

STRUCTURAL RELIABILITY ANALYSIS OF A REINFORCED CONCRETE BEAM STRENGTHENED WITH CARBON FIBER

Vinícius D'Agostini Pittarello

vpittarello@hotmail.com

Gipso Street, 88215-000, Bombinhas – Santa Catarina, Brazil

Flávia Gelatti

flaviagellati@univali.br

University of Vale do Itajaí

Uruguaí Street, 88302-901, Itajaí – Santa Catarina, Brazil

Alverlando Silva Ricardo

alverlando.ricardo@hotmail.com

Federal University of Alagoas

AL-145 Highway, 57480-000, Delmiro Gouveia - Alagoas, Brazil

Abstract. Concrete beams and strengthening design procedures are commonly specified in normative codes, using simplified deterministic procedures. This prescriptive methodology has as a consequence in practice in which structural safety is indeterminate, since the innumerable sources of uncertainty when designing the project can result in significant deviations from reality. As an alternative, the safety of these beams can be quantified through the application of structural reliability theory. The present work analyzes safety of a reinforced concrete beam strengthened with carbon fiber. For this, the Monte Carlo method is applied to determine the reliability index and probability of failure of the structural element. The steel reinforcement design of the beam was performed according to NBR 6118 [1], and in sequence a load increase analysis was conducted to justify the strengthening. The fiber strengthening was designed according to two methods, the available in ACI 440.2R-17 [11] and in Machado [3]. In the specific ACI method two set of material parameters were tested, the recommended on ACI 318-14 [2] and the designated on NBR 6118 [1]. As a result of the analysis with both methods it was observed that in some cases the reliability indexes were unanimously superior to the coefficients targeted. In the case of the use of parameters of the Brazilian standard in the ACI 440.2R-17 [11] design methodology, there were cases in which the same indexes were not satisfactory. Because the central verification in the ACI 440.2R-17 [11] design procedure is the steel tension, a different assumption in its yield limit changes considerably the results and the probabilistic response.

Keywords: Structural reliability, RC beams, Strengthening, Carbon fiber Reinforced Polymer

1 Introduction

In several situations, reinforced concrete beams may not have the resistance necessary to withstand the loads applied. This fact may occur, for example, due to modifications on its use, failures on process of design and execution of the structural element, and even due to corrosion of its steel reinforcement. In these cases, to avoid demolish the structure and rebuild her, one can use structural strengthening with the objective to increase its resistance.

The design of the structural strengthening can follow different paths, as the recommended by the ACI 440.2R-17 [11], or the methodology presented by Machado [3]. Although these designs use the existing theories for the behavior of beams, it is necessary to verify if the safety of the reinforced structure meets the requirement of established acceptable standards.

An important factor and that must be taken into account on the design process of the structural strengthening is the variability that de loads, geometries and material resistances can exhibit. These parameters are random variables that are generally described by probability distributions and their respective characteristics (mean and standard deviation). So there is the possibility that the tensions due to applied loads exceed the resistance offered by the structural element.

The structural reliability takes advantage of the probabilistic tools that verify, using statistical parameters, the probability of failure that a structural element can offer, along with a reliability index, being this a way to assess the level of safety of the structure.

This study presents the verification of reliability indexes and probabilities of failure related to flexion, for reinforced concrete beams designed by NBR 6118 [1], coupled with carbon fiber strengthening, by means of variation of loads and material properties.

2 Structural reliability

The methodology by which the structures are conceived obey code prescriptions that are different for almost every country. These codes are the link between academic research and practice, having the objective of predict minimum design requirements and safety, being the former a matter that holds a great deal of importance and that, recently, have been extensively discussed in different studies with various approaches, e.g Santos, Stucchi and Beck [4] and Ricardo [5].

The development of structural safety has a historical background, having its evolution based on trial and error. This learning process was slow at the cost of human lives and buildings. The reliability began to have a better understanding after the utilization of softwares and computes with more process capability, which allowed more precision in the prediction of the behavior of structures. However, the load and resistance uncertainties are still present and they contribute to raise the risk and the probability of occurrence of an undesired event, as proposed by Ellingwood [6].

The basic problem of reliability can be described by a limit state function (LSF), that means, que equation that governs a certain failure mode, having the following format (Eq. 1), where R is the structural resistance of the system, and S is structural effect of the applied load, as proposed by Beck [7].

$$g(R, S) = R - S \quad (1)$$

Through the LSF one can obtain relation between the statistical distributions of the loads and resistance, creating a tridimensional plane that contains the failure domain ($G < 0$) and the survival domain ($G > 0$), as can be seen in Fig. 1.

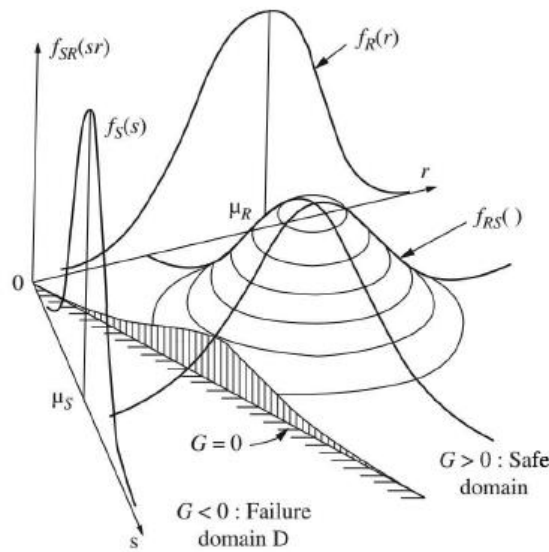


Figure 1. Failure and survival domain by Melchers and Beck [8]

3 Methodology

3.1 Structural element analyzed

The reinforced concrete beam used for this study is an adapted version of the element presented by Paliga [9], with a cross section of 12 x 40 cm, and 400 cm of spam (L). The Carbon Fiber Reinforced Polymer (CFRP) is located at the very bottom of the beam. The lateral view of the beam is presented in Fig. 2. The beam had its steel reinforcement determined according to NBR 6118 [1], and more information on geometry and reinforcement can be found on Tab. 1.

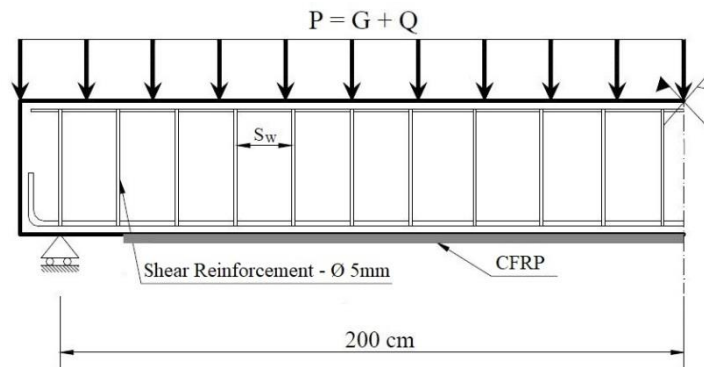


Figure 2. Lateral view of the beam (adapted from Paliga [9])

Table 1. Material and steel reinforcement properties

f_{ck} (MPa)	Dead load (kN/m)	Live load (kN/m)	Concrete cover (mm)	f_{yk} (MPa)	d effective (cm)	d' (cm)	d'' (cm)	x (cm)	A_s design (cm ²)	A_s effective (cm ²)
20	7,5	2,5	20	500	37	2,75	3	7,26	1,95	2,4

3.2 Statistics of random parameters

For the reliability analysis is necessary to have the statistical data of the random variables used in the LSF. Combining the information from Santos, Stucchi and Beck [4], and Ribeiro e Diniz [10], the Tab. 2 presents a summary of the necessary statistical information.

Table 2. Statistical data for the random parameters.

Variable class	Variables	Symbol/ Unit	Distrib.	μ_x	σ_x	References
Loads	Dead	G/kN.m	Normal	Gk	0,1 μ_x	[4]
	Live - 50 years	Q/kN.m	Gumbel	0,93 Qk	0,2 μ_x	[4]
Material resistances	Concrete	fc/MPa	Normal	1,17 fck	0,15 μ_x	[4]
	Steel for reinforcement	fy/MPa	Normal	1,08 fyk	0,05 μ_x	[4]
	Carbon fiber	ffe/MPa	Normal	1,176 ffk	0,05 μ_x	[10]
	Young's modulus for carbon fiber	Ef/MPa	Normal	Efk	0,05 μ_x	[10]
	Height (beam)	h/cm	Normal	h	0,045 μ_x	[4]
Geometric data	CG dist. from bar (lower fiber - beam)	d'/cm	LN	d' nom	1,1	[4]
	Uncert.- loads	Model uncertainties	θS	LN	1	0,05
Uncert.- resistances	Model uncertainties - Flexure	θR	LN	1	0,05	

3.3 Conducted Analyses

The analyses were divided into three parts, in that context two design were conducted using the ACI 440.2R-17 [11] methodology and one using the methodology by Machado [3]. Is was also investigated the utilization of material parameters recommended by ACI 318-14 [2] and by NBR 6118 [1] for the Young's modulus and yield tension for steel, and ultimate compressive strain for concrete.

For the reliability indexes and probability of failure of the strengthened beam two LSF were used. Both follow the idea of difference between resistant and soliciting moment in the section of the beam, as can be seen in Eq. 2.

$$g(x) = \theta R \cdot M_{Resistant} - \theta S \cdot M_{Soliciting} \quad (2)$$

The first LSF is derived from the moment resistant equation presented in ACI 440, introduced in Eq. 3 and named LSF 1.

$$g(x) = \theta R \times \left((A_s \times f_y \times d - \frac{\beta_1 \times x}{2}) + (A_f \times f_{fe} \times d_f - \frac{\beta_1 \times x}{2}) \right) - \theta S \times (M_g + M_q) \quad (3)$$

In the Eq. (3) the variables are the following:

A_s is the reinforced steel cross section;

f_y is the yield strength of structural steel;

d is distance between the center of gravity and the extreme top fiber of the steel section;

x is the neutral axis height;

A_f is the strengthening cross section;

df is distance between the center of gravity and the extreme fiber of the strengthening section;
 β is the relation between the depth of the compression portion of the cross section and the neutral axis.

The resistant moment is composed by the contribution of the strength of steel solicited and the strengthening material in tension, while the soliciting moment is the resulting effect of applied dead and live loads. The second equation is presented in Paliga [9], here called Eq. 4 and nominated LSF 2. More information on its variables can be found on the above reference.

$$g(x) = \theta R \times ((As1 \times fy \times (d - \delta g \times x)) + (Af \times Ef \times \varepsilon f - (h - \delta g \times x)) + (As2 \times Es \times \varepsilon s2 \times (h - \delta g \times x)) - \theta S \times (Mg + Mq)) \quad (4)$$

For the sake of simplification, it was adopted $\delta g = 0,4$, following the recommendation of Paliga [9]. Just like in Eq. 3, the Equation 4 has the soliciting moment composed by the applied dead and live loads. For the resistant moment there are portions due to steel in tension, of the carbon fiber, and, unlike the previous LSF, this equation presents the contribution of the steel in tension called ($As2$).

4 Results and discussion

4.1 Minimal additional load for the strengthening

For the determination of the applied load for which is necessary the structural strengthening, a reliability analysis was conducted for the beam, considering its original cross section and steel reinforcement, and the LSF presented in Santos, Stucchi and Beck [4], referred as Equation 1. Increments of 10% in the live load were performed while the dead load remained unaffected. The successive increments reached 200% of the live load.

Using the Monte Carlo method, the reliability index (β) for the original beam, without any load increment, resulted in 5,292. When comparing with the target value recommended by ACI 318-14 [2] of 3,5 the original beam has a considering high reliability index. It is possible that the conservative value of β is related to utilization of safety coefficients of loads and materials. In addition, the use of an effective steel area (As), about 18,75% higher than the necessary by design, as can be seen in Table 1, could be another important factor for the high β .

Based on the performed analysis, it was concluded that is necessary to apply 125,5% of the live load for the reliability index β to reach 3,49, and considerably close to the reference value of 3,5. Based on this value, the carbon fiber strengthening was designed for situations with load increment of 130% and above.

4.2 Design with ACI 440.2R:2017 – ACI parameters

The first methodology used for the design of the strengthening is recommended by ACI 440.2R:2017 [11]. Just as presented by the method, the values for the following parameters were used as indicated in the American standard: Young's modulus for steel, yield strength of structural steel and ultimate compressive strain for concrete – all parameters that assume different values when used in the Brazilian code context (Machado [3]), see Table 3.

Table 3 – Parameters values in the Brazilian and American standard

Parameter	NBR 6118:2014	ACI 440.2R:2017
Es	210.000 MPa	200.000 MPa
fy	500 MPa	414 MPa
ε_c	3,5 ‰	3,0 ‰

The results for the strengthening design are presented in Table 4. One can notice that the raise in the strengthening area between the values of 130% and 150%, and between 150% and 200% is considerably different. This conclusion is justified by the verification of tension in service for the steel reinforcement. Up till the live load increment of 150% the verification was satisfied with the minimum strengthening area. For load values over 150%, the strengthening area needed to be higher in order to verify this design step.

An additional analysis can be done from the resistant (ϕM_n) and soliciting moment (M_u) that define the equilibrium of the strengthened section of the beam, the resulting data are in Table 4.

Table 4 – Data of strengthening, resistant and soliciting moment, and its difference

Load increment (%)	N° layers	Width of fabric (cm)	Strengthening area (mm ²)	ϕM_n (kN.m)	M_u (kN.m)	ΔM (kN.m)
130	1	5,0	12,50	36,90	36,4	0,50
140	1	5,5	13,75	37,48	37,2	0,28
150	1	6,0	15,00	38,06	38,0	0,06
160	1	8,5	21,25	40,90	38,8	2,10
170	1	11,5	28,75	44,19	39,6	4,59
180	2	7,5	37,50	43,20	40,4	2,80
190	2	9,0	45,00	45,57	41,2	4,37
200	2	10,5	52,50	47,91	42,0	5,91

The increase in the strengthening area is reflected in the behavior of the resistant moment (ϕM_n), with its raise from the load increment of 160% in relation to the soliciting moment (M_u).

From the load increments of 170% to 180% it can be seen a great reduction of the difference between the two moments of the beam section (ΔM). This fact is explained by the increase in the number of carbon fiber layers, an important factor that have a direct influence in the tension in service for the steel reinforcement.

Finally, another relevant factor that must be taken into account is the restriction in the carbon fiber fabric width from the moment that two layers are necessary. If that restriction was not present, it would be possible to reduce ΔM for situation with load increment from 180%.

4.3 Reliability indexes and probabilities of failure according to LSF 1

The reliability analysis was conducted for the design values of the strengthening based on the LSF 1, referred in Equation 3, and the parameters presented in Table 2. The resulting values, obtained by the Monte Carlo simulation, are presented in Table 5.

All the design resulted in higher values for β than the reference index of 3,5. Also, the results meet the demand of the Eurocode 0 [12], that demands an index of 3,8 for this structural element.

That allows the conclusion that the strengthened beams are safe from the reliability point of view, under the proposed analysis. One can notice that the reliability indexes obtained up till the load increment of 150% remained fairly constants, and from the load increment of 160% on, exhibited a considerable increase (this can be seen more clearly in Fig. 3).

This progressive elevation of β can be attributed to the rise on the strengthening area so the tension in service verification of the steel reinforcement can be satisfied. It is worth reminding that the steel present a tension of, at least, 80% of its yield strength. This guarantee that the carbon fiber, because of the element deformation, remains in the elastic range, and does not suffer a brittle rupture, considering that the material does not present a yielding plateau.

Analyzing the values of β between the increases in all series, it can be observed that the variation pattern follows the same patters from ΔM observed in Fig. 3. This is due to the form of Limit State Function used (see Eq. 3), that characterizes the failure when the resisting moment is less than the soliciting moment.

Table 5 – Summary of the reliability analysis

Load increment (%)	β	Probability of Failure	CoV [Monte Carlo]	Number of simulations
130	3,86	5,72E-05	0,01	75664
140	3,84	6,04E-05	0,01	124412
150	3,83	6,38E-05	0,01	151759
160	4,23	1,17E-05	0,01	59355
170	4,64	1,72E-06	0,01	59255
180	4,37	6,16E-06	0,01	54415
190	4,61	2,02E-06	0,01	57266
200	4,82	7,16E-07	0,01	57411

That way, the lower the difference between these two values, the lower will also be the associated reliability index, just as presented in the scenario with load increase of 180%. In this situation, there is drop in the reliability index due to the utilization of two layers for the strengthening, consequently bringing the values of the moments (resistant and soliciting) closer.

4.4 Reliability indexes and probabilities of failure according to LSF 2

The same values obtained in the design of the strengthening by the ACI methodology, with the American recommend parameters, were subjected to a reliability analysis with the LSF 2, referred in Eq. 4. The results of the Monte Carlo simulation are presented in Table 6.

All the resulting values of the reliability analysis with the equation indicated in [9], for this design, resulted in reliability indexes higher than the reference value of 3,5, and that allowed the conclusion that the structures are safe.

Table 6 – Summary of the reliability analysis

Load increment (%)	β	Probability of Failure	CoV [Monte Carlo]	Number of simulations
130	3,57	1,76E-04	0,01	51903
140	3,54	1,96E-04	0,01	52157
150	3,51	2,18E-04	0,01	48316
160	3,83	6,21E-05	0,01	52285
170	4,17	1,51E-05	0,01	55060
180	3,89	4,93E-05	0,01	49482
190	4,08	2,21E-05	0,01	52652
200	4,25	1,07E-05	0,01	50818

The Fig. 3 presents the compilation of the reliability indexes for the two Limit State Functions, and bring forth the conclusion that the resulting values of the equation from the ACI 440 are higher in all scenarios.

This fact is at least curious, since the equation presented in Paliga [9] take into account the same aspects of resistance that the first equation, and still add the contribution of the steel reinforcement in compression, even though it's a small portion.

This finding can be explained by some factors like the simplicity of the LSF 1. The LSF 2 brings the material properties explicitly, like with the strengthening material strain, and specially its Young's

modulus, that was considered as a random variable for the determination of the reliability indexes.

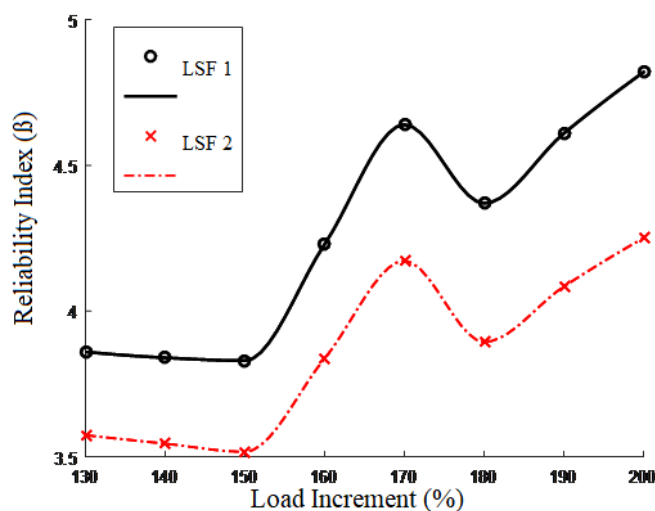


Figure 3 – Comparison between limit state function

4.5 Design with ACI 440.2R:2017 – NBR parameters

After the study with the parameters of the American standard, the same method of strengthening design was performed, but using the parameters of Young's modulus for steel, yield strength of the steel, and concrete ultimate strain of the Brazilian standard, as previously shown in Table 3. The values obtained for the design are shown in Table 7.

In this analysis, the increase in the characteristic yield strength of the steel - from 414 MPa to 500 Mpa - meant that the 130% load increase required no structural strengthening for bending. The reinforcement area values have reduced considerably due to this same change. In other words, the contribution of the resistant moment from steel increased as its characteristic strength increased by 17.2% (from 414 to 500 MPa).

It is noteworthy that the values obtained in the strengthening design could not be executed due to the limitations of nominal dimensions existing in the market, but, for analysis reasons, will be used in the following evaluations.

Table 7 – Data of strengthening, resistant and soliciting moment, and its difference

Load increment (%)	N° layers	Width of fabric (cm)	Strengthening area (mm ²)	ϕM_n (kN.m)	M_u (kN.m)	ΔM (kN.m)
130	0	0	0	36,78	36,4	0,38
140	1	5	1,25	37,60	37,2	0,40
150	1	10	2,50	38,18	38,0	0,18
160	1	20	5,00	39,32	38,8	0,52
170	1	25	6,25	39,89	39,6	0,29
180	1	30	7,50	40,46	40,4	0,06
190	1	40	10,00	41,58	41,2	0,38
200	1	45	11,25	42,14	42,0	0,14

Table 7 shows that the reduction of strengthening area values for load increments was relatively considerable in comparison with the values from Tab. 4.

Unlike the design performed with the American standard parameters, the reinforcement area values for the design with Brazilian parameters had less fluctuations in their results. This is due to the fact that the verification of service tension for steel had no influence on the reinforcement area to be used.

It becomes apparent that the design using the parameters of the Brazilian standard results in strengthening areas that make the resistant moment very close to the soliciting moment, supposing that there would be material savings when solving the same beam.

4.6 Reliability indexes and probabilities of failure for according to LSF 1

The results obtained for the reliability analysis with the LSF 1 are presented in Table 8. As seen in the item of strengthening design with parameters of the Brazilian standard, according to the method, it was not necessary to use any strengthening for the load increment of 130%, thus not being possible to determine the reliability analysis for this case.

Analyzing the other cases, it is visible that with the values of 140% and 150% of load increment, it was not possible to reach the reference reliability index of 3.5, as suggested by Santos, Stucchi and Beck [4].

Because this design adopted 500 MPa as the characteristic yield strength of steel - NBR parameter -, it was not necessary to add any strengthening material so that the service tension for the steel remained within the design limit value, making the ΔM approach to 0.

Table 8 – Summary of the reliability analysis

Load increment (%)	β	Probability of Failure	CoV [Monte Carlo]	Number of simulations
130	-	-	-	-
140	3,43	2,97E-04	0,01	61781
150	3,44	2,90E-04	0,01	65917
160	3,56	1,85E-04	0,01	71850
170	3,56	1,86E-04	0,01	75050
180	3,56	1,87E-04	0,01	71312
190	3,65	1,30E-04	0,01	111936
200	3,64	1,35E-04	0,01	123541

4.7 Reliability indexes and probabilities of failure for according to LSF 2

The values obtained in the design by the ACI methodology using the Brazilian parameters were also subjected to a reliability analysis, and LSF 2 was used. The values obtained are presented in Table 9. As in the previous analysis, the 130% load increment value did not need strengthening area, and consequently did not participate in the reliability analysis.

Unlike the data presented with the use of LSF 1, in this case none of the values of β reached the reference value. Through this analysis, normative parameters, and design method, the beam with its respective additional loads and reinforcements cannot be considered safe.

The trend towards a reduction in reliability indices when comparing LSF 1 and 2 is also confirmed in this analysis, where all β indices obtained for LSF 2 are lower than those obtained with LSF 1. This information can be seen in Fig. 4.

Table 9 – Summary of the reliability analysis

Load increment (%)	β	Probability of Failure	CoV [Monte Carlo]	Number of simulations
130	-	-	-	-
140	3,40	3,41E-04	0,01	66665
150	3,38	3,63E-04	0,01	52341
160	3,45	2,76E-04	0,01	50064
170	3,43	3,01E-04	0,01	48407
180	3,41	3,26E-04	0,01	47759
190	3,47	2,62E-04	0,01	49382
200	3,44	2,86E-04	0,01	49457

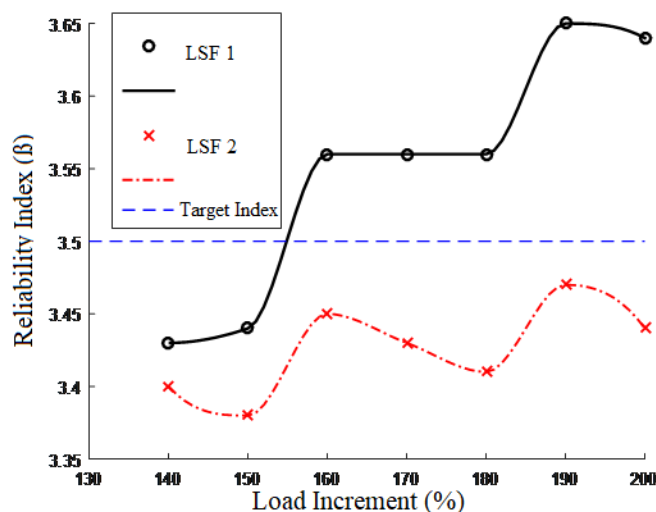


Figure 4 - Comparison between limit state function

4.8 Design with methodology of Machado [3]

As in no reliability analysis the values obtained for the design with Brazilian parameters were entirely above the reference value, another design method was used for the verification. This one presented by Machado [3] originally uses the parameters of steel Young's modulus, steel yield strength, and concrete ultimate strain of the Brazilian standard. This design routine is less conservative as it does not limit the service tension of steel to control failure. The values obtained are presented in Table 10.

Unlike with the design performed with the ACI methodology using the Brazilian parameters (see Table 7), the values obtained with the process presented by Machado [3] were significantly higher, although the widths presented are not executable.

The resistant moment values for this case are equal to the soliciting moment, making the factor ΔM for all designs equals to 0. This is due to the iterative process used to determine the required strengthening width, and consequently its area. It was chosen to use the values resulting from the design process, without rounding, to later perform the safety analysis that the process offers.

Table 10 – Results for the strengthening area

Load increment (%)	N° of layers	Fabric width (mm)	Strengthening area (mm ²)
130	1	7,20	1,80
140	1	10,78	2,69
150	1	14,61	3,65
160	1	18,67	4,67
170	1	22,98	5,75
180	1	27,55	6,89
190	1	32,39	8,10
200	1	37,50	9,37

4.9 Reliability indexes and probabilities of failure for according to LSF 2

Since, based on the previous analyzes, the LSF 2 was the one that presented the lowest reliability index values, i.e. the one with the most restrictive parameters, it was chosen for the determination of the reliability indexes for the design performed by the method presented by Machado [3]. Table 11 shows the values obtained for the reliability analysis.

Table 11 – Summary of the reliability analysis

Load increment (%)	β	Probability of Failure	CoV [Monte Carlo]	Number of simulations
130	3,63	1,41E-04	0,01	167390
140	3,61	1,51E-04	0,01	68712
150	3,59	1,63E-04	0,01	55165
160	3,57	1,76E-04	0,01	52473
170	3,55	1,90E-04	0,01	50841
180	3,53	2,06E-04	0,01	50846
190	3,51	2,22E-04	0,01	50574
200	3,49	2,40E-04	0,01	50841

In this case, all load increments, except 200%, had a higher reliability index than the reference value, indicating that these beams are safe, as can be seen graphically in Figure 5.

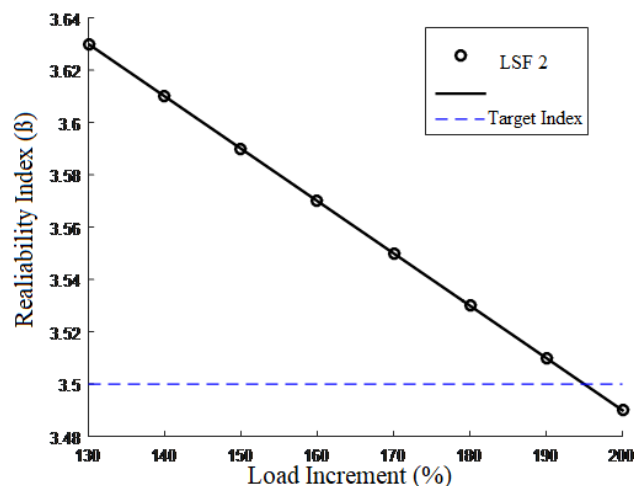


Figure 5 – Comparison between Reliability Index x Load increment

Although the value of ΔM is 0 for the design conducted by Machado's method [3], the curve presented by Figure 5 presents a behavior of reduction of the linear reliability index, as an increase in the load increment is applied. This indicates that certainly for all load increase values greater than 200%, the β indices would be below the reference value.

5 Conclusions

Initially analyzing the Limit State Functions used, one can see a propensity of LSF 1 to generate larger values of β than LSF 2 for the same strengthening material design, as verified in the two cases where the two LSFs were used. This result can be justified by the fact that the LSF 2 presents more random variables, besides explicit parameters such as the strain of the reinforcement material and its Young's modulus, which contributes to the increase of the variability of the element and its consequent reduction of the reliability index.

As for the strengthening design methods, the use of ACI 440.2R:2017 [11] with the American parameters resulted in more conservative values in strengthening area. This difference is attributed to the increase of carbon fiber area to be used so that the steel service tension remains below the limit stress, which is 80% of the yield strength. According to Beber [13], this verification must be performed in order to control crack formation, where, otherwise, localized failures in the strengthening material may occur due to the differential displacement between two sides of a crack, causing the carbon fiber to break in the transverse direction.

The reliability index values for these designs presented values that exceed the reference index $\beta = 3.5$ for all load increments, although analyzed using the two limit state functions.

The use of Brazilian parameters for this same method was not effective, since the designs resulted in lower strengthening area values than previously calculated, but mainly did not reach the reference value of $\beta = 3.5$ in several cases.

The design performed by the method presented by Machado [3], and using the material parameters of the Brazilian standard, resulted in slightly larger strengthening area values than the design performed by ACI. On the other hand, due to the simplicity of the method, there is no verification of some effective strain of the materials, since it is assumed the ultimate deformation for the concrete, like the verification of service stress of steel. The reliability index values for this method were above the reference index until the load increment of 190%. Load increments values above this limit show lower than minimum indexes in a curve with linearly decreasing behavior as load increases. The behavior seen in this analysis may have been due to the use of the calculated strengthening areas, rather than those executable, as these would be slightly larger due to rounding.

For strengthening design, it is recommended not to merge design methodology from one standard with material behavior considerations from another. As seen in the evaluations, although under the Ultimate Limit State perspective the beams were in equilibrium, several cases presented unsatisfactory reliability indexes.

The caveats made here point to the need for further studies concerning the reliability of carbon fiber reinforced beams, and the association of methodology adapted to materials with different provisions from those adopted in Machado [3].

References

- [1] ASSOCIAÇÃO BRASILEIRA DE NORMAS TÉCNICAS. NBR 6118: Projeto de estruturas de concreto – procedimento. Rio de Janeiro, 2014.
- [2] AMERICAN CONCRETE INSTITUTE. Building Code Requirements for Structural Concrete – ACI 318-14. United States of America, 2014.
- [3] MACHADO, Ari de Paula. Reforço de estruturas de concreto armado com fibras de carbono: Características, dimensionamento e aplicação. São Paulo: Pini, 2002.
- [4] SANTOS, D. M.; STUCCHI, F. R.; BECK, A. T.. Reliability of beams designed in accordance with Brazilian codes. *Revista Ibracon de Estruturas e Materiais*, [s.l.], v. 7, n. 5, p.723-746, out. 2014. FapUNIFESP (SciELO). <http://dx.doi.org/10.1590/s1983-41952014000500002>.
- [5] A. S. Ricardo. Análise da confiabilidade estrutural de elementos de aço em situação de incêndio. Master's Thesis, Federal University of Santa Catarina, 2015.
- [6] ELLINGWOOD, Bruce Russel. LRFD: Implementing structural reliability in professional practice. Elsevier, Baltimore, p.106-115, 9 jan. 1998. Disponível em: <[https://doi.org/10.1016/S0141-0296\(98\)00099-6](https://doi.org/10.1016/S0141-0296(98)00099-6)>. Acesso em: 10 out. 2017.
- [7] BECK, André Teófilo. Curso de confiabilidade estrutural. São Carlos: USP, 2014.
- [8] MERLCHERS, Robert E.; BECK, André T.. Structural reliability analysis and prediction. 3. ed. Hoboken: Wiley, 2018.
- [9] PALIGA, Charlei Marcelo. Análise probabilística de vigas de concreto armado recuperadas à flexão, através do método de Monte Carlo utilizando um modelo de elementos finitos. 2008. 249 f. Tese (Doutorado) - Curso de Engenharia Civil, Universidade Federal do Rio Grande do Sul, Porto Alegre, 2008. Disponível em: <<http://hdl.handle.net/10183/13455>>. Acesso em: 20 mar. 2018.
- [10] RIBEIRO, S.e.c.; DINIZ, S.m.c.. Reliability-based design recommendations for FRP-reinforced concrete beams. *Engineering Structures*, [s.l.], v. 52, p.273-283, jul. 2013. Elsevier BV. <http://dx.doi.org/10.1016/j.engstruct.2013.02.026>.
- [11] AMERICAN CONCRETE INSTITUTE. Guide for the Design and Construction of Externally Bonded FRP Systems for Strengthening Concrete Structures – ACI 440.2R-17. United States of America, 2017.
- [12] EUROPEAN COMMITTEE FOR STANDARDIZATION, EN 1990:2002 – Eurocode 0: - Basis of structural design, ECS. Brussels, 2002.
- [13] BEBER, Andriei José. Comportamento estrutural de vigas de concreto armado reforçadas com compósitos de fibra de carbono. 2003. 289 f. Tese (Doutorado) - Curso de Engenharia Civil, Universidade Federal do Rio Grande do Sul, Porto Alegre, 2003. Disponível em: <<http://hdl.handle.net/10183/2974>>. Acesso em: 27 jun. 2018.