtain the optimum design.

This methodology simplifies the calculations because it is practical and objective, however, it does not allow to measure the degree of reliability of the sections or the structure as a whole. In turn, Reliability-Based Design Optimization (RBDO) and Reliability-Based Robust Design Optimization (RBRDO or R²BDO) are characterized by an optimization problem where design uncertainties are statically addressed, allowing to measure the degree of structural reliability. In RBDO, some constraints associated with the failure probability or reliability index is present in its formulation. Moreover RBRDO, provides a project with good performance and reliable, has a low sensitivity to the uncertainties of the problem.

Some authors who studied methodologies for the application of reliability-based design optimization can be cited: Liu and Kiureghian [1] who evaluated the efficiency and robustness of five optimization methods involving analyzes by the finite element method, it is worth mentioning that for optimization was used the Sequential Quadratic Programming (SQP) method, which proved to be very practical and efficient; Almeida [2] applied the probabilistic analysis optimization to reinforced concrete frames structures and compared it with the deterministic optimum, and declare to be the first to apply the SQP to such optimization, besides incorporating the physical and geometric nonlinearity to the problem; Andrade [3] who applied stochastic optimization to planes and spaces structures, following the normative prescriptions of NBR 6118, however, limiting himself to the analysis of a single floor separately; Motta [4] and Motta [5] presented an analysis of 2D and 3D trusses and frames, and plates applied to robust optimization; and finally, Alves [6] who used the reliability-based design optimization applied to 2D and 3D reinforced concrete frames, considering geometric nonlinearities and a simplified approaches to consider physical nonlinearities.

Moreover, it is known that many real engineering problems have more than one goal to be optimized, so this work will also apply Multiobjective Optimization (MO) techniques, where the proper approach to problem solving is constituted of a class of strategies based on the called Pareto concept. NBI ("Normal Boundary Intersection") will be used, an efficient algorithm developed by Das and Dennis [7] that achieves efficient Pareto point distributions for bi-objective problems. The results will also be compared with the classical approaches: Weighted Sum (WS) method and Min-Max method. Also, as a reference for multiobjective optimization, there is Motta [8].

The main goal of the present research is to develop a computational tool to efficiently obtain robust optimum pareto designs of reinforced concrete framed structures under uncertainties The goal this work is to compare the three distinct multiobjective approaches by applying the robust methodology to two reinforced concrete structures. The first structure being a frame 2D with one floor and a uniformly distribute loading and the second being a frame 2D with three floors and also with uniformly distributes loadings on its beams. Details of such structures will then be presented.

For reliability constraints, the First Order Reliability Method (FORM) was used to find the reliability index. For structural analysis was used the finite element method (MEF), with a code implemented by Alves [6] in Python 2.7 language, plus own code for MO methods and pre-existing libraries for reliability analysis (PyRe) and optimization (SciPy.optimize), in the same language.

2 Structural analysis

The cases studied in this work are small size frames 2D with one and three floors, and have vertical uniformly distributes loads only, which, consequently, make them relatively small displacements, allowing the considering of linear analysis of reinforced concrete structures, according to NBR 6118: 2014. The finite element method with linear analysis was used, where, for the basic element (linear bar elements), the assumptions of the beam elements presented by the Euler-Bernoulli theory, which considers that the cross sections remain planes after deformation and therefore do not consider any deformation due to stress shear, were also considered.

In summary, this type of analysis search to find the behavior of structural elements, which is characterized by nodal displacements of the system due to the action of point loads applied to these nodes. The other loads are simulated through equivalent nodal loads that have the same effect on the structure as the original load. Once these displacements are calculated, it is possible to obtain the supporting reactions and internal efforts in the elements.

To perform the structural analysis in this work was adopted a code presented by Alves [6] developed in Python and validated with several examples presented by Logan [9] and by some commercial structural analysis software.

It was decided to use the mentioned code because the process of reliability analysis and optimization can be very computationally expensive, as a consequence, the structural analysis should be as fast as possible, and the procedure for calculating the internal displacements and efforts in this code generated a significant gain in processing time, as well as being very versatile and can be easily adapted to solve various types of structures.

3 Reliability analysis

The normative prescriptions currently used for reinforced concrete structure projects are based on semi-probabilistic analysis. This approach, however, becomes questionable when it comes to optimized designs, for example, because they tend to be located around constraints where minor disturbances may lead to violation of design constraints (MOTTA [5]). Therefore, the reliability analysis, which allows the evaluation of probability of structural failure associated with the design criteria, has been gaining ground.

The probabilistic analysis of a structure is based on the idea that there is always a probability that it will fail, either by rupture of the component materials or by failing to meet the normative requirements for maximum deformations, crack openings, etc., once it takes into account the uncertainties inherent in materials and their strengths, loads and their variability, and structure geometry. This probability can be roughly quantified by the reliability index (β).

For the reliability analysis it is necessary to define the stochastic variables, also called random and their distribution type. In Brazil, there are not studies that analyze the most appropriate probability distributions yet, however, there are several studies published outside on this topic. Therefore, this paper will follow the premises presented by the Joint Committee on Structural Safety (JCSS) [10]. In addition to random variables, it is also necessary to define the failure function.

Each probability distribution has its Probability Density Function (PDF) which, together with a set of random variables ***x*** and a failure function *G(**x**)*, can define the failure probability (P_f) as:

$$P_f = \int_F f_x(\mathbf{x})dx \therefore F = \{\mathbf{x} : G(\mathbf{x}) < 0\} \tag{1}$$

where F is the failure region of the structure or structural element and f_x is the PDF of the failure function, unknow a priori.

Because it is a robust optimization, random samples are generated based on the distributions of each random variable to calculate the *robustness measures* that are the means and standard deviations in the objective functions. Samples with size N = 100 were chosen to obtain the mean and standard deviation of the functions of interest. After a parametric study, this sample size was chosen because it gives good results with a lower computational effort. In summary, the arithmetic mean (M_a) and standard deviation (SD) of the function of interest, e.g. the cost or the maximum displacement, are calculated through the Equations (2) and (3).

$$M_a = \frac{x_1 + x_2 + \ldots + x_N}{N} \quad . \tag{2}$$

$$SD = \sqrt{\frac{\sum_{i=1}^{N}\left(x_i - M_a\right)^2}{N}} \quad . \tag{3}$$

where x_i are the values of the function of interest evaluated for the sample points. It is important to mention that for gradient based optimization process the seed, of the random number generator, must be fixed for smooth statistics responses.

For the calculation of the reliability index in each constraint was used the First Order Reliability Method (FORM) which will be presented below.

3.1 FORM

To avoid the use of the numerical integral presented in Eq. (1), the first order reliability method (FORM) was used, which uses the iterative technique. It is said of the first order because the method makes a linear approximation of the failure function at the point of greatest probability of failure.

The FORM transforms the distributions of the variables involved into standardized normal distributions whose mean is zero and the standard deviation is unitary. This causes the initial problem to become an equivalent problem in a reduced standard space.

The space corresponding to the standardized stochastic variables has radial symmetry, as seen in the concentric circles of Fig. 1. The reliability index (β) is then defined in this subspace as the shortest distance between the failure surface ($G(x) = 0$) and the origin of the reduced space coordinate system.

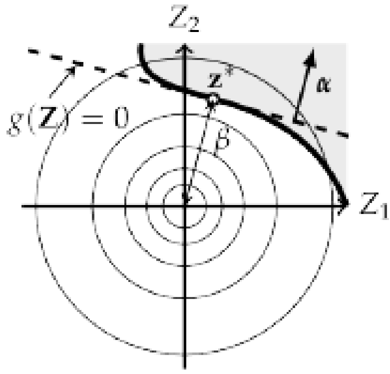

Figure 1. Subspace of standardized random variables

The failure probability is calculated by Eq. (4).

$$P_f = \phi(-\beta) \tag{4}$$

where Φ is the cumulative probability density function for a standard random variable. Details of the procedure can be found in Melchers and Beck [11] or Motta [4].

In this work was adopted the PyRe (Python Reliability) library, developed in Python 2.7 language, to use FORM in reliability analysis. This library was chosen because of its ease and versatility, and was properly tested and validated by Alves [6].

4 Multiobjective Optimization

Many real engineering problems have more than one goal to optimize, and several criteria to meet, these problems are called Multiobjective Optimization. The mathematical formulation of the optimization problem is to find a set of $\boldsymbol{n}$ design variables contained in a vector $\mathbf{x}$, such that:

$$\begin{aligned} &\text{Minimize } \mathbf{F}(\mathbf{x}) \\ &\text{Subject to: } g_i(\mathbf{x}) \geq 0 \qquad i = 1, 2, ..., m \\ &\qquad\qquad h_k(\mathbf{x}) = 0 \qquad k = 1, 2, ..., n \\ &\qquad\qquad \mathbf{x_L} \leq \mathbf{x} \leq \mathbf{x_U} \end{aligned} \tag{5}$$

where:

$$\mathbf{F}(\mathbf{x}) = [\, f_1(\mathbf{x}),\, f_2(\mathbf{x}),\, f_3(\mathbf{x}), ...,\, f_{nobj}(\mathbf{x})] \tag{6}$$

is the objective function vector, the vector components $\mathbf{x}$ are the design variables, $g_i(\mathbf{x})$ and $h_k(\mathbf{x})$ are the inequality and equality constraint functions, respectively, and the vectores $\mathbf{x_L}$ and $\mathbf{x_U}$ are, respectively, the lower and upper limits of the design variables. And inside g_i are the constraint functions related to reliability analysis of the failure functions (G), that depends on the reliability index.

The methods presented here for solving such problems consist in transforming the problem with several objectives into another with only one objective. For scalar optimization, it was applied the Sequential Quadratic Programming (SQP) method, which approximates the problem to a sequence of convex quadratic subproblems. This approximation is repeated repeatedly until the values of the design variables converge, which means that the difference between two consecutive iterations is smaller than an adopted value (tolerance). All this procedure can be found in detail in Motta [5], and this work was performed automatically within the minimize function present in the Scipy library.

Usually, it is not possible to find a project that is ideal for all objectives instead there are several solutions to the problem, each representing a relationship between the objectives. To find these solutions the concept of Pareto can be employed. Pareto points have the property that when moving in the decreasing direction of a function, the other functions have their value increased (MOTTA [5]) or kept.

The Pareto front contains the points that represent an optimal compromise (trade-off) between the respective evaluations of objective functions. Figure 2, taken from Bates [12], shows this concept, highlighting Pareto solutions and discarded inferior solutions.

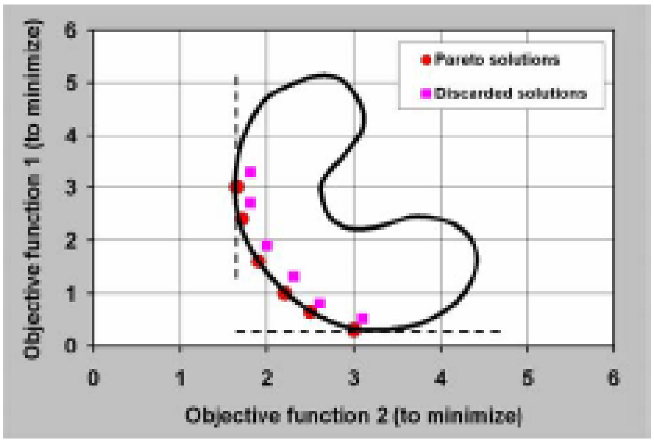

Figure 2. Pareto solutions for multiobjective optimization

There are several methods to obtain these points, the following will be studied in this work: Weighted Sum (WS), Min-Max and the Normal Boundary Intersection (NBI) methods, which will be described below.

4.1 WS method

This is the most employed method due to its simple use. The Weighted Sum Method (WS) is based on minimizing the sum of objective functions, normalized and weighted by a weighting coefficient vector B_j, and repeating for different weights. Thus, the problem becomes a single objective function, algebraically represented by:

$$F = B_j^T \frac{\mathbf{f}}{\mathbf{f_0}} = \sum_{k=1}^{nobj} B_{j,k} \frac{f_k}{f_{0k}} . \tag{7}$$

Where the elements of $B_{j,k}$ are normalized as follows:

$$\sum_{k=1}^{nobj} B_{j,k} = 1, 0 \le B_{j,k} \le 1 \tag{8}$$

and f_{0k} is the objective function *k* in the initial design.

Each B_j different gives a Pareto point, but problems may arise when the viable region contour in the objective function space is non-convex, as shown in Fig. 3. Where it is not possible to find a solution that is in the non-convex region.

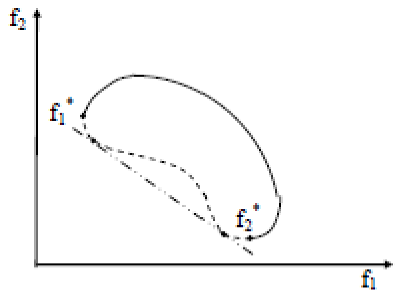

Figure 3. Viable region in objective function space

Typically, this methodology does not provide uniform Pareto points for uniform weight distribution B.

4.2 Min-Max method

A method based on the weighted sum method, the Min-Max method was created to minimize the problem of obtaining uniformly distributed points, differing in the normalization of objective functions, found in Hwang et al. [13].

To normalize the objective functions will require two more parameters: $\max f_k$ and $\min f_k$, which are obtained through individual optimizations solutions of isolated objective functions. It apply the variables set x_k^*, resulting from each optimization *k* alone, to each objective function and then find the maximum value of function ($\max f_k$) and the minimum value ($\min f_k$).

The normalized objective functions will be:

$$\overline{f_k} = \frac{f_k - \min f_k}{\max f_k - \min f_k}, k = 1, ..., nobj . \tag{9}$$

If $\max f_k = \min f_k$ for some objective *k*, this can be disregarded.

Then the following problem is proposed:

$$\min(\gamma) \tag{10}$$

where

$$\gamma = \max(B_k \overline{f_k}), k = 1, ..., nobj \tag{11}$$

subject to the same constraints as Eq. (5), in addition to the following restrictions:

$$B_k \overline{f_k} \leq \gamma \text{ para } k = 1, ..., nobj \, . \tag{12}$$

Solving this problem for various sets of vectors ***B***, a new optimization subproblem is formulated, so a new Pareto point is found.

4.3 NBI method

The Normal Boundary Intersection (NBI) method was introduced by Das and Dennis [9] in order to find efficient contour points in the objective functions space (viable space) that allow the construction of a smooth curve. When the points are over a sufficiently convex contour part, these are Pareto points, but when they are in a concave part, there is no assurance that they are Pareto points, however, they contribute to the definition of the Pareto front.

Details of the methodology can be found in the references Motta [8] and Das and Dennis [9], but in summary, first the minimum local vector of objective functions, represented in Eq. (13), must be found.

$$\mathbf{F}^{*} = [\, f_1^{*}, f_2^{*}, ..., f_k^{*}, ..., f_{nobj}^{*}]^T \tag{13}$$

where each f_i^{*} represents an individual local minimum.

After, the points of Convex Hull Individual Minima (CHIM) are defined by the convex combinations of $\mathbf{F}(\mathbf{x}_i^{*}) - \mathbf{F}^{*}$, stored in the matrix form, **Φ**, called "pay-off". So, the CHIM will be:

$$\left\{ \mathbf{\Phi B} : \mathbf{B} \in \Re^{nobj}, \sum_{i=1}^{nobj} B_i = 1, B_i \geq o \right\} \tag{14}$$

where

$$\Phi_{i,j} = f_i(x_j^{*}) - f_i^{*}, i = 1, ..., nobj; \, j = 1, ..., nobj \, . \tag{15}$$

The NBI method aims to find part of the contour *δf*, as illustrated in the example shown in Fig. 4, which contains Pareto points, from the intersection of the quasi-normal line to the CHIM, pointing to the origin, whose line is defined from the midpoint of the CHIM as shown in Eq. (16).

$$n_i = \frac{1}{nobj} \sum_{j=1}^{nobj} \Phi_{i,j} \, . \tag{16}$$

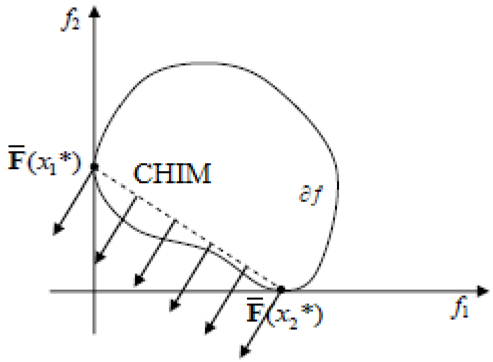

Figure 4. The viable set image about the mapping of *f* into the objective functions space

Then, $\mathbf{\Phi B}+t\mathbf{n}$, with $t \in \Re$, represents the set of points on $\mathbf{n}$, which form a quasi-normal line to the CHIM. And mathematically, to find the intersection of the quasi-normal line to the ECMI and the contour that defines the space (*δf*), one must solve the following problem:

$$\max_{\mathbf{x},t} t \tag{17}$$

subject to the constraints of Eq. (5), plus the following constrain:

$$\mathbf{\Phi B}+t\mathbf{n} \geq \overline{\mathbf{F}}(\mathbf{x}) \tag{18}$$

where $\mathbf{F}(\mathbf{x})$ was replaced by $\overline{\mathbf{F}}(\mathbf{x}) = \mathbf{F}(\mathbf{x}) - \mathbf{F}^*$, as it was considered that at the origin is the utopia point $\mathbf{F}^*$ and thus all functions are non-negative.

5 Case studies

Two examples of 2D reinforced concrete framed structure will be studied, where the three mentioned multiobjective optimization methods were applied, to obtain robust optimum results, and the results obtained by those methods compared. To applying the robust methodology the mean and standard deviation measures that were calculated as shown in Section 3, respectively Eq. (2) and Eq. (3) were used.

5.1 Frame with one floor

It is a three bar frame with uniformly distributed loading applied along its beam, as shown in Fig. 5, and was taken from Coêlho [14], who in her work presented the analysis of this frame by sundry optimization methods and with several values for α. Here RBRDO will be used, as already mentioned, and only for α = 1 and L = 5.5 meters.

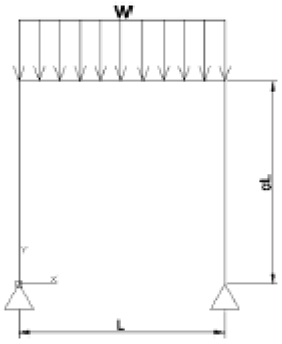

Figure 5. Three bar frame

Reliability. The default value for the reliability level recommended by JCSS [10] has been assumed for the ultimate limit state check of most new structures, β = 4.2.

Design variables. The dimensions of the columns (x_1) and beam (x_2) were adopted as the *continuous* design variables of this problem which have boundaries between 0.01 m and 1 m. Note that the sections of both elements were considered square.

Objective functions. One of the objectives is to decrease the mean of the total concrete volume of the structure, the function being defined as: $f_{obj1}(\mathbf{x}) = mean(2\alpha L\mathbf{x}_1^2 + L\mathbf{x}_2^2)$. The second objective is to minimize the standard deviation of the maximum displacement that occurs in the middle of the beam span. Such deformation is calculated by the finite element method, through the library implemented by Alves [6].

Random variables. The list of all random variables is shown in Table 1.

Table 1. Random Variables from Example 1

Variable	Symbol	Probability density function	Unit	Mean	Standard deviation	V
Column dimension	x_1	Normal	m	Design variable	-	0.025
Beam dimension	x_2	Normal	m	Design variable	-	0.025
Load	w	Lognormal	tf/m	1.5	2.25	1.5
Resistent stress	σ	Lognormal	kgf/cm²	9100	1547	0.17
Span	L	Deterministic	m	5.5	-	-
Width/height ratio of the frame	α	Deterministic	-	1.0	-	-

Constraints. The constraints adopted, in addition to the constraints required for each multiobjective optimization method, have the format of Eq. (19) and depend on the evaluation of the limit state functions of Eq. (20), which represent the maximum stress at the top of the column and the maximum tensions of the beam in the meeting with the pillar and in the middle of the span, to obtain the reliability indexes. The variables *Ni* and *Mi* of Eq. (20) are, respectively, the normal forces and bending moments acting on the top of the pillar (1), the beam support (2) and the middle of the span (3).

$$g_i^T(\mathbf{x}) = \beta_i - \beta_{alvo}(\mathbf{x}) \geq 0, i = 1,2,3. \tag{19}$$

$$G_i(x) = \sigma - \left(\frac{N_i}{x_1^2} + \frac{M_i\left(\frac{x_1}{2}\right)}{\frac{x_1^4}{12}} \right), i = 1,2,3. \tag{20}$$

Results. The problem was solved by the three multiobjective optimization methods presented, with 11 Pareto points for each method, the values found are in Table 2 and represented in Fig. 6.

Table 2. Example 1 results

WS		*Min-Max*		*NBI*	
Mean(V) (m³)	Sdt(δ_{max}) (m)	Mean(V) (m³)	Sdt(δ_{max}) (m)	Mean(V) (m³)	Sdt(δ_{max}) (m)
1650.0290	2.1005e-09	1238.6877	2.6415e-09	1650.0290	1.7992e-09
40.3228	2.1762e-07	66.0204	9.1819e-08	1320.5223	2.0452e-09
29.2872	3.4966e-07	47.8807	1.4606e-07	991.0667	2.8265e-09
23.7266	4.7998e-07	38.7503	1.9915e-07	661.6977	4.5132e-09
19.9893	6.2291e-07	32.6291	2.5686e-07	335.5219	3.9620e-08
17.0959	7.9189e-07	27.9058	3.2443e-07	30.0089	2.9098e-07
14.6343	1-0075e-06	23.9030	4.0965e-07	6.1357	3.4899e-06
12.3661	1.3112e-06	20.2323	5.2783e-07	4.0630	6.9170e-06
10.0834	1.8106e-06	16.5616	7.1795e-07	3.1998	1.0357e-05
7.4382	2.9513e-06	12.3502	1.1345e-06	2.7050	1.3800e-05
2.3779	1.9929e-05	2.3779	1.7246e-05	2.3779	1.7246e-05

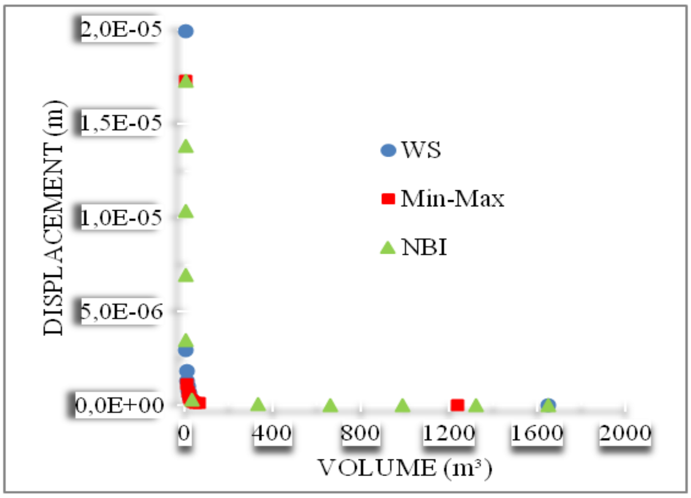

Figure 6. Pareto points for frame 1

The points found by the weighted sum method were concentrated in a single region because, as stated, the method does not provide a uniform distribution of points. As with the Min-Max method which, in this example, also did not provide a proper distribution of points. However, when considering NBI more evenly distributed points were obtained, this method allows regular distribution of Pareto points for even a small set of vectors of the β parameter already mentioned, regardless of the number of objective functions.

It can be observed that the Pareto points form a curve of approximately 90 degrees very close to the axes, that is, small variations of displacement up to an order smaller than 1e-08 cause a large volume variation. It can also be said that the global optimum is located at the point closest to the origin, in the region where the weighted sum method points were located.

The Table 3 shows comparative values between the methods, where the number of iterations and the number of functions evaluations of the optimizations are found. In that table, each function evaluation represents the objective functions and the constraint functions evaluation, for a given design point, i.e. the statistic calculation (for 100 random points) and one reliability analysis for each constraints. It can be concluded that the NBI, despite being the costliest, obtain better distributed points and is the most complete method. On the other hand, the weighted sum method has the smallest number of iterations and the smallest number of function evaluations.

Table 3. Method comparison in example 1

Method	Iterations	Function evaluations
WS	110	457
Min-Max	184	748
NBI	276	1535

5.2 Frame with three floors

This second problem was also taken from the reference Coêlho [14] where was considered the deterministic optimization. It is a frame with three floors with uniformly distributed loading applied along its beams, as shown in Fig. 7.

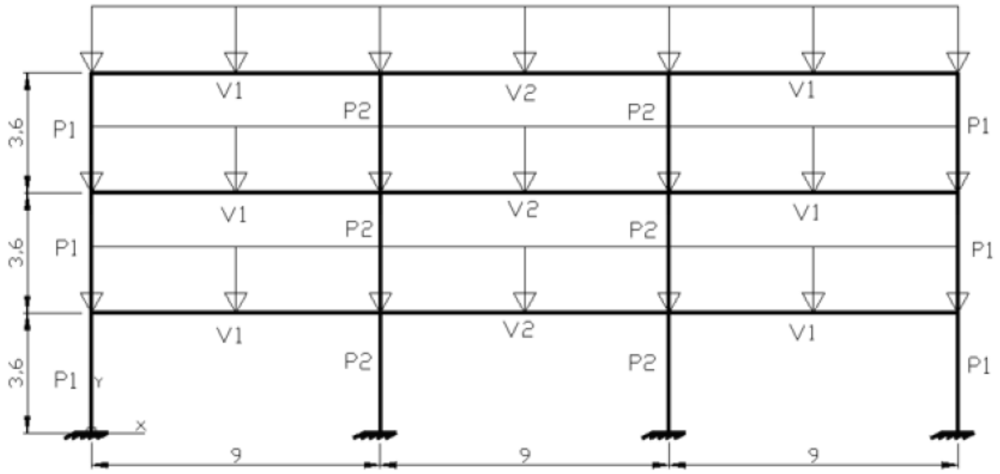

Figure 7. Frame with three floors

Two groups of beams and pillars were adopted: type 01 pillars are those in the corner and type 02 columns are in the center; the beams of the extreme spans are of type 01 and those of the central span of type 02. To obtain the efforts on the beams were discretized into two elements of the same size in each span, so the structure has a total of 30 bars (12 pillars + 18 beams).

Reliability. The default value for reliability level, recommended by JCSS [10] for the ultimate limit state check, β = 4.2, was also assumed.

Design variables. The *continuous* design variables adopted for the problem were the dimensions of the beams and columns of each group (b_{V1}, h_{V1}, b_{V2}, h_{V2}, b_{P1}, h_{P1}, b_{P2}, h_{P2}) and the steel areas of the elements (A_{sV1}^{+}, A_{sV1}^{-}, A_{sV2}^{+}, A_{sV2}^{-}, A_{sP1}, A_{sP2}). These variables have the limits indicated in Table 4.

Table 4. Limits values of design variables for example 2

Description	Upper limit	Lower limit
Beams base	20	50
Beams height	35	90
Columns base	30	60
Columns height	30	90
Steel beams area	5.67	22.81
Steel columns area	11.34	88.36

Objective functions. The first objective (f_{obj1}) is to minimize the mean of the total cost of the structure, which is calculated as shown in Eq. (21). The second objective (f_{obj2}) is to minimize the standard deviation of the maximum displacement, which is also calculated by the finite element method, through the library implemented by Alves [6].

$$f_{obj1} = mean\left(\sum_{i=1}^{n_p}(C_c\mathbf{b}_i\mathbf{h}_i + C_aW_{a_i} + 2C_f(\mathbf{b}_i + \mathbf{h}_i))L_i + \sum_{j=1}^{n_v}(C_c\mathbf{b}_j\mathbf{h}_j + C_aW_{a_j} + 2C_f(\mathbf{b}_j + \mathbf{h}_j))L_j\right). \tag{21}$$

$$f_{obj2} = \max(std(\mathbf{d})). \tag{22}$$

where:

- n_p and n_v are the numbers of pillars and beams, respectively;
- C_c = 54 USD/m³, C_a = 0.55 USD/kg and C_f = 54 USD/m² are the costs with concrete volume, steel weight and shape area, respectively, adopted as reference work;
- **b** and **h** are the vectors that contain the bases and heights of the elements respectively;
- *L* represents the elements length;
- W_a is the steel specific weight of each piece;
- **d** is the vector that contain the displacements of the points.

Random variables. The random variables adopted as well as their respective distributions and related parameters are presented in Table 5.

Table 5. Example 2 stochastic variables

Variables	Unit	PDF	Mean	*V*
Beams base	cm	Normal	Variable	0.025
Beams height	cm	Normal	Variable	0.025
Columns base	cm	Normal	Variable	0.025
Columns height	cm	Normal	Variable	0.025
As positive	cm²	Deterministic	Variable	-
As negative	cm²	Deterministic	Variable	-
As columns	cm²	Deterministic	Variable	-
Concrete strength	MPa	Lognormal	39.38	0.10
Steel strength	MPa	Lognormal	491.2	0.05
Steel modulus of elasticity	GPa	Normal	210	0.05
Permanent load	kN/m	Normal	16.5	0.04
Overload	kN/m	Normal	-6.84	0.10

Constraints. In addition to the restrictions for the multiobjective optimization methods, the other constraints have the same format as Eq. (19), however with $i = 1, 2, ..., 8$, being associated the limit state functions shown in Eq. (23), which refer to the moments (positive and negative) and resistive shear forces on the beams and the resistive bending moments on the pillars. The ultimate limit state has been verified.

$$G_i(\mathbf{x}) = 1 - \frac{S_{Sd}(\mathbf{x})}{S_{Rd}(\mathbf{x})} \geq 0, i = 1, 2, ..., 8\,. \tag{23}$$

Where S_{Sd} refers to the requesting efforts of sizing and S_{Rd} resistant design efforts.

Results. Similarly, this problem was solved by the three multiobjective optimization methods presented, with 11 Pareto points for each method. It was found the values in Table 6 which are illustrated in the graphs in Fig. 8.

Table 6. Exemple 2 results

WS		Min-Max		NBI	
Mean*(Cost)* *(USD)*	Sdt($\delta_{\max}$) *(m)*	Mean*(Cost)* *(USD)*	Sdt($\delta_{\max}$) *(m)*	Mean*(Cost)* *(USD)*	Sdt($\delta_{\max}$) *(m)*
22504.7781	5.2880e-05	20385.2918	8.5643e-05	20385.2918	8.5643e-05
20381.2527	7.1139e-05	16543.0767	2.0173e-04	18439.3429	1.2376e-04
16377.6990	1.8269e-04	15255.6824	2.9567e-04	16461.1318	1.5647e-04
20238.6464	7.1500e-05	14431.4860	3.8947e-04	14723.6231	2.2956e-04
18173.4901	1.0370e-04	13371.9460	4.4589e-04	13231.7004	3.4387e-04
20019.0143	7.3443e-05	12767.0534	5.3029e-04	12052.3068	5.1063e-04
16255.0088	1.8940e-04	12274.1373	6.4454e-04	10696.2509	6.4779e-04
16230.1793	1.9042e-04	11902.2627	8.0111e-04	10375.6097	9.5859e-04
16173.1041	1.9281e-04	11633.5718	1.1411e-03	9705.0277	1.2107e-03
12889.0817	3.5953e-04	10921.9801	1.4414e-03	9668.0383	1.5692e-03
10231.8318	1.5287e-03	9975.1077	1.7417e-03	9519.7681	1.9090e-03

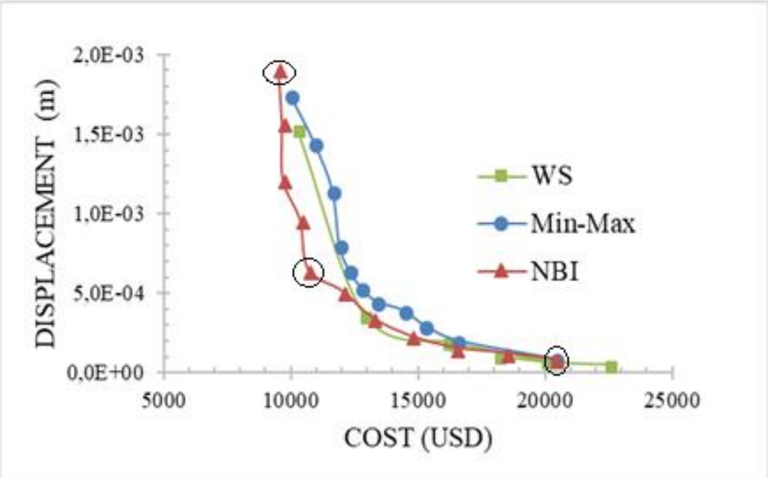

Figure 8. Pareto points for example 2/

As in the previous example, the Weighted Sum method presented points more concentrated in a single region, while the Min-Max method, in this example, was able to minimize this effect. Finally, the results of the NBI method for example 2 also obtained a more uniform distribution of points.

The comparison between the multiobjective optimization methods can be seen in Table 7. Again, each function evaluation includes the objective functions and the constraints functions evaluations as explained in previously example. The table shows that the NBI method is the costliest. However, this method obtained a smooth curve with well distributed Pareto points, which was not obtained by the other methods.

Table 7. Methods comparison in example 2

Method	Iterations	Function evaluations
WS	335	5736
Min-Max	702	14574
NBI	1109	20905

Table 8 display the number of iterations for each Pareto point in each method, where usually the points closest to the lowest cost have the largest number of iterations. This is due to the fact that the reliability constraints become actives.

Table 8. Number of iterations

Method	Lowest disp. (P1)	P2	P3	P4	P5	P6	P7	P8	P9	P10	Lowest cost (P11)
WS	48	23	13	29	36	30	10	10	07	36	93
Min-Max	10	22	13	8	24	20	49	23	326	34	173
NBI	10	17	19	35	37	66	142	274	145	106	258

It was also made the analysis of three points found by the NBI method (indicate in Fig. 8), calculating their displacement histograms, shown in Fig. 9. The points are: the minimum displacement point, the minimum cost point and an intermediate point that can be considered as a global optimum, which is in the slope change of the Pareto curve found and has low values for both cost and displacement.

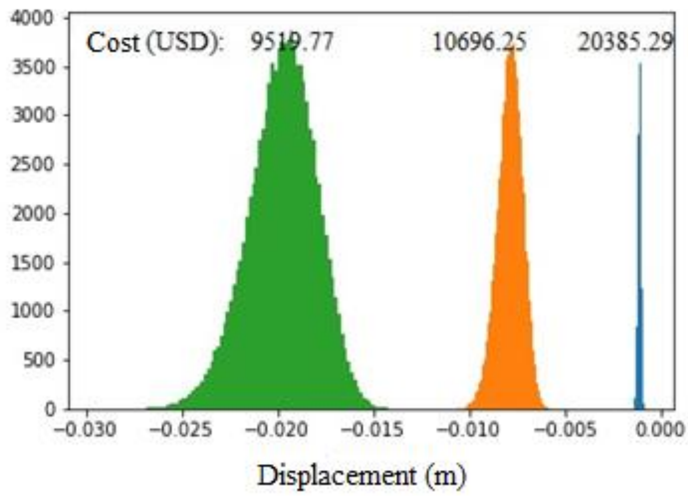

Figure 9. Histograms of displacements at three points

As can be observed that the mean and the standard deviation of the displacement are proportional, that is, the minimum point for the standard deviation has the smallest mean of the displacement (in modulus). It is also proved that the point that minimizes the cost has the largest displacement and therefore the largest standard deviation.

6 Conclusions

Two statistical objectives of reinforced concrete framework with reliability-based constraints were optimized using code developed in the Python language. The methods for obtaining the Pareto curve fulfilled their objectives by finding curves with the acceptable reliability level recommended by the JCSS [10], making possible to choose the optimal overall design, according to the needs of the designer.

From the methods used for multiobjective optimization, the NBI obtained better results, even being the costliest. It is the most complete, finding smooth curves and evenly distributed Pareto points, increasing the need for its use with increasing complexity of the problem.

The code used proved to be very efficient, obtaining coherent results, as well as being free and publicly licensed libraries, and easy to use, allowing implementation for other methods and objectives, according to the interest of other authors and future research.

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