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Solution of Time-Variant Risk Optimization Problems using Two-Level Active Learning Kriging Approach

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Abstract. Risk optimization is a general approach for structural optimization regarding uncertainties. Different life-cycle costs are considered, including expected failure costs, whose calculation requires the computation of failure probabilities. Although more comprehensive than concurrent approaches, the literature about this topic is scarce. Time-dependency can be considered, broadening the scope of the analysis, but further complicating the solution. In this work, a numerical framework for solving time-dependent risk optimization problems is proposed. It consists in a Monte Carlo simulation based approach, where two adaptive coupled metamodels are employed. In the first level, objective functions are approximated, and in the second, the limit state functions related to the computation of the failure probabilities. An iterative procedure is developed for selecting candidate points to each surrogate model's design of experiment. The accuracy and generality of the method is shown in an example including system-reliability and load-path dependent failures.

Keywords: Risk-Based Optimization; Time-Dependent Reliability; Adaptive Kriging

1 Introduction

Different approaches have been proposed in the literature to solve design optimization problems considering structural reliability. In reliability-based design optimization or RBDO [1–3], a deterministic objective function involving material and manufacturing costs is minimized under reliability constraints. This approach is a natural extension of deterministic design optimization, where deterministic constraints are replaced by probabilistic design constraints. A different problem is obtained when structural reliability is part of the objective function. In life-cycle cost or risk optimization [4–6], the objective function is formulated in terms of total expected costs, which includes expected costs of failure. These, in turn, are given by the product of failure costs by failure probabilities. Risk optimization allows one to find the optimal point of balance between safety and economy in structural designs. Risk optimization also allows different failure modes to compete with each other.

Comprehensive literature reviews [3, 7–9] reveal that the RBDO problem has received much more attention than the life-cycle cost or risk optimization problems. Several very efficient methods have been proposed for solving RBDO. In particular, several methods were designed to overcome the nested optimization loops arising from the use of First Order Reliability Method (FORM) for structural reliability evaluation. In contrast, not much is found in the literature about solving risk optimization problems. Moreover, it is worthwhile to emphasize that the underlying reliability problems are *time-variant*, due to the presence of stochastic loading, strength degradation (corrosion, fatigue), consideration of inspection and maintenance, etc., which adds another level of complexity.

Assessing the reliability of engineering structures under random load processes, and with consideration of resistance degradation, requires time variant reliability formulations. Unfortunately, analytical or semi-analytical solutions of time-variant reliability problems are limited to very specific cases [10]. The up-crossing rate solution is limited to scalar loads with Gaussian distribution. The out-crossing rate solution is limited to polyhedral failure domains. Fast probability integration is subject to convergence problems. Load combination solutions are mainly limited to discrete (pulse-like) processes. Time integrated or extreme value solutions neglect resistance degradation, and so on. Hence, most often time-variant reliability problems have to be solved by Monte Carlo simulation. This has a significant impact in computational costs, which makes the outer optimization loop impractical. Hence, general methods for solving time-variant risk optimization problems shall involve: a) speeding Monte Carlo simulation via dedicated techniques; and/or b) using surrogate models to simplify (approximate) the underlying time-variant reliability problem.

With respect to the first point, Gomes and Beck [11] proposed a Monte Carlo-based method which involves finding the roots of the limit state function, in the design space, for each sample. Rashki et al. [12] and Okasha [13] proposed efficient solutions for risk optimization involving random design variables. These solutions are based on the ranked weighted average simulation of Rashki et al. [14]. Regarding the second point, Echard et al. [15] proposed an active learning method, combining Kriging and Monte Carlo simulation, for reliability analysis. A similar approach was adopted to RBDO by Dubourg et al. [16], Moustapha et al. [17]. Wang and Chen [18] presented an equivalent stochastic process transformation approach for solving general time-variant reliability problems. This approach was employed by Li et al. [19] to solve RBDO problems.

Based on the above observations, this paper proposes a general procedure for solving time-variant risk optimization problems, based on adaptive Kriging [15, 20, 21]. The proposed scheme has some similarities with Li et al. [19]; however, herein it is applied for solving time-variant risk optimization problems. Moreover, equivalent stochastic process transformation is not employed herein. In the proposed approach, two adaptive Kriging surrogate models are constructed, namely one for approximating the objective function and one for approximating the limit state function. An expected improvement function is employed to efficiently choose additional support points for the surrogate models. Typical time-variant reliability problems, involving random loads and stochastic corrosion degradation, are used as examples to illustrate efficiency and accuracy of the proposed approach.

2 Reliability Problem Statement

In a context where structures degrade in time, or when loads are described as stochastic processes, it may be important to calculate not only instantaneous probabilities of failure, but the probability of a failure occurring within a certain time interval, sometimes referred to as the *cumulative probability of failure* in the literature. Consider a set $\boldsymbol{X}(t,\omega)$ of $M = p + q$ elements representing the uncertainties of a given problem, where $X_j(\omega)$, $j = \{1,\ldots,p\}$ are random variables, typically describing geometric characteristics and material properties, and $X_k(t,\omega)$, $k = \{p+1,\ldots,p+q\}$ are random processes. In this notation, ω is the outcome in the space of outcomes Ω. Moreover, let $\boldsymbol{d}$ be a vector that gathers together all the system's design parameters. It may include parameters describing moments of random variables, in case tolerances on design dimensions are included in the analysis [22]. A limit state function $g(\boldsymbol{d},t,\boldsymbol{X}(t,\omega))$ defines, for a given $\boldsymbol{d}$, safe states if it is greater than zero and failure if it is smaller than zero, so that the boundary between desirable and undesirable structure responses is given by the limit state surface of equation $g(\boldsymbol{d},t,\boldsymbol{X}(t,\omega)) = 0$:

$$D_f(\boldsymbol{d},t) = \{\boldsymbol{d},\boldsymbol{X}(t,\omega) : g(\boldsymbol{d},t,\boldsymbol{X}(t,\omega)) \leq 0\} \qquad \text{is the failure domain,}$$

$$D_s(\boldsymbol{d},t) = \{\boldsymbol{d},\boldsymbol{X}(t,\omega) : g(\boldsymbol{d},t,\boldsymbol{X}(t,\omega)) > 0\} \qquad \text{is the safe domain.} \tag{1}$$

For each limit state in the problem, the instantaneous probability of failure P_{f_i} at a time $t = \tau$ is calculated as:

$$P_{f_i}(\boldsymbol{d};\tau) = \mathbb{P}\left(g(\boldsymbol{d},\tau,\boldsymbol{X}(\tau,\omega)) \leq 0\right) = \int_{D_f(\boldsymbol{d},\tau)} f_{\boldsymbol{X}}(\boldsymbol{x})\mathrm{d}\boldsymbol{x}, \tag{2}$$

where $\mathbb{P}(\bullet)$ indicates the probability of the event $\bullet$ and $f_{\boldsymbol{X}}$ is the joint probability density function of the random variables $\boldsymbol{X}$ for a given configuration $\boldsymbol{d}$ at a time τ.

In the problems studied herein, the quantity of interest is the so-called *cumulative probability of failure* $P_{fc}(t_1,t_2)$ which is defined for a given configuration $\boldsymbol{d}$ as the probability of occurrence of a structural failure within the time interval $[t_1,t_2]$:

$$P_{f_c}(\boldsymbol{d};t_1,t_2) = \mathbb{P}\left(\exists \tau \in [t_1,t_2] : g(\boldsymbol{d},\tau,\boldsymbol{X}(\tau,\omega)) \leq 0\right) \tag{3}$$

Not many approaches have been suggested to compute P_{f_c}, the most widely used being probably the so-called *out-crossing approach*. Popular methods include the *PHI2 approach* [23] and the *asymptotic PHI2 method* [24]. The accuracy of such methods is significantly limited by the consideration of approximations in its formulation The introduction of the first-order reliability method (FORM) in the problem solution is known to lead to spurious results in presence of multiple design points or highly non-linear limit states. A comprehensive review about time-dependent reliability can be found in Melchers and Beck [10]. In order to avoid the aforementioned limitations, a direct Monte Carlo simulation approach is considered instead in this paper. The adoption of such simulation techniques may lead to excessive computational burden, and thus will be coupled to surrogate modeling, as shown in the sequel.

2.1 Simulation-based estimation of the cumulative failure probability

The adopted simulation approach basically consists in drawing sample trajectories of the limit state function over the time interval of interest, and then counting the number of such trajectories for which failure occurs within each time step. In order to do so, the random processes involved in the problem must first be discretized, *i.e.* represented by a finite set of correlated random variables [25]. In this work, the *expansion optimal linear estimation* (EOLE) method, after Li and Der Kiureghian [26], is employed.

Let $X(t,\omega)$ be a scalar Gaussian random process, with mean $m(t)$, standard deviation $\sigma(t)$ and autocorrelation coefficient function $\rho_X(t_1,t_2)$. An arbitrary number of time points P are selected in the interval $[0,\mathcal{T}]$, so that $t_1 = 0$ and $t_P = \mathcal{T}$. The EOLE expansion is then given by:

$$X(t,\omega) \approx m(t) + \sigma(t)\sum_{i=1}^{r}\frac{\xi_i(\omega)}{\sqrt{\lambda_i}}\phi_i^T C_{t,t_i}(t), \tag{4}$$

where $\{\xi_i(\omega), i = 1, \ldots, P\}$ are independent standard normal variables, $\{\phi_i, \lambda_i, i = 1, \ldots, r\}$ are the eigenvectors and eigenvalues of the correlation matrix $\boldsymbol{C}$ sorted in decreasing order, with $\boldsymbol{C}_{ij} = \rho_X(t_i, t_j), i, j = \{1, \ldots, P\}$. The order of the expansion is defined by the number of terms $r \leq P$ that are kept after truncating the series. One usually chooses r in such a way that a significant part of the spectrum of $\boldsymbol{C}$ is retained, i.e. for an $\varepsilon \ll 1$:

$$r = \min_{k \in [1,..,P]} \left\{ k, \sum_{i=1}^{k} \lambda_i \geq (1 - \varepsilon) \text{ tr } \boldsymbol{C} \right\} \tag{5}$$

where tr $\boldsymbol{C} = \sum_{i=1}^{P} \lambda_i$ is the trace of the correlation matrix.

Once the random processes are discretized, it is possible to sample trajectories of the limit state function itself. Consider the limit state $g(\boldsymbol{d}, t, \boldsymbol{X}(t, \omega))$ for a given $\boldsymbol{d}$ in the time interval $[0, \mathcal{T}]$. Samples of the random processes $X_k(t, \omega)$, $k = \{p + 1, \ldots, p + q\}$ are built from the EOLE expansions, and the time independent random variables $X_j(\omega)$, $j = \{1, \ldots, p\}$ are sampled once and remain the same throughout the whole trajectory. Let G be an array length N, where N is the number of time instants in which the continuous time is discretized. The values obtained in the simulation are stored in this array, where each position $i = 1, ..., N$ corresponds to a time $t_i = (i - 1) \cdot \Delta t$, where $\Delta t = \frac{\mathcal{T}}{N-1}$ is the sampling step, considering a uniform discretization. For each time interval $[t_i,\, t_{i+1}]$, a counter k_{i+1} is defined. Every time g presents the first outcrossing in the interval $[t_i,\, t_{i+1}]$, all the counters k_n, with $n = i + 1,\, \ldots\, , N$ are increased (i.e. all the remaining counters on the time interval after the outcrossing are increased). A brute Monte Carlo estimation for the cumulative probability of failure until an arbitrary instant t_i, i.e. $P_{fc_{MC}}(0, t_i)$, is given by:

$$P_{fc_{MC}}(0, t_i) = \frac{k_i + k_0}{N_{MC}}, \tag{6}$$

where k_0 counts the number of failures at $t = 0$.

3 Risk Optimization Formulation

Defining a structural configuration which is safe and cost efficient at the same time is a challenge for the structural designer. Unfortunately, structures will always be associated with a probability of failure. Thus, when its total life cost is of interest, a comprehensive approach should account for the expected cost of failure. In risk optimization, different cost terms, associated with different phases of the structure life, are considered. The function to be minimized is the total life cost $C_T(\boldsymbol{d})$, defined by:

$$C_T(\boldsymbol{d}) = C_I(\boldsymbol{d}) + C_O(\boldsymbol{d}) + C_{I\&M}(\boldsymbol{d}) + C_{EF}(\boldsymbol{d}), \tag{7}$$

where $\boldsymbol{d} \in \mathbb{D}$ is a given design configuration. This cost is composed of various terms, namely the **I**nitial design costs C_I, **O**peration costs C_O, **I**nspection and **M**aintenance costs $C_{I\&M}$, and the **E**xpected cost of **F**ailure C_{EF} defined as:

$$C_{EF} = \sum_{j=1}^{N_{ls}} P_{f_j} C_{f_j}, \tag{8}$$

where $j = \{1, \ldots, N_{ls}\}$ enumerates different limit states associated with a possible failure that occurs with a probability P_{f_j} and whose cost is C_{f_j}. Design and reliability constraints can also be considered, so that the optimization problem can be cast as:

$$\begin{aligned} \boldsymbol{d}^* &= \arg\min_{\boldsymbol{d} \in \mathbb{D}} C_T(\boldsymbol{d}), \\ \text{subject to:} \quad P_{f_j} &\leq \bar{P}_{f_j}, \quad j = \{1,\, \ldots\, , N_{ls}\}, \end{aligned} \tag{9}$$

where $\bar{P}_{f_j}$ are target failure probabilities that shall not be exceeded, for each limit state.

Constraints are often unnecessary in this type of problem, since the probabilities of failure are directly defined in the objective functions. Although reliability constraints can be considered in order to comply with standards, the solution domain $\mathbb{D}$ may also be limited by bound constraints, so that only possible structural configurations are studied.

The consideration of only initial and expected failure costs is a common practice in risk optimization, neglecting the inspection and maintenance terms [27, 28]. Aissani et al. [29] explain that the failure cost is particularly important, because it largely affects the optimal solution in an uncertain context, whereas other terms can usually be regarded as deterministic. Several works indicate that maintenance costs could be high over a structure's lifespan, but generally have weak dependence on the design variables [30]. The same is observed with inspection costs, as shown by Gomes et al. [31]. Therefore, in this work, only initial and failure cost terms will be considered in the objective functions.

Since the life cycle of a structure may be considered to span over years or decades, the costs to be optimized cannot be directly treated. Economic changes over time would make the considered values unrepresentative. In order to account for this effect, the structural life time can be discretized, and all costs brought to present value considering discount rates over each period (e.g. yearly discount rates). This way, cumulative failure probabilities associated with each given period can be considered to compose the expected cost of failure as follows [32]:

$$C_{EF}^{PV}(\mathcal{T}) = \sum_{j=1}^{N_{ls}} \sum_{n=1}^{\mathcal{T}} \frac{P_{fc_{jn}} C_{f_{jn}}}{(1+\eta)^n} \tag{10}$$

C_{EF}^{PV} is the expected cost of failure in present value, $P_{fc_{jn}}$ and $C_{f_{jn}}$ are, respectively, the so-called *cumulative probability* and cost of failure of the j-th limit-state in year n, and η is the discount rate, herein adopted as 1% per year. In the remainder of this paper, instead of Eq. (8), Eq. (10) will be used to compute the expected cost of failure in Eq. (7).

4 Proposed Framework

The solution of the problem in Eq. (7) relies on optimization techniques which would usually require thousands of calls to the objective function C_T. Furthermore, the evaluation of a single cost $C_T(\boldsymbol{d})$ requires to solve a time-variant reliability analysis using Monte Carlo simulation. The associated cost amounts to millions of calls to the limit-state function. Solving naively this problem as introduced above would therefore be extremely time-consuming. This becomes even more intractable when the limit-state function involves expensive-to-evaluate computational models.

To address this challenge surrogate modeling is used in this paper. The basic idea is to replace a time-consuming black box model by an analytical proxy that can be evaluated millions of times at practically no cost. Several surrogate modeling techniques have been introduced in the literature to solve optimization and reliability analysis problems, *e.g.* response surface models [33], polynomial chaos expansions [34–37], support vector machines [38, 39], neural networks [37, 40, 41] or Kriging [16, 17, 37, 42, 43]. In this work, we are interested in Kriging as it features a built-in error measure that arises from epistemic uncertainty and which allows for the development of active learning techniques. Such techniques allow one to reduce the computational cost of building the surrogate model by controlling its accuracy only in confined regions of the input space.

In summary, the proposed framework is based on two coupled surrogate models. In the inner loop, an adaptive Kriging surrogate of the limit state equations is built. This allows us to effectively estimate various cumulative failure probabilities for different design choices using the simulation-based approach presented in Section 2.1. In the outer loop, the optimization is carried out using EGO. The computation of the various costs $C_T(\boldsymbol{d})$ in this stage is performed using the inner Kriging model. The two stages are decoupled yet interdependent, henceforth the accuracy of the inner metamodel is crucial for the proposed methodology to provide valid results. This accuracy is enforced by using an adaptive scheme (EGRA) with a tight convergence criterion.

5 Example

Consider the truss composed by circular bars 1 and 2, as shown in Figure 1. Two time-variant loads $H(t)$ and $V(t)$ are applied on the upper node. Three failure modes are considered: tensile rupture of bar 1 (g_{t1}), buckling of bar 1 (g_{b1}), and buckling of bar 2 (g_{b2}). Thus, a time-variant system reliability problem is defined considering the limit state equations associated to these failure modes:

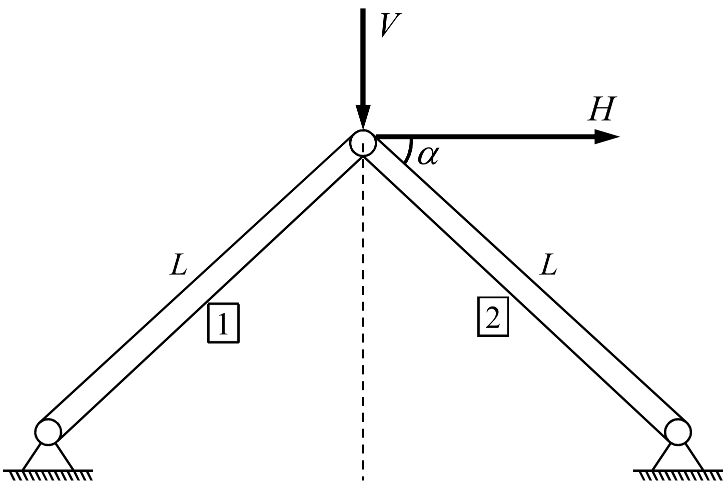

Figure 1. Two-bar truss scheme

$$
\begin{aligned}
g_{t1}(\boldsymbol{X},t) &= A_1\sigma_u - \left[\frac{H(t)}{2\cos\alpha} - \frac{V(t)}{2\sin\alpha}\right] \\
g_{b1}(\boldsymbol{X},t) &= \frac{\pi^2 E I_1}{L^2} - \left[-\frac{H(t)}{2\cos\alpha} + \frac{V(t)}{2\sin\alpha}\right] \\
g_{b2}(\boldsymbol{X},t) &= \frac{\pi^2 E I_2}{L^2} - \left[\frac{H(t)}{2\cos\alpha} + \frac{V(t)}{2\sin\alpha}\right] \\
g_{sys}(\boldsymbol{X},t) &= \min(g_{t1}, g_{b1}, g_{b2})
\end{aligned}
$$

where A_i is the area of the i-th bar in m^2 and L is the length of the bars in m. The truss is symmetric. The two bars have the same Young Modulus E, defined as a normal random variable with $\mu_E = 70$GPa and $COV_E = 0.03$, and the same ultimate tensile strength, defined as a normal random variable σ_u, with $\mu_{\sigma_u} = 24.5643$MPa and $COV_{\sigma_u} = 0.1$. This value of ultimate stress was set so as to result in a tight compromise between the three different failure modes. The probability that random variables reach negative values is very small and can be neglected in this example. This problem is load-path dependent, i.e. the structure can violate different limit states or fail at different times depending on the trajectory that the loads follow in time. To illustrate the load path dependent problem, consider that the radius of the cross sections are $r_1 = 4$mm for the first bar, and $r_2 = 5.2$mm for the second bar. Figure 2 shows three possible load paths, as well as the limit state equations, evaluated at the mean $\mu_{\boldsymbol{X}}$. Suppose that at time $t = t_0$ the loads are at point A, and at $t = t_f > t_0$, the loads correspond to point B. If the loads follow Path 1, the structure fails due to buckling of the first bar. If the loads follow Path 2, the horizontal load is increased first, and the structure fails by tensile rupture of bar 1. Now, if the loads follow Path 3, which corresponds to a concomitant increase in both loads, point B is safely reached, and there is no failure. Thus, the load-path dependency of the problem is demonstrated.

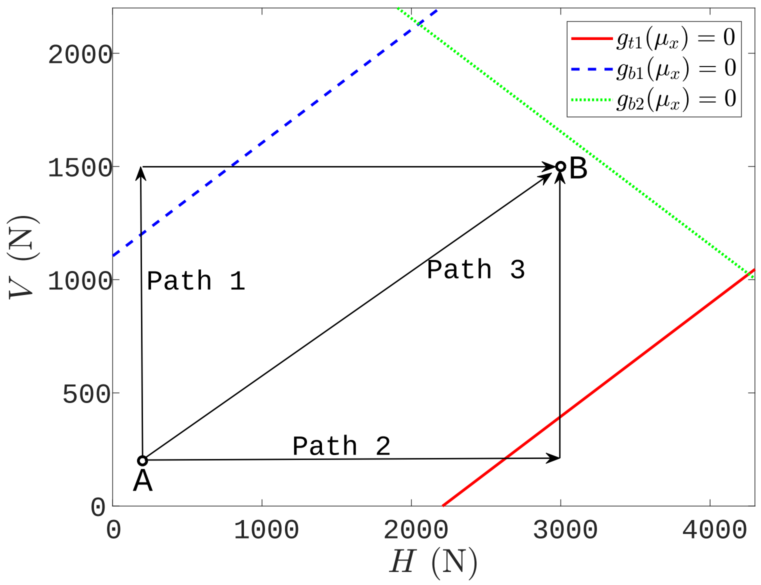

Figure 2. Load Paths

When the loads are stochastic processes, there is an infinite number of possible trajectories, and evaluating structural reliability depends on considering such trajectories, which only adds complexity to the problem. Load-path dependent problems cannot be solved by usual techniques, such as time-integration (extreme value analysis) or load combination, as discussed in Melchers and Beck [10]. However, load path-dependent reliability problems can be solved by explicit simulation of load process realizations, as proposed in this work.

Consider now that one is interested in the optimal areas for the two bars, aiming at minimizing total costs in a risk-optimization scenario. Forces $V(t)$ and $H(t)$ are stochastic Gaussian processes with means 1 kN and 2 kN, respectively. Both loads have a COV of 0.2 and a correlation length of $\lambda_V = \lambda_H = 1$ month. The auto-correlation function of the random processes is given by:

$$R(x, \lambda) = \exp\left[-\left(\frac{x}{\lambda}\right)^2\right] \tag{11}$$

The loads are independent of each other and of the other random variables. A time interval of 10 years is studied, so that the objective function of the problem can be stated as:

$$\begin{aligned} C_T(r_1, r_2) &= C_I(r_1, r_2) + \sum_{i=1}^{10} C_f P_{fc_i}(r_1, r_2) \\ \text{s.t. } 4\,\text{mm} &\leq r_1 \leq 6\,\text{mm} \\ 4\,\text{mm} &\leq r_2 \leq 6\,\text{mm} \end{aligned} \tag{12}$$

The initial costs are proportional to the volume of the structure $C_I(r_1, r_2) = 10^5\big(A_1(r_1)+A_2(r_2)\big)L$, and the cost of system failure is 10 times higher. An annual discount rate of 2% is also considered. Different failure costs could be associated to different limit states, without any change in the solution procedure.

Table 3 shows the results for the optimization problem, comparing 10 runs of the approach proposed in this work (denoted by 'EGO') and a reference obtained with 20 generations of 30 particles of a PSO algorithm, performed without the aid of surrogate models. The standard deviations of the obtained results are denoted between parenthesis.

As seen from table 3, the results obtained with both methodologies are remarkably consistent, with less than 1% discrepancy between the optimum design radii and associated total cost.

Table 1. Mean and COV of optimization results and reference

	r_1(mm)	r_2(mm)	C_T	N_{calls}
EGO	4.37(0.01)	5.32(0.01)	5.16(0.01)	17(5.1)
PSO	4.35	5.29	5.13	600

6 Conclusion

Expected life-cycle cost, or risk optimization, allows one to find the optimal points of compromise between safety and economy in structural desing. Typically, the underlying reliability problem is time-variant, and its solution is far from trivial. Problems involving strength degradation or load-path dependency usually require solution by Monte Carlo simulation, with a large computational burden, especially in an optimization context. To address efficiently and accurately this type of problems, a nested Kriging approach with active learning is proposed in this paper. The strategy is based on constructing two adaptive Kriging surrogates. One surrogate is built so as to mimic the objective (cost) function, starting from a design of experiment built with LHS in the space of the design variables, which is further enriched as the optimization problem is solved using the EGO approach. Another Kriging surrogate model is built for the limit state functions, starting from a first design of experiment built with LHS in the augmented space of both design and random variables. The surrogate is then enriched using the EGRA strategy. An analytical example was studied, with satisfactory accuracy and convergence with a few calls to the objective function.

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