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# PARAMETRIC OPTIMIZATION OF TLCD-MAIN STRUCTURE COUPLED SYSTEM SUBJECT TO ARBITRARY STOCHASTIC EXCITATION

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**Abstract.** The vibration levels in slender structures, such as walkways, bridges, high towers, and wind turbines, are receiving more importance with free span increase. To preserve the structure lifespan, it is necessary to study additional mechanical devices capable of reducing the vibrational level of the main structure. Tuned Liquid Column Damper (TLCD) is a kind of passive absorber composed by a U-shaped tube filled with liquid, commonly, water. This study performs a parametric optimization using response maps to obtain TLCD optimum parameters to several arbitrary stochastic excitation. First of all, the optimum parameters of TLCD coupled to the main structure using response maps were compared to the analytical solution. Finally, the same procedure was reproduced to obtain an optimal parameter for other arbitrary stochastic excitation. The result obtained show a reasonable accuracy compared to the analytical solution. With this validation performed, we obtain the optimum values for Kaimal wind spectrum.

**Keywords:** TLCD, Passive Control, Optimization, Structural Vibration, Dynamic Vibration Absorber

## 1 Introduction

The vibration levels in slender structures, such as walkways, bridges, high towers, and wind turbines are getting more attention with free span increase. Slender structures are characterized by having low frequencies and damping, not avoiding unexpected responses. To preserve the structure lifespan, it is necessary to study additional mechanical devices capable to reduce the vibration level of the main structure. These equipment, known as vibration absorbers, are important for the structure's health. They help to prevent disasters on structures under natural hazards such as wind and earthquake.

The devices can be classified as active, passive, hybrid or semi-active. Tuned Liquid Column Damper (TLCD) is a type of passive absorber composed by a U-shaped tube filled with liquid, commonly, water. TLCD is a non-linear mechanical system due to the turbulent head-loss in oscillatory conditions [1]. Restoring and damping forces occurs due to the liquid mass when it oscillates with opposite phrase to the structure, settled by tuned parameters[2]. The TLCD holds researchers attention, since it presents low cost, easy handling, and lightweight. The headloss depends on the opening of the orifice in its centre[3, 4].

TLCD equipment has been receiving attention among researchers. Balendra et al [5] present an application of TLCD to buildings, a comparison between TLCD and TMD showed that the reduction of response by the TLCD could behave in a similar mode compared to the TMD. Gao et al. [6] present an analytical solution to harmonic excitation using an equivalent damping approach. Balendra et al. [5] show that for a better accomplishment the TLCD frequency has to be tuned as the tower frequency. However, although the TLCD system is not totally tuned, it still obtaining good performances. The authors also expose this lead to advantage since the structural frequency keeps changing throughout its lifespan.

Yalla and Kareem [3] investigate a set of optimum parameters for the TLCD. Mensah and Duenas-Osorio [4] examine a wind turbine structural response coupled to TLCD, and they observe reliability enhances. Shum [7] presents the optimal parameters of a TLCD-structure system using a non-linear closed form solution for white noise excitation. Alkmim [8] extend optimal parameters of TLCD-structure for other kinds of random excitation. Silva and Morais [9] compare numerical FE fluid-structure coupled solution by reporting to experimental results with a good agreement. There are several application of TLCD considering seismic behaviours [10–13], wind excitation [14–17], and base interactions [18, 19].

This study performs a parametric optimization using response maps to obtain TLCD optimum parameters to several arbitrary stochastic excitations. First of all, the optimum parameters of TLCD coupled to the main structure subject to white noise using response maps were compared to the analytical solution. Finally, the same procedure was reproduced to obtain an optimal parameter for Kaimal-spectrum stochastic excitations.

## 2 Tuned Liquid Column Damper to Main Structure

Figure 1 presents the schematically representation of the coupled system (main structure + TLCD). Deduced from the Bernoulli equation or the energy principles [20], governing equation of liquid column relative motion is given by:

$$\rho AL\ddot{w} + \frac{1}{2}\rho A\varepsilon\,|\dot{w}|\,\dot{w} + 2\rho Agw(t) = -\rho Ab\ddot{u}, \tag{1}$$

where $w$ is denoted to liquid relative displacement, $u$ is structure horizontal displacement of main structure, $\varepsilon$ is head loss coefficient, $\rho$ is fluid density, $A$ is cross sectional area, and $g$ local gravity considered constant. The geometric parameters of TLCD reservoir are defined by the variables fluid column height $h$, the horizontal length $B$, and the total length $L = B + 2h$. The expression $-\rho AB\ddot{u}(t)$ is the fluid coupling term.

In order to perform the structural vibration control, the main system is reduced to a SDOF model (single degree of freedom). The linearized equation of motion of the reduced main structure equipped

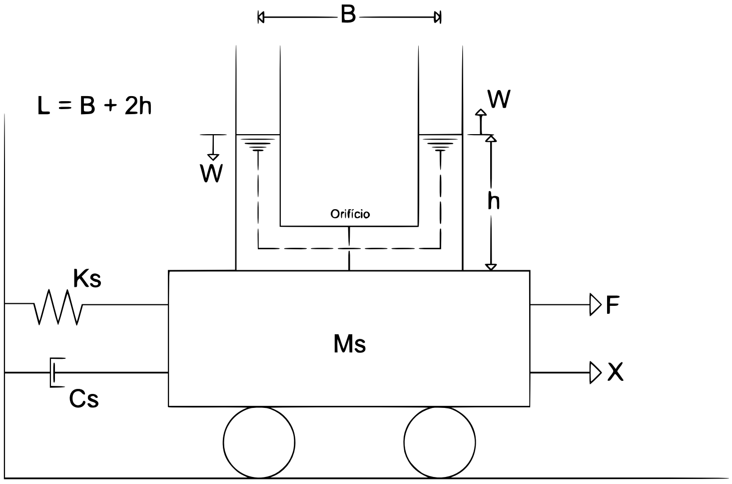


Figure 1. Schematic representation of coupled system main structure and TLCD.

with TLCD for lateral vibration is given by:

$$(m_s + m_w)\ddot{u} + \rho A b \ddot{w} + c_s \dot{u} + k_s u(t) = F(t), \tag{2}$$

where $m_s$ is structure mass, $k_s$ structural stiffness, $c_s$ structural damping coefficient, $m_w = \rho AL$ fluid mass, $c_w = 0.5\rho A\varepsilon\,|\dot{w}(t)|$ fluid damping coefficient, $k_w = 2\rho Ag$ fluid stiffness, and $F(t)$ the excitation load applied at main structure. In literature, the principal dimensionless parameters are defined as mass ratio $\mu = m_w/m_s$ and aspect ratio $\alpha = B/L$

The fluid damping is non-linear, but it can be replaced by an linear equivalent damping ratio [6, 7, 20]. The fluid damping coefficient is defined as $c_w = 0.5\rho A\varepsilon\,|\dot{w}(t)| = 2m_w\xi_w\omega_w$ where fluid damping coefficient $\xi_w = 2/(3\pi)\varepsilon w_0 L$ may be similar to $2m_w\xi_w\omega_w$, where $w_0$ is the response amplitude of the liquid and $\varepsilon$ the headloss coefficient. Besides, $F$ is the force acting on the primary system where the coefficient is $\Delta = 1$ for base excitation and $\Delta = 0$ for primary system excitation [3]. Grouping Eq.(1) and (2), the dimensionless motion equation is obtained by:

$$\begin{bmatrix} 1+\mu & \alpha\mu \\ \alpha\mu & \mu \end{bmatrix} \begin{pmatrix} \ddot{u} \\ \ddot{w} \end{pmatrix} + \begin{bmatrix} 2\omega_s\xi_s & 0 \\ 0 & 2\omega_w\xi_w \end{bmatrix} \begin{pmatrix} \dot{u} \\ \dot{w} \end{pmatrix} + \begin{bmatrix} \omega_s^2 & 0 \\ 0 & \omega_w^2\mu \end{bmatrix} \begin{pmatrix} u \\ w \end{pmatrix} = \begin{pmatrix} \delta_{st} \\ \Delta\mu\delta_{st} \end{pmatrix}, \tag{3}$$

with $\delta_{st} = F_0/k_s$.

## 3 Map Response

The steady-state responses of the 2DOF coupled system under excitation of an harmonic force ($F = F_0 e^{i\Omega t}$) are provided by:

$$\begin{Bmatrix} u & w \end{Bmatrix}^T = \begin{Bmatrix} U_0 & W_0 \end{Bmatrix}^T e^{i\Omega t} = Z_0 e^{i\Omega t} \tag{4}$$

Substituting Eq. (4) in Eq. (3) we obtain:

$$K_d(\Omega) Z_0 = F_0 \rightarrow Z_0 = H(\Omega) F_0, \tag{5}$$

where the dynamic matrix $K_d(\Omega) = \mathbf{K} + i\Omega\mathbf{C} - \Omega^2\mathbf{M}$ and the impedance function $H(\Omega) = (K_d(\Omega))^{-1}$. The dynamic matrix $K_d(\Omega)$ is divided by $\omega_s^2$. Then, the frequency ratio $r$ is defined as $r = \Omega/\omega_s$, and the tuning ratio as $\gamma = \omega_w/\omega_s$. Thus, the maximum dynamic response $U_o(r)$ of the main structure is given by:

$$\max(U_0) = f(\mu, \alpha, \xi_s, \xi_w, \gamma, F_0, r), \tag{6}$$

where the response is function of dimensionless parameters mass ratio $\mu$, aspect ratio $\alpha$, tuning ratio $\gamma$, the structural $\xi_s$ and fluid $\xi_w$ damping, and the force excitation amplitude $F_0$ a single value of frequency $r$. For a stochastic excitations, this analysis of maximum dynamic response must be done analysing the displacement variance $\sigma_u^2$.

Slender structures have low damping rates. When thinking about the worst case scenario, it is assumed that there is no structural damping ($\xi_s = 0$). Colheirinhas [21] explain that the absorber damping ($\xi_w$) does not have a direct influence on the setting of $\mu$ and $\gamma$ but only in the amplitude response $\sigma_y^2$. Hence, the fluid damping $\xi_w$ has not a considerable influence on the TLCD design.

From equation (6), the dynamic displacement response was obtained by $U_0/F_0 = f(\mu, \gamma, r)$ given a constant alpha, $\forall r$. Thus, a response map ($\Upsilon$) is defined as displacement variance $\sigma_u^2$ for a given combination of $\mu$ and $\gamma$ for a constant $\alpha$. The equation below summarizes it, as follows:

$$\begin{aligned} \Upsilon : \mathbb{R} \times \mathbb{R} \times \mathbb{R} &\longrightarrow \mathbb{R} \\ (\mu, \gamma, \alpha \doteq \bar{\alpha}) &\longrightarrow \sigma_u^2 = 2 \int_0^{\Delta\Omega} H^* F_0 H d\Omega, \end{aligned}$$

where the displacement variance integral is defined for interval $[0, \Delta\Omega]$ large as possible to enable its determination in feasible computing time.

The minimization goal is to find ($\mu$, $\gamma$) combinations that result in the minimum dynamic response displacement of the main system. The present approach can determine the optimum parameters of the coupled system (main structure + TLCD) subject to random excitation with a broadband spectrum, e.g. wind actions.

Supposing a white noise excitation, $F_0$ has a constant value:

$$\sigma_u^2 = 2F_0 \int_0^{\Delta\Omega} |H|^2 \, d\Omega \tag{7}$$

Figure 2 presents the typical displacement FRF of main structure $H_u(\Omega)$. We can note that the area $\max(H_1^2(\Omega))\Delta\Omega$ is greater than the integral of $|H_1(\Omega)|^2$. The displacement's variance is proportional of $\max H_1(r)$, in other words, ($\sigma_u^2 \propto max(H_1)$). Then, minimum of variance displacement ($\min(\sigma_u^2)$) is obtained with the same combination of $(\mu, \gamma)$ obtained by the minimum of $\max(H_1(r))$.

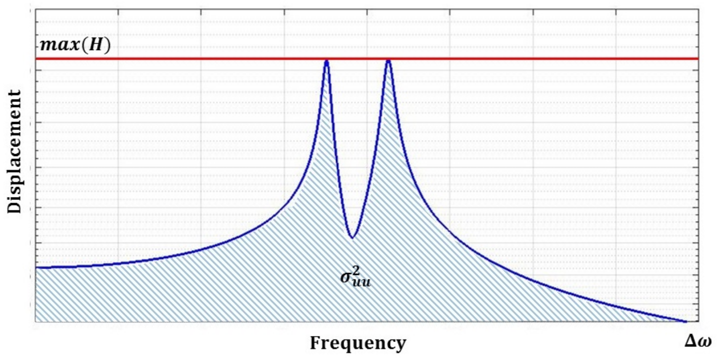


Figure 2. Displacement function of response frequency $H_u(\Omega)$ as function of frequency $\Omega$ for coupled mechanical system (main system + TLCD).

## 4 Results

The numerical were performed for a wind turbine SDOF model studied earlier by Alkmim et al [22], the system parameters are: $k_s = 470685 N/m$, $m_s = 34975 kg$. The absorber aspect ratio ($\alpha$) was setled as 0.8, $\xi_s = 0\%$.

### 4.1 Validation with white noise excitation

In order to verify the optimization procedure, a simple case with a closed-form solution is investigated considering a white noise excitation ($S_{WN}(\omega) = S_0$). Yalla and Kareem [3] achieved the closed expression for optimum tunning ratio ($\gamma_{opt}$) and damping ratio($\xi_{opt}$) for an undamped structure case ($\xi_s = 0$).

$$\gamma_{opt} = \frac{\sqrt{1+\mu(1-\alpha^2/2)}}{1+\mu} \tag{8}$$

$$\xi_{opt} = \frac{a}{2}\sqrt{\frac{2\mu(\alpha^2\mu/4-\mu-1)}{\alpha^2\mu^2+\alpha^2\mu-4\mu-2\mu^2-2}} \tag{9}$$

In this approach, the response map is obtained by two methods. The first method integrate Eq.(7) by trapezoidal rule (Matlab command trapz) (Fig. 3a). The second method find the maximum value of displacement FRF $(\max(H_u(\Omega))$ (Fig. 3b). Figures 4 and 5 present the optimum combination of frequency ratio $\gamma$ and mass ration $\mu$ on response map that minimize $H_u(\Omega)$ with a comparison obtained with a response map grid $100 \times 100$ discretization and using an analytical fluid damping ratio $\xi_w$ (Eq. 9) or an automatized $\xi$ search minimization technique. The Figures 4 and 5 compare the results obtained by two techniques (with analytical solution) [3]. On Figure 4 the results show the influence of the optimum liquid damping ratio$\xi_{w,opt}$. It appears that the optimum values obtained by Eq.(7) observe a good adjust to the analytical solution. Besides, the Figure 5 shows the results of optimum damping ratio ($\xi$) found on maps.

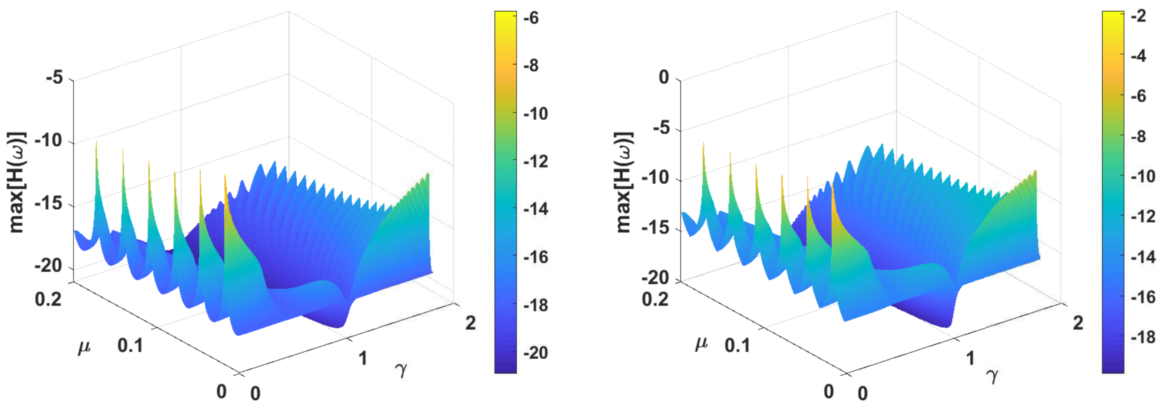


Figure 3. Respose map for white noise with $\alpha = 0.8$ and $\xi_s = 0$. ($a$) Method 1: Trapezoidal rule (trapz) and ($b$) Method 2: Hmax

### 4.2 Validation with Kaimal spectrum excitation

Although white noise spectrum covers a wide range frequency band with a constant value, other broadband spectrum types can physically represent wind profiles essentially. One model is the Kaimal spectrum described by:

$$S_{Km}(\omega) = [\frac{4S_0^2(L_k/v_{hub})}{1+(6\omega(L_k/v_{hub}))}]^{5/3}, \tag{10}$$

where $L_k = 340.2$ m is a scale parameter associated to the wind turbine high and $v_{hub} = 16m/s$ is the wind velocity average. The map response for this spectrum is presented in figures 6a and 6b using

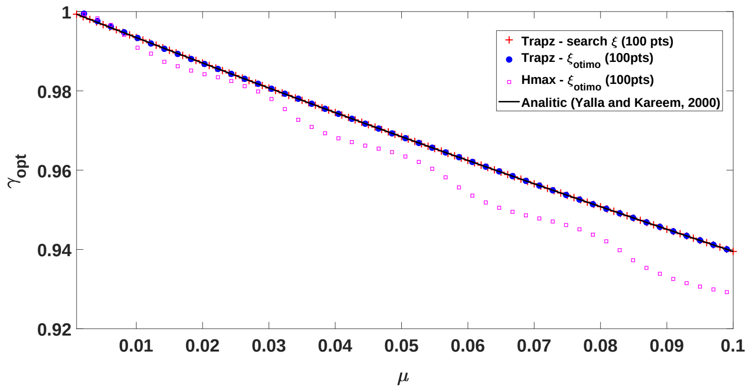


Figure 4. Optimum values of mass ratio $\mu$ and frequency ratio $\gamma$ to minimize response map subject to white noise excitation ($\alpha = 0.8$)[3].

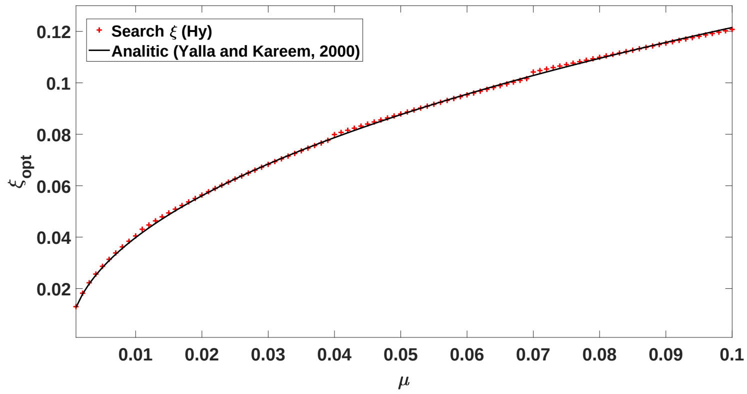


Figure 5. Optimum values of mass ratio $\mu$ and damping ratio $\xi$ to minimize response map subject to white noise excitation ($\alpha = 0.8$)[3].

the trapezoidal rule (**trapz**). On Figure 7 is presented the optimum values of mass ratio $\mu$ and tunning ratio $\gamma$ compared with Alkmim et al [22], which show great results. On this case, its is presented $\xi_w$ is a constant value, and then the proposed solution with the optimum values represented by the "searched $\xi$ (Hy)" curve. The solution of Fig 7 shows the relation with $\xi_{opt}$ points with mass ratio ($\mu$) comparing to analytical solution. As show in Alkmim et al (2018), the damping is less sensible to a stochastic excitation change. Moreover, the tuning ratio show a difference by report to literature inferior than $1\%$ that could be improve with an increase of map response grid.

## 5 Conclusion

This study performs a parametric optimization using response maps to obtain TLCD optimum parameters to several arbitrary stochastic excitations. First of all, the optimum parameters of TLCD coupled to the main structure using response maps subject to white noise spectrum were compared to the analytical solution. Finally, the same procedure was reproduced to obtain an optimal parameter for a wind stochastic excitations (Kaimal spectrum), using a technique of parametric optimization, called map response.

This paper presents an optimization theory that allows on identifying TLCD optimum parameters for arbitrary stochastic excitations: white noise and Kaimal spectrum. Damping forces occur due to the liquid mass oscillation with a phase shift with respect to the motion of the main structure. This motion has a non-linear particularity on the fluid damping, and linearization is presented.

The optimum parameters are settled to occur on the valley of the response map. On this area is

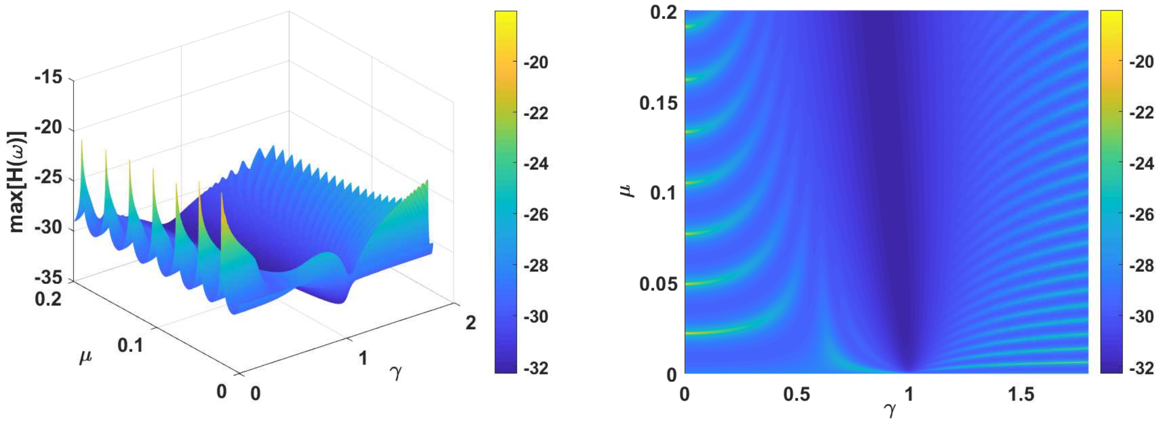


Figure 6. Map Response Kaimal Spectrum with $\alpha = 0.8$. Method 1: Trapz $(a)$Isometric View and $(b)$Top View

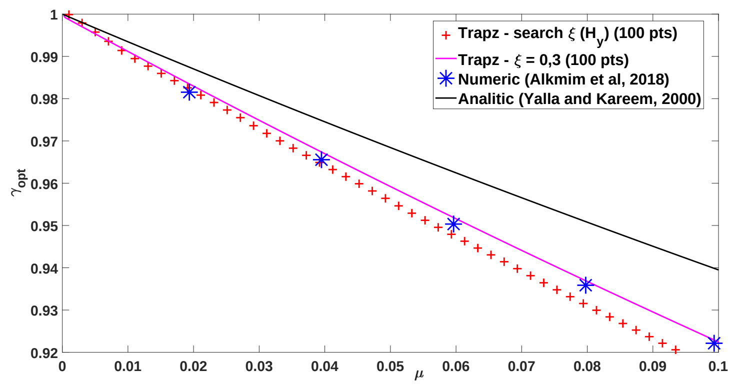


Figure 7. Optimum values for Kaimal spectrum (Alkmin et al, 2018) of Kaimal Spectrum with Alkmim et al data ($\alpha = 0.8$)[22].

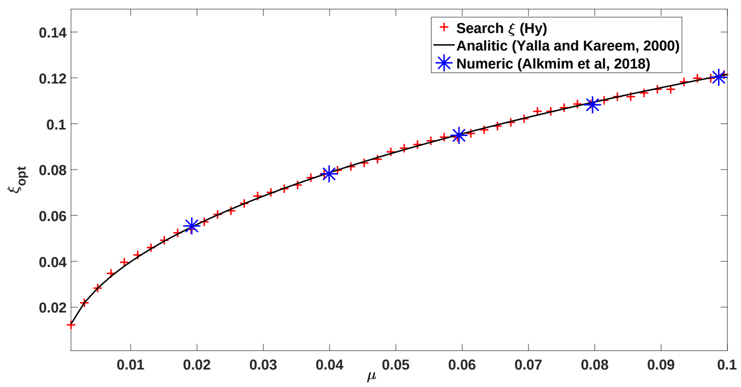


Figure 8. Optimum values for Kaimal spectrum (Alkmin et al, 2018) of mass ratio $\mu$ and damping ratio $\xi$ best points fitted with analitic[3].

observed the values of $\mu$ and $\gamma$ for a specific $\alpha$ and $\xi_w$. The results of the best region fit with the results expected in the literature [3, 22]. Besides, when the structure is subjected to white noise, the results show great dependence of the fluid damping ($\xi_w$) among the optimum parameters. On the $H_{max}$ case, it is demonstrated that teh method does not show the expected accuracy.

For white noise spectrum, the optimum values of tuning ratio $\gamma$ and mass ratio $\mu$ obtained show a reasonable accuracy compared to analytical solution [3]. With this validation performed, we obtain the optimum values $(\gamma, \mu)$ for Kaimal wind spectrum. These parametric optimum values are compared to Alkmin et al [22] with a good approximation. Further works are necessary to validate these results such as experimental validation and investigations considering other wind models.

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