PARAMETER SENSITIVITY ANALYSIS IN STEEL FIBER REINFORCED CONCRETE

Marcello Congro[1,2]
Fernanda L. G. Pereira[3]
Lourdes M. S. Souza[2]
Deane Roehl[1,2]

marcellocongro@tecgraf.puc-rio.br
fernanda.pereira@uerj.br
lmsilvadesouza@tecgraf.puc-rio.br
deane@tecgraf.puc-rio.br

Civil and Environmental Engineering Department / Pontifical Catholic University of Rio de Janeiro – PUC-Rio – Brazil
Tecgraf Institute / Pontifical Catholic University of Rio de Janeiro – PUC-Rio – Brazil
Civil Engineering Department / State University of Rio de Janeiro – UERJ – Brazil
Rua Marquês de São Vicente, 225 – Gávea – Rio de Janeiro – Brazil
Rua São Francisco Xavier, 524 – Maracanã – Rio de Janeiro – Brazil

Abstract. *In steel fiber reinforced concrete, the random dispersion of discrete fibers in the cementitious matrix and the distinct lengths, cross sections and volumetric contents cause deviations in the global behavior of a given structure. Traditionally, the association of probabilistic and numerical approaches reproduces the variability effects in the material mechanical behavior. However, the accomplishment of these simulations is strongly dependent on statistical study of experimental tests. In recent years, computational intelligence techniques emerge as powerful tools for parameter identification and calibration in engineering. However, parameter sensitivity analysis is a predecessor stage required for applying these new concepts into an Artificial Neural Network (ANN). In this sense, this paper presents a sensitivity study of macroscale parameters in steel fiber reinforced concrete by using the Design of Experiments (DOE) methodology. This technique is an important method to analyze the influence of one or more parameters on the given output of the ANN. An experimental database available in literature provides the input data for the ANN: water-cement ratio, volumetric content, diameter and length of steel fibers. The parameter outputs are the Young modulus (E), tensile strength (ft) and fracture energy (G) of the material. Response surface plots are provided in order to identify the relevant experimental parameters for the description of the mechanical behavior of the composite predicted by the ANN. Thereby, sensitivity analysis in Artificial Intelligence methods becomes an attractive approach to verify the mechanical behavior of concrete, establishing the most suitable parameters that will generate a reliable neural network.*

Keywords: Steel fiber reinforced concrete; Sensitivity analysis; Design of Experiments (DOE).

1 Introduction

In Structural Engineering, several mathematical models are employed to model real-world phenomena. However, in most of the cases, these simulations can be highly complex, making it difficult to understand the relationships between the input and output parameters of a given model. In this sense, sensitivity analysis emerge as an essential tool for model building, especially for material behavior modeling.

For computational simulations involving composite materials in a macroscale approach, the elastic properties (Young's modulus and Poisson's ratio), tensile strength and fracture energy need to be assessed in order to understand the material mechanical behavior. At this scale, there is no explicit representation of the heterogeneities such as fibers or aggregates in the finite element model and the required parameters for the analyses represent the combined effects of the cementitious matrix and steel fibers. Consequently, these simulations consider equivalent properties for material modeling. However, it is important to study which material parameters are truly relevant for the description of the mechanical behavior of the composite.

Due to their excellent mechanical properties, fiber reinforced cement based materials have been widely used. The addition of randomly distributed short fibers in the brittle matrix can significantly reduce the brittleness of the composite material (Naaman, 1998; 2003; Congro *et al*., 2017; 2019). Considering the anisotropic behavior of the composite due to fiber random dispersion within the cement matrix, finite element simulations coupled to probabilistic analysis attempt to reproduce the nonlinear behavior of concrete after the first crack (Naaman *et al*., 1974; Karinsrki *et al*., 2016; Ríos *et al*., 2017; Congro *et al*., 2019). However, many numerical studies based on probabilistic methodologies are strongly dependent from experimental tests. This assessment can inspect the macroscale behavior of a given structural system; however, they are usually costly and time consuming. Some obstacles can also be included in this scenario, such as how to get the materials for each experiment or how to access the mechanical properties and reproduce specific loading conditions.

In parallel, Artificial Intelligence (AI) has emerged as a new field for research and applications in many areas, such as knowledge and intelligent database systems in the last decades. According to Lu *et al*. (2012), the research in AI field has been developed since 1956 and wraps several disciplines in Science, such as Computer Science, Mathematics, Biology and Physics. This joint study is performed in order to reproduce the intelligent function of human brain. With the increasing advancement of computer technology and data processing during the last years, AI started to be a more practical field, strongly connected to problem-solving systems, especially in engineering applications. In this scenario, several studies have been performed by using those techniques, especially for complex problems that depend on professional expertise or experience (Lee & Mosalam, 2004; Sandemir, 2009; Naderpour *et al*., 2010; Arslan, 2010; Yuan *et al*., 2014; Babanajad *et al*., 2017).

This paper proposes a parameter sensitivity analysis of an artificial neural network (ANN) by using the Design of Experiments (DOE) and Response Surface Methodology (RSM). The main goal is to evaluate the relationship between each input variable of the network and the mechanical parameter outputs to be used in finite element simulations. ANN is generated considering an experimental database collected from literature. Water-cement ratio, steel fiber length, diameter and volumetric content of steel fiber reinforced concrete in tensile tests gather the input data for the network. Giving those variables, concrete parameters under tensile behavior can be predicted, namely Young's modulus, tensile strength and fracture energy of the material. Therefore, it is a valuable tool to predict the mechanical response of the composite, evaluating the most relevant input data for the neural network.

2 Design of Experiments (DOE)

Experiments are used to study the performance of processes and systems, and can be schematically represented by Figure 1. According to Montgomery (2013), a process is a combination of machines, methods, people, and resources that transforms an input into an output that has one or more observable response variables.

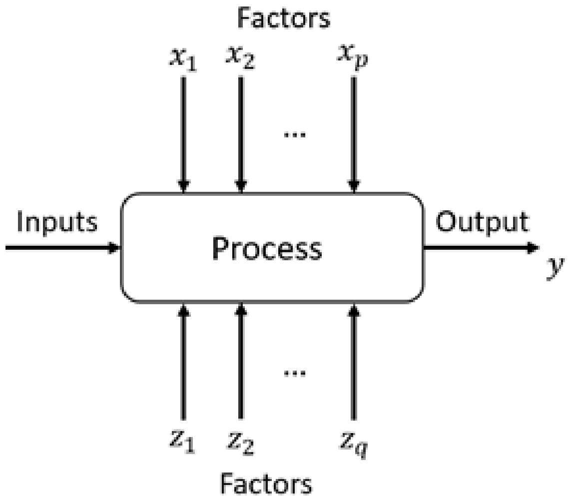


Figure 1. General model of a process (Adapted from Montgomery, 2013).

The main goals of experiments are to determine which variables (factors) are most influential in the response y (Myers *et al*., 2004; Montgomery, 2013). Experiments often are related to several factors, and DOE emerges as a structured method for settling the relationship between factors and process outputs (Yoon, 2007). Hence, it is a good procedure for planning experiments and yields reliable conclusions.

After running the experiments, it is possible to analyze the influence of each input parameter in the system response. Several DOE methods can be employed in order to study this behavior, and one of the most selected techniques is the response surface methodology (RSM). In other words, it is a collection of mathematical and statistical techniques for modeling and problem analysis in which the goal is to analyze the influence of input parameters for the given outputs.

Response surface design methodology is often used to refine models after having determined important factors using screening designs or factorial designs; or especially if there is a suspect curvature in the response surface (Khuri & Cornell, 1996). In most engineering cases, outputs have to be designed with curvature to allow the quadratic effects of each independent variable.

For example, consider an output variable y that needs to be studied in function of variables x_1 and x_2. If the expected response is denoted by $E(y) = f(x_1, x_2) = \gamma$, then the surface is called response surface, represented mathematically by Equation 1.

$$\gamma = f(x_1, x_2) \tag{1}$$

In several RSM problems, the relationship between the response and the independent variables is unknown. In this way, the first step is to find a suitable approximation for the relationship between y and the independent variables. A low-order polynomial in some region of the independent variables is often employed, such as first or second-order models. Equation 2 presents the approximating function for second-order models, usually applied when there is a curvature in the system.

$$y = \beta_0 + \sum_{i=1}^{k} \beta_i \, x_i + \sum_{i=1}^{k} \beta_{ii} \, x_i^2 + \sum \sum \beta_{ij} \, x_i \, x_j + \varepsilon \tag{2}$$

Almost all RSM problems use one or both of these models. Myers *et al.* (2004) and Montgomery (2013) point out that, for a relatively small region, such as the one analyzed during this paper, they usually work well.

In surface plot interpretation, when the border of the surface is horizontal (parallel to one of the axes), then there is no main effect. Each level of the factor affects the response in the same way, and the response mean is the same across all factor levels. On the other hand, when the line is not horizontal, then there is a main effect. Different levels of the factor affect the response differently. The steeper the slope of the line, the greater the magnitude of the main effect.

3 Methodology

This paper proposes an Artificial Neural Network (ANN) to predict parameters of direct tensile tests of cementitious composites reinforced with randomly dispersed straight steel fibers. A parameter sensitivity study was conducted using the Design of Experiment (DOE) method. Firstly, experimental results from literature are collected to generate the network input data. Secondly, ANN is trained considering specific configurations and architecture, returning the mechanical parameters of the composite material: Young's modulus, tensile strength and fracture energy. These are the most relevant parameters to be included in the finite element numerical simulations for direct tensile tests modeling. Then, a sensitivity analysis of the network is performed by using DOE and RSM. Response surface plots are provided in order to verify the influence of each parameter in the macro behavior of concrete. Figure 2 presents a workflow for the analyses developed during this study.

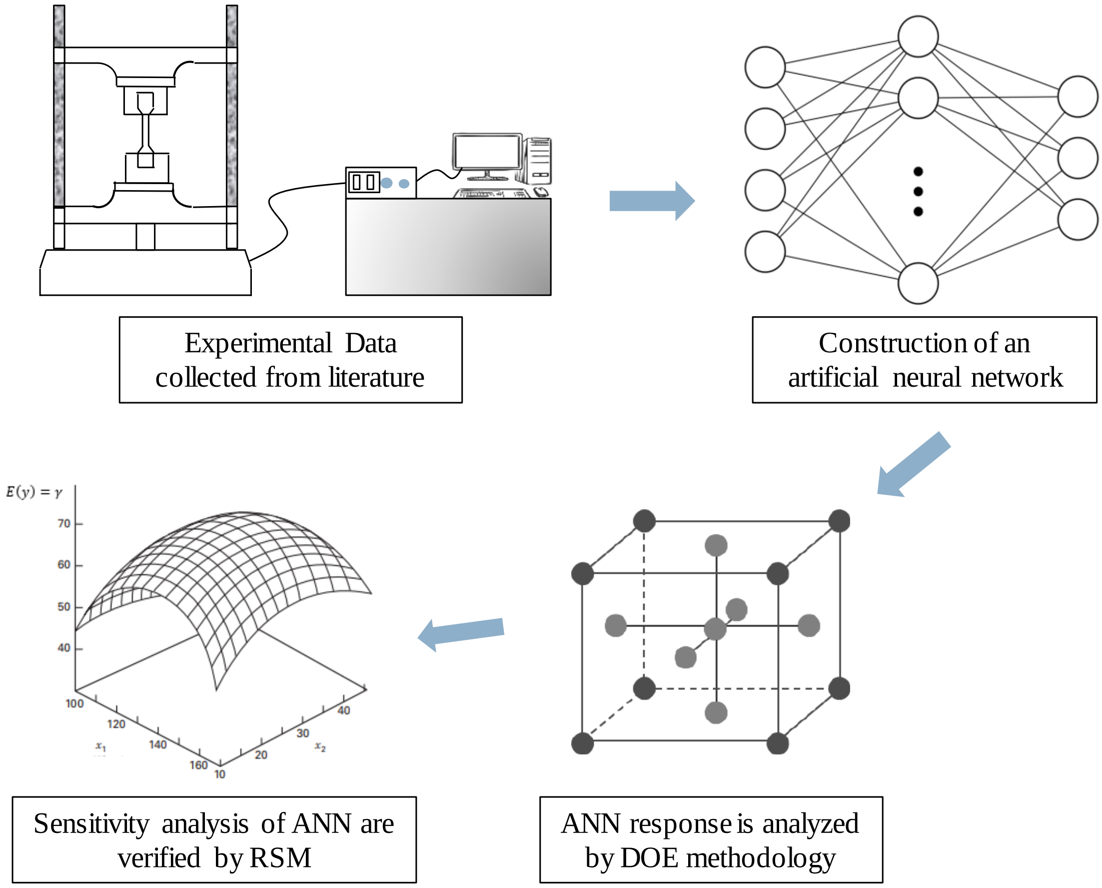


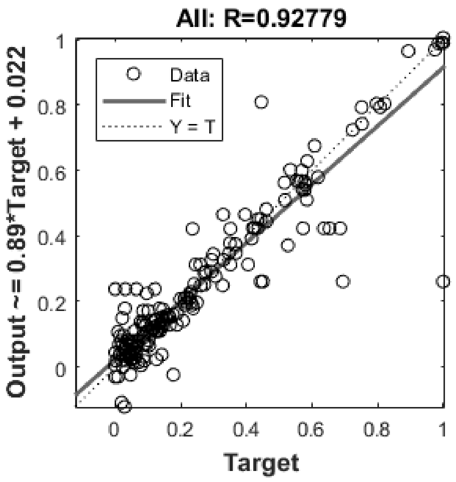
Figure 2. Workflow for sensitivity analysis developed during this paper.

As a quasi-brittle material, Young's modulus, tensile strength and fracture energy are the necessary input variables for predicting the global behavior of concrete in tension in a FEM model. Tensile strength is increased in this particular case due to the presence of fibers. On the other hand, fracture energy is related to fracture mechanisms in concrete, governing the material post-crack phase after matrix first crack.

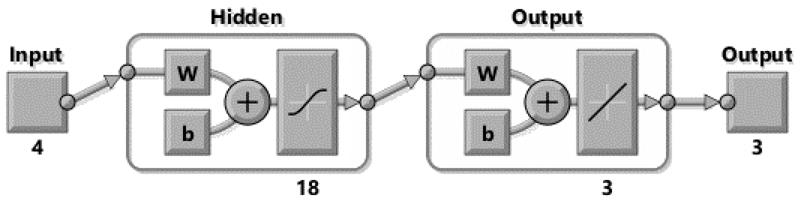
Considering the experimental database, a neural network with 90 samples was constructed with the following input data: water-cement ratio, steel fiber volumetric content (%) and steel fiber length and diameter (mm). Input data was introduced in the ANN after a linear normalization procedure. The samples were randomly divided in three distinct groups: 70% for training and 30% for validation and testing. This study uses the multilayer perceptron backpropagation algorithm to estimate the correct targets. Given the random variability from Artificial Intelligence methods, several rounds were performed. The number of neurons, hidden layers and the training algorithm were set after multiple rounds and previous studies of ANN architectures and training configurations. The criteria for network selection is the highest value of global regression (R^2). All simulations and tests for the artificial neural network stage were performed by Deep Learning Optimization Toolbox in MATLAB ®. Table 1 summarizes the network architecture and their respective configurations. In addition, Figures 3(a) and 3(b) present the global regression value for the selected ANN and the schematic architecture of the network.

Table 1. Configurations for the selected neural network.

ANN Architecture and Configurations	
Number of Neurons	18
Training Algorithm	*Trainlm*
Transfer Function	*Tansig*
Number of Hidden Layers	1



(a)



(b)

Figure 3. (a) Global regression value for the ANN; (b) ANN architecture.

RSM analyses were carried out in Minitab 18 ® software. Default configuration of statistical analysis considers a 95% confidence interval for the simulations, with the adoption of a central circumscribed composite design (CCC). Table 2 displays the complete matrix for the Box-Wilson central composites design in coded factor settings. In addition, Table 3 indicates the lower and upper bounds for each input parameter of the ANN database, used as model for the DOE methodology.

Table 2. Complete matrix for central circumscribed composite design (CCC).

OrdemPad	A	B	C	D
1	0	0	0	0
2	0	0	-2	0
3	1	-1	1	-1
4	1	-1	1	1

5	0	0	0	-2
6	1	-1	-1	-1
7	0	0	0	0
8	1	1	1	-1
9	-1	-1	-1	-1
10	0	0	0	0
11	-1	1	1	1
12	1	1	-1	-1
13	1	1	1	1
14	0	-2	0	0
15	-1	1	1	-1
16	-1	1	-1	-1
17	0	0	2	0
18	1	-1	-1	1
19	-1	-1	1	1
20	2	0	0	0
21	0	2	0	0
22	0	0	0	0
23	-1	1	-1	1
24	0	0	0	0
25	0	0	0	0
26	0	0	0	0
27	-1	-1	-1	1
28	-1	-1	1	-1
29	0	0	0	2
30	1	1	-1	1
31	-2	0	0	0

Table 3. Upper and lower bounds for the ANN model employed in the DOE methodology.

Bounds/ Input Parameters	**Water-Cement Ratio (-)**	**Fiber Volumetric Content (%)**	**Fiber Length (mm)**	**Fiber Diameter (mm)**
Upper Bound	0.65	2.00	60	1.6
Lower Bound	0.16	0.10	5	0.2

As previously reported in this paper, a sensivity analysis of each ANN input parameter is developed by using a DOE methodology, generating surface response plots to compare the relationship between each input and output variable: water-cement ratio-fiber volumetric content x Young Modulus, fiber length-fiber diameter x Young's modulus; water-cement ratio-fiber volumetric content x tensile strength, fiber length-fiber diameter x tensile strength; water-cement ratio-fiber volumetric content x fracture energy, and fiber length-fiber diameter x fracture energy.

It is important to point out that the surface plots represent the tensile behavior of steel fiber reinforced concrete with random distribution in the cement matrix inside the bounds presented in Table 3. In this way, they do not represent the behavior of concrete in the entire space of independent variables, once the database is restricted to specific boundaries. Thus, considering this relatively small region of analysis, the adoption of a second-order polynomial for DOE/RSM methodology is satisfactory for the problem. Finally, Figure 4 presents how data points from the ANN database are distributed in the parameter input space.

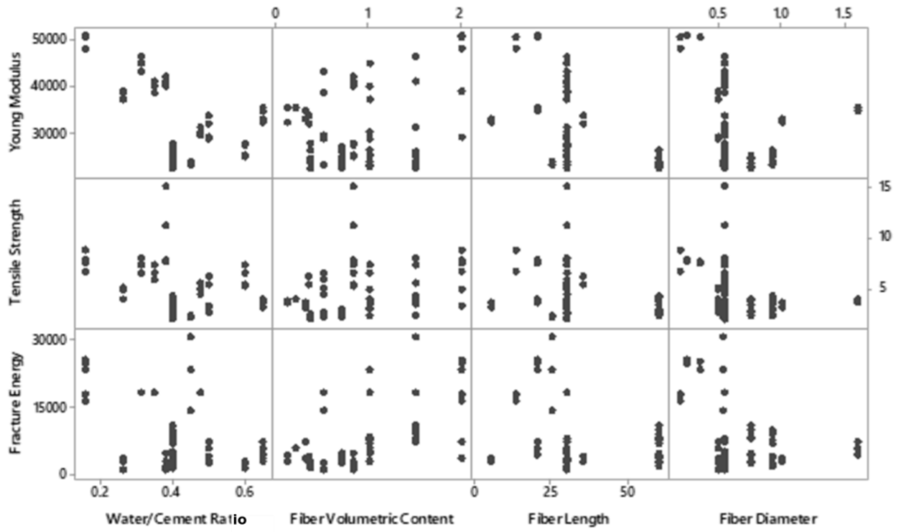


Figure 4. Distribution of ANN data points in the parameter input space.

4 Results and Discussion

As previously mentioned, the sensitivity analysis developed in this paper investigates the influence of water-cement ratio, steel fiber length, diameter and volumetric content in the macro behavior of steel fiber reinforced concrete. In practice, these parameters are assessed in a great number of experimental tests. Material behavior inferences in macroscale are endorsed by verifying how these variables impact the mechanical properties of concrete, such as elastic properties, tensile strength and fracture energy of the material.

The statistical analysis of the ANN model using DOE allows the generation of response surface plots that confirm the presence or absence of influence of each input parameter in the system response. The results of the sensitivity analysis are compared with literature reports, validating the DOE response surface plot.

4.1 Young's Modulus

Figures 5 and 6 present the response surface plots considering Young's modulus as the analysis output. Water-cement ratio and steel fiber volumetric content are the independent variables in the plot of Figure 5, while steel fiber length and diameter are the independent parameters in the plot of Figure 6.

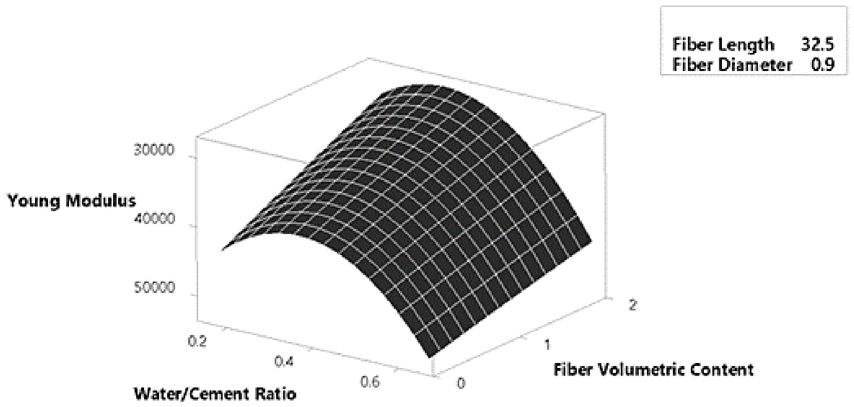


Figure 5. Response surface plot for Young's modulus considering water-cement ratio and steel fiber volumetric content as independent variables.

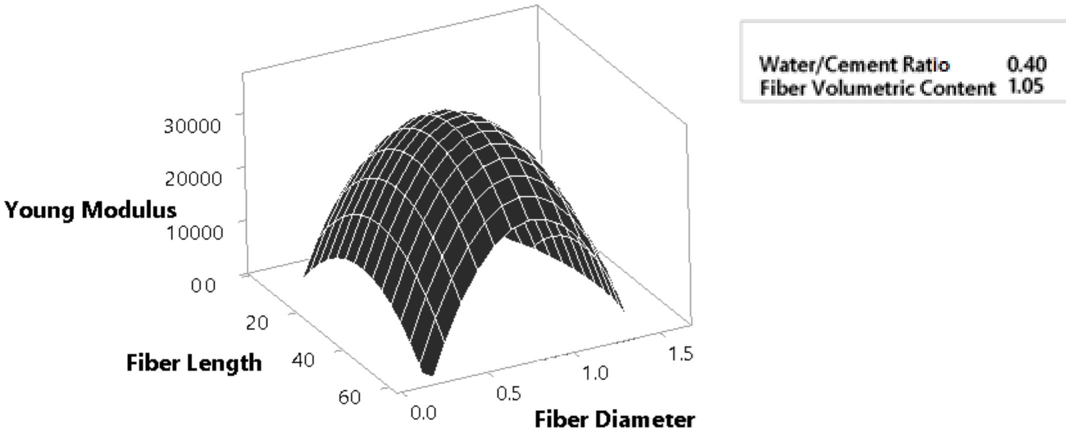


Figure 6. Response surface plot for Young modulus considering fiber length and diameter as independent variables.

According to Figures 5 and 6, the relationships between Young's modulus and all four input parameters is not constant, suggesting that they are significant for this output. Young's modulus decreases quadratically with the water-cement ratio, in agreement with response surface. This can be confirmed by previous research of Gao *et al.* (2017), Figueiredo (2011) and Bentur & Mindess (2007). In addition, Young's modulus presents a linear dependence with the steel fiber volumetric content, as also indicates Figure 7. According to Neves & Almeida (2005), the elasticity modulus shows a tendency to decrease as fiber content increases.

Figure 6 points out the increase in elastic modulus with steel fiber length and diameter until a specific point of each parameter. However, as reported by Góis (2010) and Abbas *et al.* (2014), fiber aspect ratio (i.e, the quotient between fiber length and diameter) does not change expressively the elastic modulus. In their respective works, distinct relationships between Young's modulus-fiber lengths are verified, which leads to plural behaviors of fiber reinforced concrete. In this sense, it is necessary to consider each mix properties and possible additions in order to make inferences about the material mechanical response in this case.

4.2 Tensile Strength

Figures 7 and 8 display the response surface plots considering tensile strength as the analysis output. Water-cement ratio and steel fiber volumetric content are the independent variables in the plot of Figure 7, while steel fiber length and diameter are the independent parameters in the plot of Figure 8.

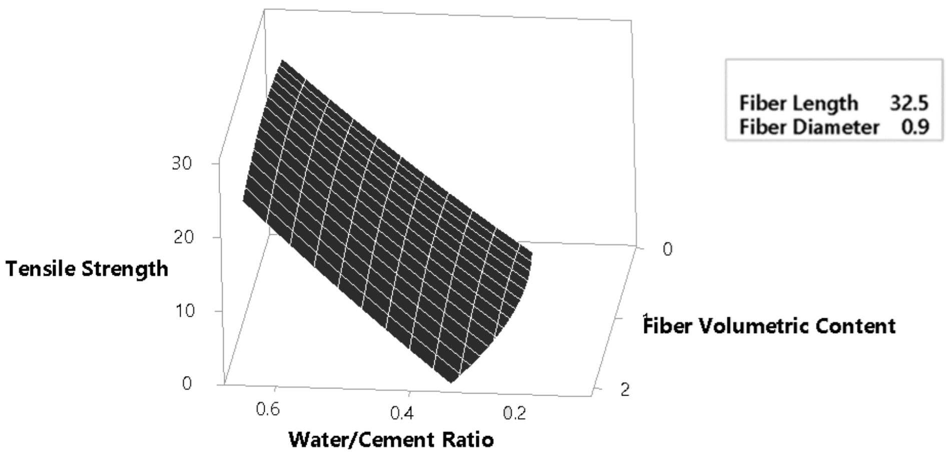


Figure 7. Response surface plot for tensile strength considering water-cement ratio and steel fiber volumetric content as independent variables.

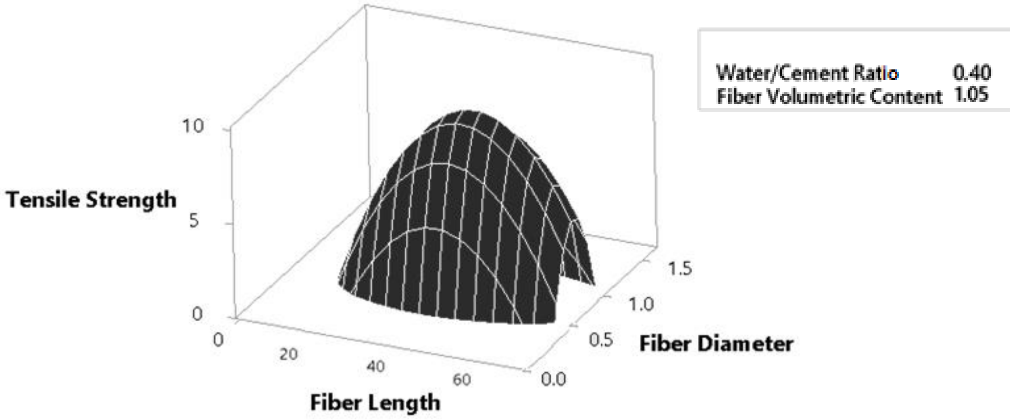


Figure 8. Response surface plot for tensile strength considering steel fiber length and diameter as independent variables.

From previous response surface plots, it is possible to infer that all input parameters affect the tensile strength behavior of the composite, within the analyzed ranges. According to Figure 8, tensile strength starts increasing until a fiber volumetric content, and then decreases from this point on. The random dispersion of steel fibers in the cement matrix is a relevant factor that leads to the reduction of tensile strength after a specific fiber content. This observation is also supported by some authors in literature, such as Bhat & Khan (2018) and Beigi *et al.* (2013). DOE response surface also points out that tensile strength rises with water-cement ratio. However, Bentur & Mindess (2007) support that tensile strength decreases with water-cement ratio. This contradictory behavior is due to specific points in the ANN database used for training. In mixtures with low values of water-cement ratio, the workability was affected by the presence of coarse aggregate, generating the interlock between aggregate and steel fibers (Zhang *et al*., 2018 *apud* Liu *et al*., 2016; Ma *et al.*, 2004). Other data points also present small values of tensile strength (near 1 MPa, for example). This can suggest that, for some input parameters, the tensile strength drops to values close to zero, but not exactly this value.

In Figure 8, it is verified that tensile strength rises with fiber diameter and fiber length growth. After a specific value, the tensile strength declines when both parameters decrease. Previous experimental research of Góis (2010) and Abbas *et al.* (2014) endorse this comment about steel fiber reinforced concrete, although mixture properties and test configurations impact the experimental results.

4.3 Fracture Energy

Figures 9 and 10 present the response surface plots considering fracture energy as the system output. Water-cement ratio and steel fiber volumetric content are the independent variables in the plot of Figure 9, while steel fiber length and diameter are the independent parameters in the surface of Figure 10.

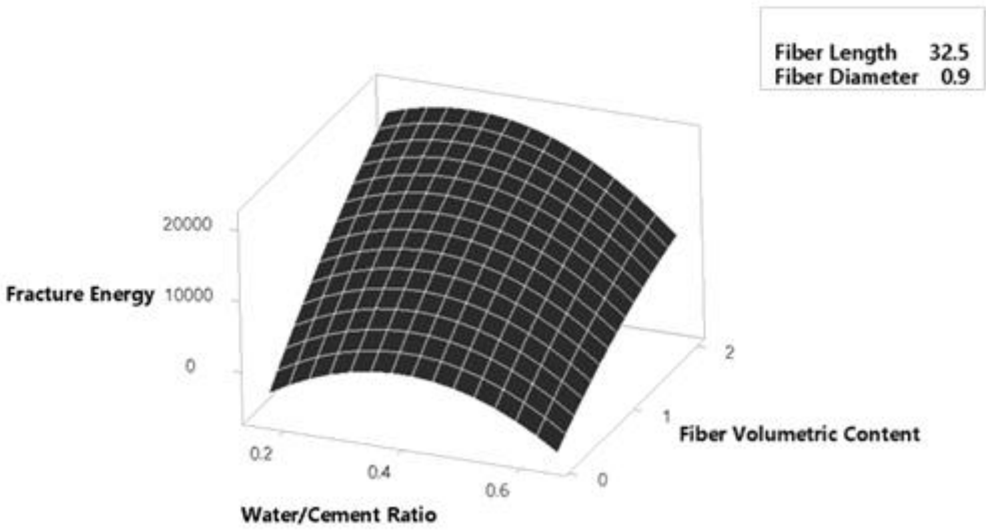


Figure 9. Response surface plot for fracture energy considering water-cement ratio and steel fiber volumetric content as independent variables.

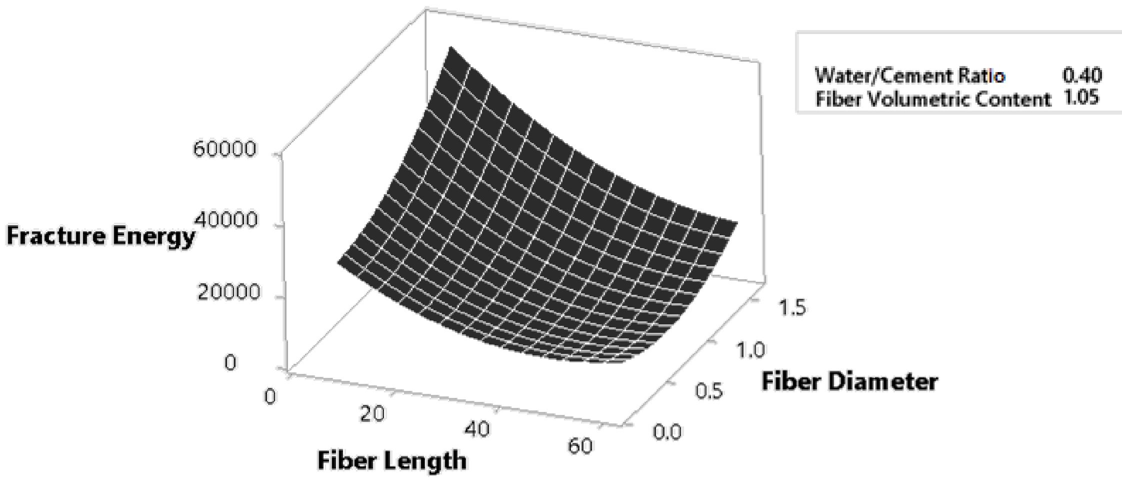


Figure 10. Response surface plot for fracture energy considering steel fiber length and diameter as independent variables.

According to Figure 10, there is an increment in the fracture energy with respect to fiber volumetric content. This behavior is also discussed in previous works (Figueiredo *et al*., 2000; Kim *et al*., 2010; Figueiredo, 2011), since a greater number of fibers will be inserted in the cement matrix. In direct tensile tests of steel fiber reinforced specimens, load carrying capacity is directly proportional to fiber content (Naaman, 2018). On the other hand, as water-cement ratio increases, fracture energy values decline. Beigi *et al.* (2013) and Han *et al*. (2019) support these observations in their respective works in steel fiber reinforced concrete experimental analyses.

Fiber length and diameter do not present a constant relationship with fracture energy parameter, since the DOE surface presents a curvature. Figueiredo (2011) conducted an experimental analysis where fracture energy increases with the fiber length up to a critical length at a specific fiber volumetric content; for greater values the fracture energy decreases. Assessing fiber diameter-fracture energy interaction, Figueiredo (2011) confirms that the fracture energy reduces as fiber diameter decreases up to a specific diameter. Beigi *et al.* (2013) also come to the same results after comparing those variables in experimental procedures carried out on steel fiber reinforced concrete.

5 Conclusions

This study carried out a sensitivity analysis with the primary goal of verifying which experimental variables are relevant for the mechanical response of steel fiber reinforced concrete predicted by an artificial neural network (ANN). Sensitivity analysis is an important stage for developing numerical models, especially when experimental tests include a great number of parameters. Design of Experiments (DOE) methodology was employed to assess system sensitivity. An experimental database of tensile experimental tests available in literature with discrete straight steel fibers in random dispersion in the cement matrix was considered for the generation of an ANN able to predict the mechanical parameters of fiber reinforced concrete, such as Young modulus, tensile strength and fracture energy. Water-cement ratio, fiber length, diameter and volumetric content gather the input data for the artificial intelligence method. Next, a statistical analysis of the ANN was performed by a DOE methodology. Minitab ® software carried out all simulations with a Central circumscribed composite design (CCC) and a range of confidence of 95%. Response surface plots considering a second-order model were generated in order to verify which input parameters of the network affect the mechanical behavior of the composite, analyzing Young's modulus, tensile strength and fracture energy outputs.

The results show that water-cement ratio, fiber length, fiber diameter and steel fiber volumetric content have an impact in Young modulus, tensile strength and fracture energy in steel fiber reinforced concrete. The response surface methodology (RSM) provided surfaces with distinct curvatures indicating the effects of each of input parameters in the system response. Finally, it is concluded that these variables are relevant for modeling the mechanical behavior in traction of fiber reinforced concrete. In addition, the DOE methodology can be an interesting tool to perform parameter sensitivity analysis in fiber reinforced cement-based materials field. This stage is essential to investigate and model the mechanical behavior of composites.

6 References

ABBAS, S; SOLIMAN, A. M.; NEHDI, M.L. 'Exploring mechanical and durability properties of ultra-high performance concrete incorporating various steel fiber length sand dosages'. Construction and Building Materials, v. 75, p. 429-441, 2014.

ALEXANDRIDIS, A.; TRIANTIS, D.; STAVRAKAS, I.; STERGIOPOULOS, C. 'A neural network approach for compressive strength prediction in cement-based materials through the study of pressure-stimulated electrical signals'. Construction and Building Materials, v. 30, p. 294-300, 2011.

ARSLAN, M. H. 'Predicting of torsional strength of RC beams by using different artificial neural network algorithms and building codes'. Advances in Engineering Software, v. 41, p. 946-955, 2010.

ATICI, U. 'Prediction of the strength of mineral admixture concrete using multivariable regression analysis and an artificial neural network'. Expert Systems with Applications, v. 38 (8), p. 9609–9618, 2011.

BABANAJAD, S. K.; GANDOMI, A. H.; ALAVI, A. H. 'New prediction models for concrete ultimate strength under true-triaxial stress states: An evolutionary approach'. Advances in Engineering Software, v. 110, p. 55-68, 2017.

BEIGI, M.; BERENJIAN, J.; OMRAN, O. L.; NIK, A. S.; NIKBIN, I.M. 'An experimental survey on combined effects of fibers and nanosilica on the mechanical, rheological, and durability properties of self-compacting concrete'. Materials & Design, v. 50, p.1019-1029, 2013.

BENTUR, A., MINDESS, S. 'Fibre reinforced cementitious composites'. 2nd edition, Francis and Taylor, UK, 2007.

BHAT, K.M.U.D.; KHAN, M. Z. ‘Effect of Steel Fibre Reinforcement on Early Strength of Concrete’. International Journal of Trend in Scientific Research and Development, v. 2, p. 198-225, 2018.

CONGRO, M.; MEJIA, C.; ROEHL, D.; MARANGON, E. ‘Fracture Modeling of Fiber Reinforced Concrete in a Multiscale Approach’. Composites Part B: Engineering, v. 174, 106958, 2019.

CONGRO, M.; MEJIA, C.; ROEHL, D. ‘XFEM and Interface Element Models Applied to Fracture of Fiber Reinforced Concrete Beams’. Proceedings of XXXVIII CILAMCE – Ibero Latin-American Congress of Computational Methods in Engineering. Florianópolis, Brazil, 2017.

FIGUEIREDO, A. ’Concreto Reforçado com Fibras’. 248 p. Tese (Doutorado) - Curso de Engenharia Civil, Universidade de São Paulo, São Paulo, 2011.

FIGUEIREDO, A. ’Concreto: Ciência e Tecnologia’. São Paulo: Geraldo C. Isaia, v. 1. Instituto Brasileiro do Concreto – IBRACON, 2011.

GAO, D.; ZHANG, L.; NOKKEN, M. ‘Compressive behavior of steel fiber reinforced recycled coarse aggregate concrete designed with equivalent cubic compressive strength’. Construction and Building Materials, v. 141, p.235-244, 2017.

GÓIS, F. ’Avaliação Experimental do Comportamento de Concreto Fluido Reforçado com Fibras de Aço: Influência do fator de forma e da fração volumétrica das fibras nas propriedades mecânicas do concreto’. 156 p. Dissertação (Mestrado) - Curso de Engenharia Civil, Universidade Federal de Alagoas, Maceió, 2010.

HAN, J.; ZHAO, M.; CHEN, J.; LAN, X. ‘Effects of steel fiber length and coarse aggregate maximum size on mechanical properties of steel fiber reinforced concrete’. Construction And Building Materials, v. 209, p.577-591, 2019.

KARINSKI, Y. S.; DANCYGIER, A. N.; NAVON, Z. ‘Experimental verification for a probabilistic model of fibers distribution along a reinforced concrete bar’. Materials & Structures, v. 50, p. 1-10, 2017.

KIM, W-J.; KWAK, M-S.; LEE, J-C. ‘Fracture properties of high-strength steel fiber concrete’. Proceedings of 7th International Conference on Fracture Mechanics of Concrete and Concrete Structures (FraMCoS-7), p. 1526-1523. Jeju, South Korea, 2010.

KHURI, A. I.; CORNELL, J. A. ‘Response Surfaces: Designs and Analyses’. 2nd edition, Dekker, New York, 1996.

LAW, A.M.; KELTON, W. D. ‘Simulation Modeling and Analysis’. 3rd edition. McGrawHill, New York, USA, 1999.

LEE, T-H.; MOSALAM, K. M. ‘Probabilistic fiber element modeling of reinforced concrete structures’. Computers & Structures, v. 82, p. 2285-2299, 2004.

LIU, J. Z.; HAN, F. Y.; CUI, G.; ZHANG, Q. Q.; LV, J.; ZHANG, L. H.; YANG, Z. Q. ‘Combined effect of coarse aggregate and fiber on tensile behavior of ultra-high performance concrete’. Construction and Building Materials, v. 121, p. 310-318, 2016.

MA, J. X.; ORGASS, M.; TUE, N. V.; DEHN, F.; SCHMIDT, D. ‘Comparative investigations on ultra-high performance concrete with and without coarse aggregates’. In: Proceedings of International Symposium on Ultra High Performance Concrete. Kassel, Germany, p. 205-212, 2004.

MONTGOMERY, D. C. 'Design and Analysis of Experiments'. 8th edition, John Wiley & Sons, Inc., New York, USA, 2013.

MYERS, R. H.; MONTGOMERY, D. C.; VINING, G. G.; BORROR, C.M.; KOWALSKI, S. M. 'Response Surface Methodology: A Retrospective and Literature Survey'. Journal of Quality Technology, v. 36, p. 53–77, 2004.

NAAMAN, A. E., MOAVENZADEH, F.; MCGARRY, F. J. 'Probabilistic Analysis of Fiber Reinforced Concrete'. Journal of Engineering Mechanics, v. 100, p. 397-413, 1974.

NAAMAN, A.E. 'New fiber technology: Cement, ceramic and polymeric composites'. Concrete International, v. 7, p. 57-62, 1998.

NAAMAN, A. E. 'Engineered steel fibers with optimal properties for reinforcement of cement composites'. Journal of Advanced Concrete Technology, v. 1, p. 241-252, 2003.

NADERPOUR, H.; KHEYRODDIN, A.; AMIRI, G. G. 'Prediction of FRP-confined compressive strength of concrete using artificial neural networks'. Composite Structures, v. 92, p. 2817-2829, 2010.

NEVES, R. D.; ALMEIDA, J. C. O. F. 'Compressive behavior of steel fiber reinforced concrete'. Structural Concrete, v. 6, p. 1-8, 2005.

NI, H. G.; WANG, J. Z. 'Prediction of compressive strength of concrete by neural networks'. Cement and Concrete Research, v. 308, p. 1245–1250, 2000.

RAJASEKARAN, S.; LEE, S. C. 'Prediction of concrete strength using serial functional network model'. Structural Engineering Mechanics, v. 161, p. 83–99, 2003.

RÍOS, J. D.; CIFUENTES, H.; YU, R. C.; RUÍZ, G. 'Probabilistic Flexural Fatigue in Plain and Fiber-Reinforced Concrete'. Materials, v. 10, 2017.

SANDEMIR, M. 'Prediction of compressive strength of concretes containing metakaolin and silica fume by artificial neural networks'. Advances in Engineering Software, v. 40, p. 350-355, 2009.

SHINOZUKA, M. 'Probabilistic modeling of concrete structure'. Journal of the Engineering Mechanics Division, v. 98 (6), p. 1433-1451, 1972.

YOON, J. 'Application of experimental design and optimization to PFC model calibration in uniaxial compression simulation'. International Journal of Rock Mechanics and Mining Sciences, v. 44, p. 871-889, 2007.

YUAN, Z.; WANG, L-N.; JI, X. 'Prediction of concrete compressive strength: Research on hybrid models genetic based algorithms and ANFIS'. Advances in Engineering Software, v. 67, p. 156-163, 2014.