FATIGUE LIFE ESTIMATION USING FREQUENCY DOMAIN TECHNIQUE AND PROBABILISTIC LINEAR CUMULATIVE DAMAGE MODEL

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Abstract. Engineering structures are designed to withstand a variety of in service loading specific to their intended application. Random vibration excitation is observed in most of the structural component applications in the naval, aerospace and automotive industry. Likewise, fatigue life estimation for such components is fundamental to verify the design robustness assuring structural integrity throughout service. The linear damage accumulation model (Palmgren-Miner rule) is still largely used for damage assessment on fatigue estimations, even though, its limitations are well-known. The fact that fatigue behavior of materials exposed to cyclic loading is a random phenomenon at any scale of description, at a specimen scale, for example, fatigue initiation sites, inclusions, defects and trans-granular crack propagation are hardly predicted, indicates that a probabilistic characterization of the material behavior is needed. In this work, the inherent uncertainties of the fatigue life and fatigue strength of the material are characterized using the random fatigue limit (RFL) statistic method, which incorporates the maximum likelihood estimation to produce probabilistic S-N curves. Furthermore, a frequency domain technique is used to determine the response power spectrum density (PSD) function of a structural component subjected to a random vibration profile excitation. The fatigue life of the component is then estimated through a probabilistic linear damage cumulative model, where not only the time to failure is predicted but also its variability. The methodology was applied to a Titanium alloy structural component, exposed to a specific random excitation, where the predicted life using the material percentile curves 5% and 95% has shown a significant variability when compared to the common used percentile 50%. Thus, this characterization might be relevant for the definition of material design curves.

Keywords: Fatigue Life, Cumulative Damage, Random Vibration, PSD

1 Introduction

Engineering structures are designed to withstand a variety of in service loading specific to their intended application. Random vibration excitation is observed in most of the structural component applications in the naval, aerospace and automotive industry. Likewise, fatigue life estimation for such components is fundamental to verify the design robustness assuring structural integrity throughout service.

Fatigue as a technical problem became evident around the middle of the 19th century, August Wöhler perfomed systematic fatigue tests of smooth and notched railway axles in the 1850s. In addition to the introduction of the S-N diagram, a plot of the number of cycles to failure at a given stress level, his work also led directly to the concept of a fatigue (or endurance) limit which represents the theoretical maximum cyclic load a material can withstand indefinitely without risk of fatigue failure [1].

Significant advances in the fatigue research were achieved with the mean stress effect studies by Gerber and Goodman, the development of fatigue safety diagrams by Haigh, investigation of reversed loading phenomena by Bauschinger, investigation of the notch effect on fatigue limit by Heyn, formulation of empirical laws to characterize fatigue limit by Basquin, the introduction of the crack growth energy balance by Griffith, life estimation under variable loading by Palmgren and the recognition of the statistical nature of fatigue by Weibull [1;2].

In the evaluation and prediction of the fatigue life of structures the role of mathematical and statistical models are crucial, due to the high complexity of the fatigue problem an efficient estimation of the corresponding parameters represents one of the most difficult challenges for the problem assessment, additionally, the possible shortage of data, which represents a common feature in the case of fatigue experimentation due to economic and/or time reasons.

Nevertheless, the fatigue damage assessment for components subjected to random excitation is an important concern in engineering. Fatigue damage increases with applied load in a cumulative manner which may lead to fracture. Palmgren suggested the concept which is known as the linear rule, Miner in 1945, first expressed the concept in mathematical form, where the measure of the damage is the cycle ratio with the assumption of constant work absorption per cycle (n_i) and characteristic amount of work absorbed at failure (N_i). The energy accumulation, therefore, leads to a linear summation which at failure equals one, as shown in Eq. (1). Despite of the well-known limitations, such as, not accounting for load sequence and interaction effects, the Miner's rule is still dominantly used in design due to its simplicity [3].

$$D_i = \sum_{i=1}^{k} \frac{n_i}{N_i} = 1 \tag{1}$$

The basic fatigue modeling for N_i consists of reproducing the fatigue behavior of materials under alternating stresses. Due to the random character of fatigue life, if several specimens were subjected to this type of tests with the same values of stress range and stress ratio, different fatigue life estimation are obtained.

An extension of this concept is the p quantile S-N curves, also called S-N-P curves, a generalization that relates the percentile of fatigue life to the applied stress or strain [4]. The percentile curves illustrate the variability of fatigue life, meaning that at p=0.5, 50% of the specimens fail above this curve and 50% below it. The percentile curves are also used to define design curves, at p=0.01, only 1% of the failures are expected to occur below this curve. The definition of the confidence interval is need to access reliability, because of the nature of the phenomenon as the confidence interval is only one statistic in many, load and geometry variability are other examples. The confidence interval is generally defined in a range between 90% and 97.5%.

Figure 1a represents a fatigue test data due to a cyclic stress load and Figure 1b has the addition of the percentile curves.

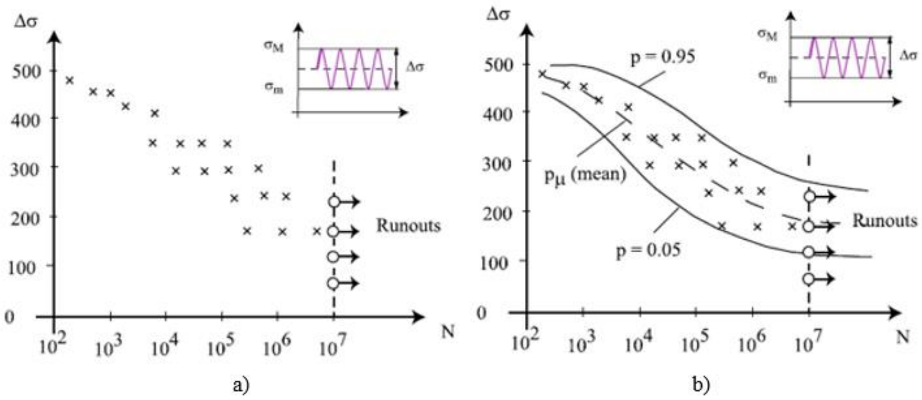

Figure 1. a) Fatigue test data on a log-log plot due to constant alternating stress level b) Fatigue test data with percentiles curves (from Castilo and Cantelli [4])

Fatigue data on ferrous and titanium alloys reveal that specimens tested below a stress level are unlikely to fail. This limiting stress level called the fatigue limit or endurance limit is observed by an accentuated curvature and an asymptotic behaviour near the fatigue limit. Most nonferrous metals such as aluminium, copper and magnesium appear not to have a fatigue limit, the post endurance S-N slop for these materials gradually continues to drop.

Nelson's work in [5] fitting fatigue curves with non-constant standard deviation associated with each stress level utilizing maximum likelihood methods and his suggestion in [6] to have fatigue limited modelled as a random parameter, where the test specimen would have different fatigue limits following a statistical distribution called "strength distribution" were the basis for the development of the Random Fatigue Limit (RFL) by Pascual and Meeker in [7]. As in a S-N testing the stress amplitude is the independent variable the fatigue life distribution is represented in figure 2 by the horizontal distribution for a specific stress level. The vertical distribution represents the fatigue strength at a specified life. The RFL model provides a description of the commonly observed increase in variability, it explicitly assumes that each specimen has its own fatigue limit based on each specified number of cycles.

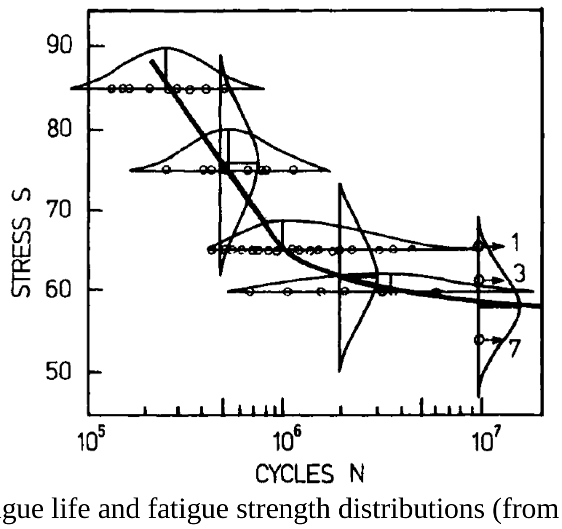

Figure 2. Fatigue life and fatigue strength distributions (from Nelson [6])

The fatigue damage traditionally determined from time signals of loading, usually in the form of stress or strain. This approach is satisfactory for periodic loading but requires very large time records to accurately describe random loading processes. Alternatively, a frequency domain approach has a substantial advantage when the finite element analysis is utilized where random loading and response are categorized using power spectral densities (PSDs). In this work the determination of the number of cycles n_i in Equation (1) is performed using the Dirlik model which ultimately allows the fatigue life estimation.

2 Random fatigue limit model

There are two main considerations in modelling the relationship between the applied stress and fatigue life, first, the standard deviation of fatigue life decreases as the applied stress increases. Second, curvature in fatigue curves suggests the inclusion of a fatigue limit in the statistical model for fatigue life. The random fatigue-limit model describes features observed during experimental testing of small samples.

The dispersion of the results is related on the heterogeneity of materials, the surface defects, the machining tolerances and metallurgical factors.

The formulation of the random fatigue limit model is shown in Eq.(2) using the notation from [8], rather than the original notation used by Pascual and Meeker [7].

The fatigue life for each specimen *i* denoted by N at a stress level S. The fatigue life for specimen is then modelled as:

$$ln(N_i) = \beta_0 + \beta_1 \log(S_i - \gamma) + \epsilon \,, \; S_i > \gamma, \tag{2}$$

where β_0 and β_1 are fatigue curves coefficients, γ is the fatigue limit of the specimen, ϵ is the error term, which is a random life variable associated with the scatter from specimens which have same value for fatigue limit.

Let $V = log(\gamma)$, then Pascual and Meeker assume V to have a probability density function (pdf) given by:

$$f_V(v; \mu_\gamma, \sigma_\gamma) = \frac{1}{\sigma_\gamma} \phi_V \left(\frac{v - \mu_\gamma}{\sigma_\gamma} \right) \tag{3}$$

In Eq. (3), μ_γ and σ_γ are the location and scale parameters for the distribution of γ, respectively. ϕ_V is either the standardized smallest extreme value (sev) or normal pdf.

Let $x = log(S)$ and $W = log(N)$. Then, for $V < x$ they assume that W given V has a pdf of the form:

$$f_{W|V}(\omega, \beta_0, \beta_1, \sigma, x, v) = \int_{-\infty}^{x} \frac{1}{\sigma} \phi_{W|V} \left\{ \frac{\omega - \{\beta_0 + \beta_1 \log[\exp(x) - \exp(v)]\}}{\sigma} \right\} \tag{4}$$

In Eq. (4), $\beta_0 + \beta_1 \log[\exp(x) - \exp(v)]$ acts as a location parameter and σ as a scale parameter, $\phi_{W|V}$ is either the standardized smallest extreme value (sev) or normal pdf. The marginal pdf of W is given by:

$$f_W(\omega, x, \theta) = \int_{-\infty}^{x} \frac{1}{\sigma \sigma_\gamma} \phi_{W|V} \left[\frac{\omega - \mu(x, v, \theta)}{\sigma} \right] \phi_V \left(\frac{v - \mu_\gamma}{\sigma_\gamma} \right) dv, \tag{5}$$

where $\theta = (\beta_0, \beta_1, \sigma, \mu_\gamma, \sigma_\gamma)$ and $\mu(x, v, \theta) = \beta_0 + \beta_1 \log[\exp(x) - \exp(v)]$. The marginal cumulative distribution function (cdf) of W can be written as:

$$F_W(\omega, x, \theta) = \int_{-\infty}^{x} \frac{1}{\sigma}\, \Phi_{W|V}\left[\frac{\omega - \mu(x, v, \theta)}{\sigma}\right] \phi_V\left(\frac{v - \mu_\gamma}{\sigma_\gamma}\right) dv, \tag{6}$$

where $\Phi_{W|V}(\cdot)$ is the cdf of $W|V$. This statistical model is referred as random fatigue-limit model (RFL).

There are no closed forms for the density and distribution functions which demands numerical evaluation. For the distribution of the random variable γ, the Weibull distribution is an adequate choice for describing the skewed downward (towards lower stress levels) strength distribution of many engineering materials. The Weibull distribution introduces two parameters, namely the location parameter η and the scale parameter β, which correspond to the location and scale parameters μ_γ and σ_γ respectively, used by Pascual and Meeker. When the RFL model incorporates these assumptions, it includes five total parameters $(\beta_0, \beta_1, \sigma_\in, \eta, \beta)$.

2.1 Model parameters estimation

Unlike conventional S-N analysis in which all specimens are tested until failure, an ordinary least-squares approach cannot be used to estimate the parameters of the probabilistic S-N model used in the RFL approach.

Pascual and Meeker used the maximum likelihood (ML) methods to estimate the parameters of the random fatigue-limit model. Statistical theory suggests that ML estimators, in general, have favorable asymptotic (large sample) properties.

Let $N_p(s)$ be the p quantile of the life distribution at stress level s. The ML is estimated for $N_p(s)$ for $p = 0.05; 0.50$ and 0.95 which are the percentile curves.

For the random fatigue-limit model defined previously with sample data $\omega_1 = \log(N_1), \ldots, \omega_n = \log(N_n)$ at log stress levels $x_1, \ldots, x_n$, respectively, the likelihood can be written as:

$$L(\theta) = \prod_{i=1}^{n} [f_W(\omega_i; x_i, \theta)]^{\delta_i} [1 - F_W(\omega_i; x_i, \theta)]^{1-\delta_i}\,, \tag{7}$$

where $\delta_i = 1\ if\ \omega_i$ is a failure and $\delta_i = 0\ if\ \omega_i$ is a censored observation.

The function $L(\theta)$ can be interpreted as being approximately proportional to the probability of observing $N_1, \ldots, N_n$ for a given set of parameters θ. Generally, it is easier to work with the log-likelihood function, maximizing the log-likelihood function produces the same set of parameters as maximizing the likelihood function. The log-likelihood function is shown in Eq. (8).

$$\mathcal{L}(\theta) = \log[L(\theta)] = \sum_{i=1}^{n} \mathcal{L}_i(\theta)\,, \tag{8}$$

where,

$$\mathcal{L}(\theta) = \delta_i \log[f_W(\omega_i, x_i, \theta)] + (1 - \delta_i)\log[1 - F_W(\omega_i, x_i, \theta)]\,, \tag{9}$$

is the contribution of the i th observation. The ML estimate $\hat{\theta}$ of θ is the set of parameters values that maximizes $L(\theta)$ or $\mathcal{L}(\theta)$.

2.2 Random loading characterization

Any periodic time history may be represented by the summation of a series of sinusoidal waves of various amplitude, frequency and phase. This is the basis of Fourier series expansion. The Fourier transform pair allows transformation between the time and frequency domains. The Fourier transform is defined as:

$$X(f) = \int_{-\infty}^{\infty} x(t)e^{-i2\pi ft}\,dt \tag{10}$$

and the inverse Fourier transform is:

$$x(t) = \int_{-\infty}^{\infty} X(f)e^{-i2\pi ft}\,dt \tag{11}$$

The autocorrelation function that defines how a signal is correlated with itself, with a time separation τ can be written as,

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)x(t+\tau)dt = E[x(t)x(t+\tau)] \tag{12}$$

The autocorrelation and the power spectral density (PSD) are related by the Fourier transform pair.

$$R(\tau) = \int_{-\infty}^{\infty} S_{xx}(f)e^{-j\omega t}\,d\omega \tag{13}$$

$$S_{xx}(f) = \int_{-\infty}^{\infty} R(\tau)e^{j\omega t}\,d\tau \tag{14}$$

The one-sided PSD $G_{xx}(f)$ defined for $0 \le f \le \infty$, is:

$$G_{xx}(f) = 2S_{xx}(f) \tag{15}$$

By definition a random time history cannot be periodic, however, if the time history is taken from an ergodic stationary Gaussian random process then it may be expressed in the frequency domain. A process is said to be stationary if the probability distributions of the ensemble are the same for all points in time. If the ensemble probability distribution function is Gaussian then the process is known as a Gaussian random process. A stationary process is called ergodic process if the statistics taken from one sample are the same as those obtained for the ensemble. For nonsationary process the statistics obtained from a sampled time history would not be representative of those of the whole random process as they would be continuously changing.

The disadvantage of the Fourier transform is in the complex manner in which the amplitude and phases are stored. For an ergodic stationary Gaussian random process, it has been observed that the phases angles are purely random with a constant probability distribution function between $-\pi$ and$+\pi$. Therefore the PSD function contains amplitude information but does not hold phase information.

The PSD gives a statistical representation of a stationary random process in the frequency domain. Spectral moments are calculated from the one-sided PSD [9;10].

$$m_n = \int_0^\infty f^n G(f) df \; = \sum_{k=1}^{m} f_k^n G_k(f) \delta f \tag{16}$$

The key statistical properties derived by S. O. Rice using the spectral moments m_0, m_1, m_2 and m_4 are presented below:

The mean square amplitude of the time history is the area under the PSD curve and therefore the Root Mean Square (RMS) is obtained by Eq. (17).

$$\sigma_{rms} = \sqrt{m_0} \tag{17}$$

The expected number of upward zero crossings is given by Eq. (18).

$$E[0] = \sqrt{\frac{m_2}{m_0}} \tag{18}$$

The expected number of positive peaks (peak frequency) can be written as:

$$E[P] = \sqrt{\frac{m_4}{m_2}} \tag{19}$$

The irregularity factor γ' helps classify a time history signal as narrow or wide band

$$\gamma' = \frac{E[0]}{E[P]} = \sqrt{\frac{{m_2}^2}{m_0 . m_4}} \tag{20}$$

2.3 Structure dynamic response characterization – Modal analysis

The dynamic behavior characterization of a component or structure can be determined both in time and frequency domain. In the time domain this involves a complicated and often lengthy transient analysis. In frequency domain the transfer function relates the amplitude of the input (force, acceleration, moment) to the amplitude of the output stress for each frequency. The frequency response function is determined by the modes and frequencies which the component or structure vibrates.

Modes are an inherent property of the component/structure determined by the material properties (mass, damping, and stiffness) and boundary conditions. Each mode is defined by a natural (modal or resonant) frequency and a mode shape.

Multiple-degree-of-freedom systems are described by Eq.(21).

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\}\,, \tag{21}$$

Where: [M] is the global mass matrix, [C] the global damping matrix, [K] the global stiffness matrix, $\{x\}$ the displacement vector, $\{\ddot{x}\}$ and $\{\dot{x}\}$ are the second and first derivatives of $\{x\}$ respectively, $\{f\}$ is the vector of external forces. Considering a undamped system with no external forces, Eq. (21) can be written as:

$$[M]\{\ddot{x}\} + [K]\{x\} = \{0\} \tag{22}$$

Assuming the harmonic oscillation, described by Eq. (23) and (24).

$$\{x\} = A.sen(\omega_i t + \theta_i) \tag{23}$$

$$\{\ddot{x}\} = -\omega_i{}^2 A.sen(\omega_i t + \theta_i) \tag{24}$$

Substituting Eq.(23) and Eq.(24) into Eq.(22), one obtain Eq. (25) and Eq. (26).

$$[M] - \omega_i{}^2 A.sen(\omega_i t + \theta_i) + [K]A.sen(\omega_i t + \theta_i) = 0 \tag{25}$$

$$A.sen(\omega_i t + \theta_i)\{[K] - \omega_i{}^2[M]\} = \{0\} \tag{26}$$

This equality is satisfaied if either $A.sen(\omega_i t + \theta_i) = \{0\}$ known as trivial solution which implies no vibration to the system or if the determinant of the matrix $[[K] - \omega_i{}^2[M]] = \{0\}$. This is an eigenvalue problem which may be solved up to *n* values of ω^2 and *n* eingenvectors. From Eq. (26) one obtain.

$$det\{[K] - \omega_i{}^2[M]\} = \{0\} \tag{27}$$

The eigenvectors are the vibration modes and the eigenvalues are the frequencies associated with that particular vibration mode.

Rather than outputting the natural circular frequencies (ω_i), the natural frequencies f_i values of the frequencies in Hertz (Hz) can be obtained from the Equation (28).

$$f_i = \frac{\omega_i}{2\pi} \tag{28}$$

2.4 Random Vibration Method and von Mises stress.

The random vibration is based on computing statistics of each of the modal reponse and combining them. In this work the excitation are assumed to be stationary random processes.

Equation (29) defines the relationship between the power spectrum density $S_{zz}(\omega)$ of the response and the base excitation $S_{ff}(\omega)$, by the transfer function $|H(\omega)|^2$, for each point of the structure.

$$S_{zz}(\omega) = |H(\omega)|^2.S_{ff}(\omega), \tag{29}$$

The modal transfer function for the modal coordinate j due to an input acceleration for the degree of freedom a and damping factor ζ can be written as:

$$H_{ja}(\omega) = \frac{1}{\omega_j{}^2 - \omega^2 + 2i\zeta_j\omega_j\omega} \tag{30}$$

Segalman et al [11,12], developed a method to calculate the von Mises root mean square as a combination of the modal stress vectors. The stress at a point of the structure can be assembled from the contributions of each mode as shown by Eq.(31):

$$\sigma_{von\ Mises}(t,x) = \sum_k q_k(t)\ \Psi_k\ (x)\ , \tag{31}$$

where q_k is the k^{th} modal coordinate and $\Psi_k(x)$ is the stress vector $\sigma(t)$ at location x associated with that mode, comprised of the six non redudant terms for the stress tensor.

Considering a quadratic function of stress, written in the following form of:

$$p^2(t) = \sigma(t)^T A\sigma(t). \tag{32}$$

In the case of von Mises stress $p^2(t)$,

$$p^2(t)\ =\ \sigma^2{}_{xx} + \sigma^2{}_{yy} + \sigma^2{}_{xx} - (\sigma_{xx}\sigma_{xx} + \sigma_{xx}\sigma_{xx} + \sigma_{xx}\sigma_{xx}) + 3(\sigma^2{}_{xx} + \sigma^2{}_{xx} + \sigma^2{}_{xx}) \tag{33}$$

where **A** is a symmetric and constant matrix defined as

$$A = \begin{bmatrix} 1 & -0.5 & -0.5 & 0 & 0 & 0 \\ -0.5 & 1 & -0.5 & 0 & 0 & 0 \\ -0.5 & -0.5 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 \end{bmatrix} \tag{34}$$

Combining Eq.(31) and Eq. (32) the von Mises stress could be written as:

$$p^2(t) = \sum_i \sum_j q_i(t)\ q_j(t)\Psi_i{}^T A\Psi_j \tag{35}$$

Taking the expected value of $p^2(t)$ we obtain the mean square value of the von Mises stress:

$$E\ \{p^2(t)\} = \sum_{i,j} E\{q_1(t)q_j(t)\}\Psi_i{}^T A\Psi_j \tag{36}$$

The root mean square value of the von Mises stress or equivalent von Mises stress in the frequency domain is an important parameter used in the determination of failure criteria through the comparison against the material strength.

2.5 Dirlik model and probabilistic linear cumulative damage

In 1985, Dirlik developed an empirical closed form solution for the wide band random problem, ther proposed pdf considered the sum of two Rayleigh and one exponential distribution developed from an extensive Monte Carlo technique [13]. Although apparently more complicated than some alternative methods, it is only a function of the four moments of area of the PSD. This method has been found to be widely applicable and contantly outperforms all of the other available methods considering the four moments of area of the PSD [9].

The Dirlik pdf model developed in [13] is given by:

$$p(s) = \frac{\frac{D_1}{Q} e^{\frac{-Z}{Q}} + \frac{D_2}{R^2} e^{\frac{-z^2}{2R^2}} + D_3 Z e^{\frac{-Z^2}{2}}}{2\sqrt{M_0}} \tag{37}$$

where the parameters D_1, D_2, D_3 , Q, R e Z are defined as function of the PSD moments as shown from Eq. (38) through Eq.(43).

$$x_m = \frac{M_1}{M_0}\sqrt{\frac{M_2}{M_4}} \tag{38}$$

$$D_1 = \frac{2(x_m - \gamma^2)}{1 + \gamma^2} \tag{39}$$

$$R = \frac{\gamma - x_m - {D_1}^2}{1 - \gamma - D_1 + {D_1}^2} \tag{40}$$

$$D_2 = \frac{1 - \gamma^2 - D_{1+} {D_1}^2}{1 - R} \tag{41}$$

$$D_3 = 1 - D_1 - D_2 \tag{42}$$

$$Q = \frac{1.25(\gamma - D_{3+} D_2 R)}{D_1} \tag{43}$$

The number of cycles at a specific value of stress *i* are obtained for a given time of vibration exposure *T* by the Eq.(44).

$$n_i = p[S_i].dS.E[P].T \tag{44}$$

The damage is then calculated using generally the 50% or B50 percentile S-N curve using Equation (1). However, due to the scatter in the material data, a probabilistic linear cumulative is proposed to caracterize the variability of the damage estimation using the different percentile fatigue life curves.

The probabilistic linear cumulative damage is then defined as:

$$D_{p_x} = \sum_{i=1}^{k} \frac{n_i}{N_{i_{p_x}}} \tag{45}$$

where x is the percentile curve. The damage is now computed not only for the commonly used median percentile but fot all confidence level interval.

The time to failure could also be calculated considering the time which the component or structure is exposed to the vibration profile relative to the damage at determined life percentile.

$$T_{failure_{p_x}} = \frac{T_{exposure}}{D_{p_x}} \tag{46}$$

3 Results and Discussions

Random vibrations at the spacecraft base are generated by propulsion system operation and by the adjacent structure's vibro-acoustic response. A fatigue life estimation was performed for a α-β Ti-6Al-4V specimen, shown in figure 3, subjected to an acceleration spectral density profile as per figure 4. The linear cumulative damage was calculated based on the S-N percentiles 0.5, 0.05 and 0.95 of the material fatigue testing data from [8].

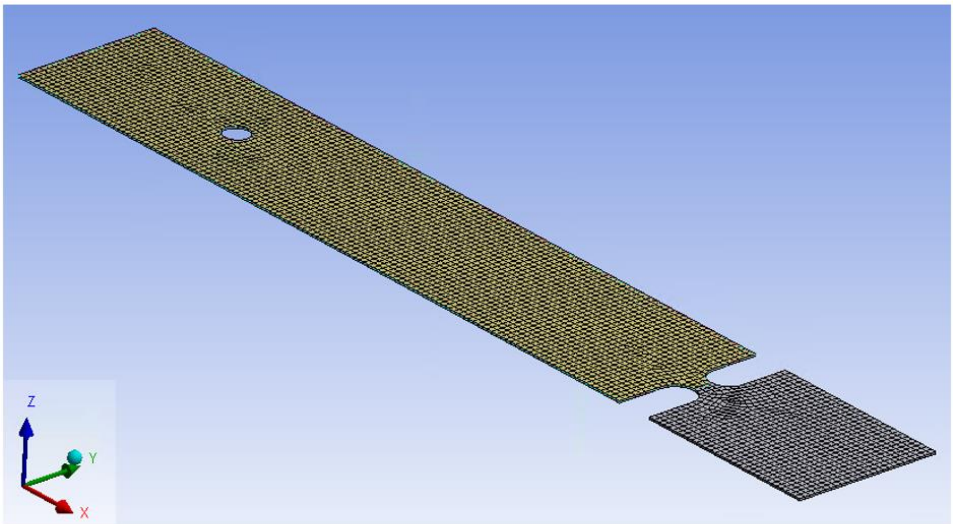

Figure 3. Acceleration PSD base excitation input

Figure 4 shows the acceleration PSD for component qualification applied at the Z - axis direction with 20.2 G rms (root mean square acceleration).

The total exposure to the vibration profile is 10 hours (36,000 s).

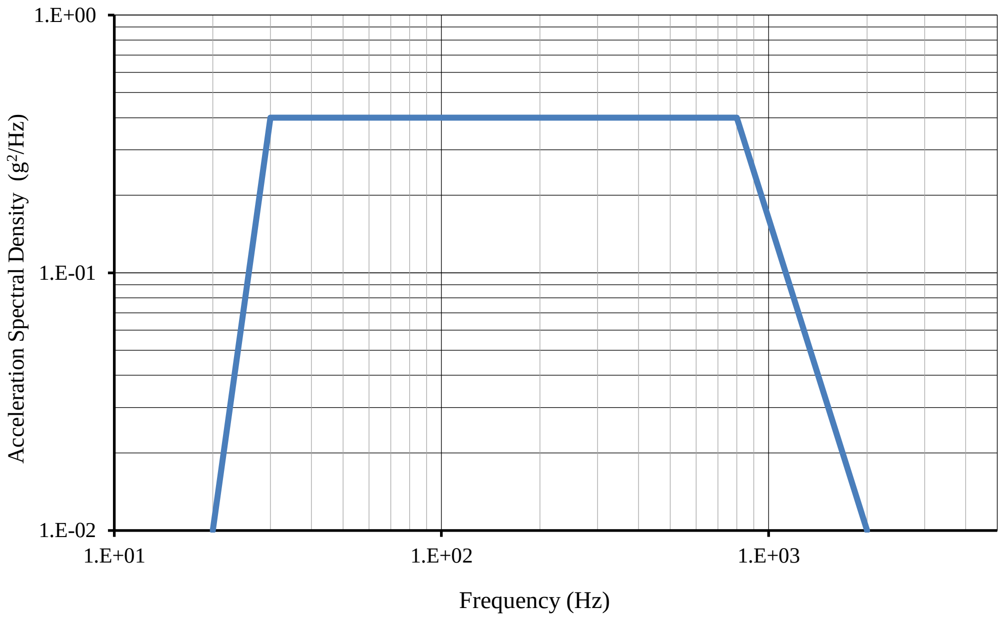

Figure 4. Generalized Random Vibration Test Levels Components

The equivalent 3σ von Mises stress plot is illustrated in Figure 5. It is notable the high stress concentration at the notch, area which has the higher likehood of failure. The stress response spectrum is obtained at the critical area of the part. Figure 6 shows the worst-case stress response spectrum, which occurs normal to the X-axis.

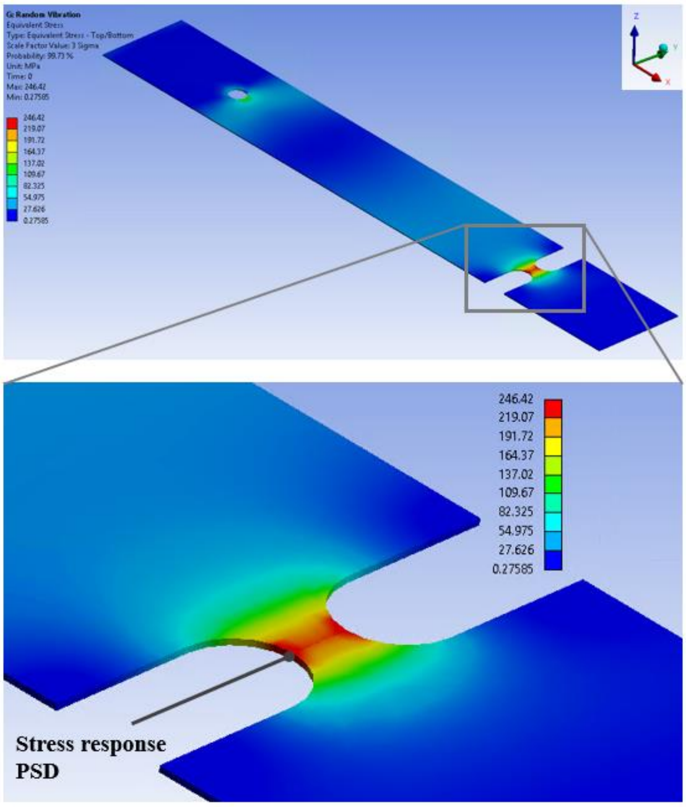

Figure 5 - 3σ von Mises stress plot

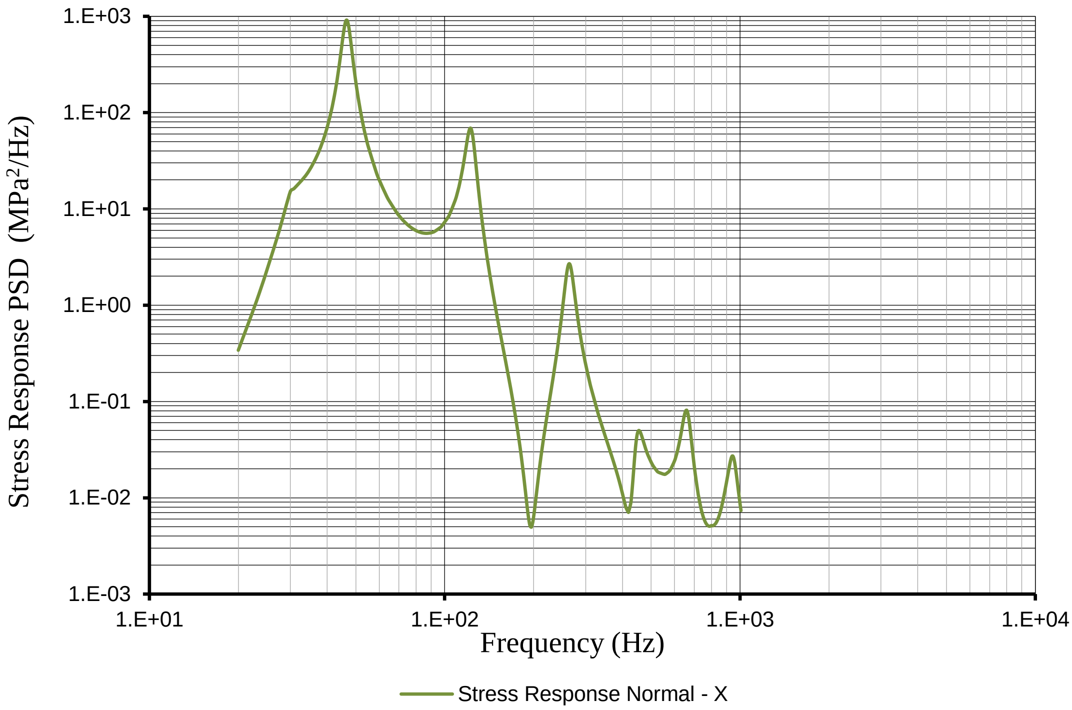

Figure 6. Stress Response PSD

The number of cycles at the different stress levels are calculated using the Dirlik probability distribution function, for the defined time of exposure to the random vibration profile.

The application of the RFL method to the Titanium alloy material data from [8] allowed the percentiles curves to be established. Figure 7 illustrates the material S-N-P curve from the fatigue test for the α-β Ti-6Al-4V titanium alloy.

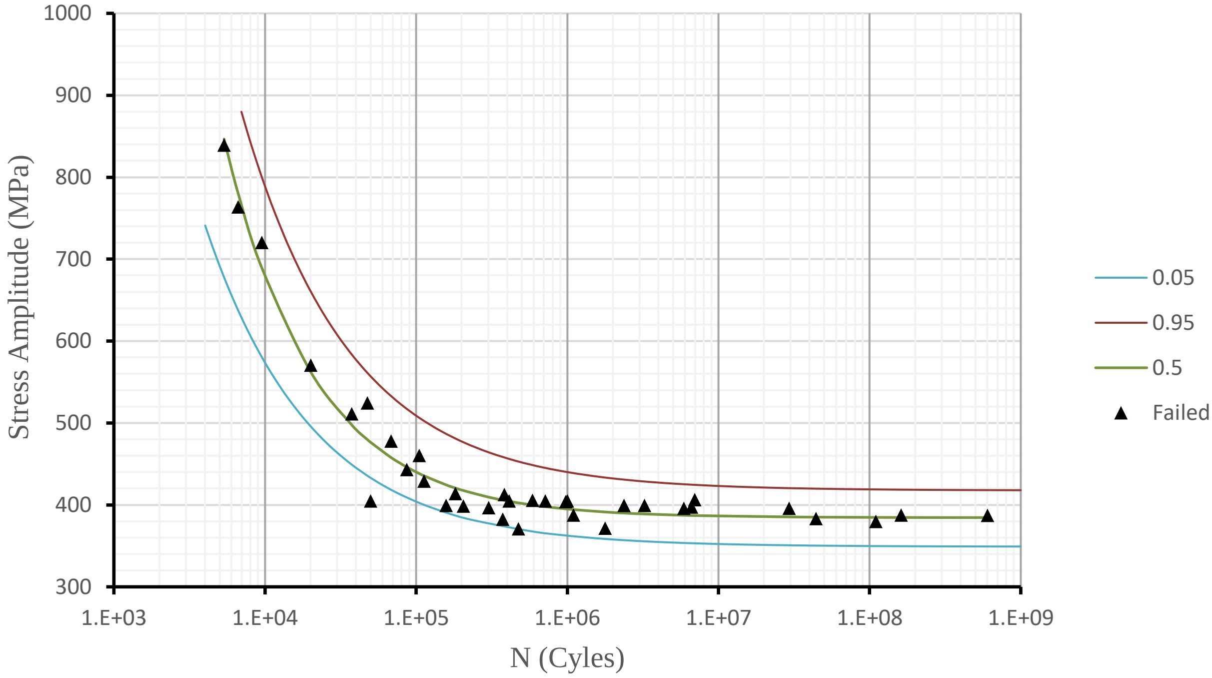

Figure 7. RFL model applied to the α-β Ti-6Al-4V showing the 0.05, 0.5 and 0.95 percentile

The damage is then calculated from the stress PSD for the different percentile curves. A summary of damage and time to failure is presented in Table 1.

Table 1. Fatigue Life and Damage relative to the S-N curve percentile

S-N curve percentiles	Damage	Time to failure (s)
0.5	0.9512	37,871
0.05	2.0877	17,243
0.95	0.2156	166,972

The equivalent von Mises 3σ stress plot is used here not as the failure criteria, where a comparison with the fatigue limit could result in a completely wrong interpretation of the fatigue phenomenon. Instead, it is used to identify the areas of the part with higher stress concentration which are likely to fail. The determination of the number of cycles at each stress level is then calculated from the stress response PSD, given the duration of the exposure to the vibration profile. Through the application of the RFL model to the material testing data, the calculated damage was highly influenced by the fatigue limit threshold. Incorporating the variability of the fatigue limit into the damage calculation, the fatigue life would be rated as survive for 50% of the samples tested under this condition, however, 5% of the samples would fail half way through the test. Likewise, the usage of the 0.95 percentile would rate the damage as only 22.6% of the one calculated for the 0.5 percentile curve. The RFL method models the probability of failure associated with each stress level, therefore, the variability of the damage and fatigue life are taken into consideration for the structures under random vibration, helping reliability assessments, as well as defining safety factors and design curves when limited material data is available.

4 Conclusions

In this work the fatigue life estimation of a structural component under random vibration was calculated using a probabilistic linear cumulative damage proposed model, through the application of the Random Fatigue Limit (RFL) model for the material characterization and frequency domain technique. For applications, as the one presented in this work, that have stresses in the fatigue limit region, the proper characterization of the material, as well, as the understand of the effects in the fatigue damage calculation are of fundamental importance for the structural integrity assessment and its reliability. Compared to the commonly used 0.5 percentile, the damage and consequently the life estimation was found to be approximately half for the 0.05 percentile, this result shows the importance of the incorporation of probabilistic models in the determination of the design curves for example. When a compromise between cost and structural performance must be achieved, the understand of the variabilities of the material data and how it reflects into the fatigue estimation for the application is extremely important for the decision-making process of the design.

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