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DETERMINATION OF ELASTIC PROPERTIES BY MEANS OF FINITE ELEMENT MODEL UPDATING

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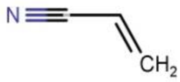
Abstract. The range of available 3D printable materials is constantly expanding and unlocking advanced applications. Thus, the using of 3D printers to manufacture industrial parts is increasing due to its adaptable manufacturing of complex geometries and reduction in material waste. Especially the correct adoption of material elastic properties is central to secure and consistent design. However, the methodologies that allow the researchers characterize the mechanical properties of the parts after the impression without damage them are scarce. In this way, this paper presents the calibration of elastic properties by means of a hybrid numerical-experimental methodology that objective the automatic updating of a finite element model regarding the experimental natural frequency. Here, natural frequencies were obtained through acoustic tests of a sample of 30x30x30 mm³ with a square periodic distribution of circular cylindrical holes manufactured by a Fused Deposition Modeling (FDM) 3D printer. The sample was positioned on a device which simulates the free-boundary condition and excited by impact using a standard hammer. 10,000 numerical models were processed in ABAQUS and the calibration was carried out employing a genetic algorithm implemented in Python. Based on experimental measurements the elastic constants were determined. This proposed methodology presents high accuracy, the natural frequencies obtained by the calibrated numerical model differed in less than 3% of the obtained ones by the experimental model and could be applied in a wide range of materials.

Keywords: Optimization, Finite Element Model, Elastic Properties

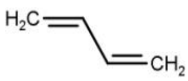
1. Introduction

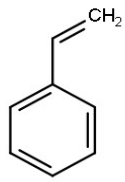
Recently developed Additive Manufacturing (AM) is an emerging technological advancement in the field of engineering, see Yap et al. [1] and Wohlers [2], due to its highly adaptable manufacturing capabilities and ease of use, for instance, for rapid prototyping of intricate geometries with metals, polymers, and fiber-reinforced composite materials. In particular, Fused Deposition Modeling (FDM), which pertains to the material extrusion family in AM, consists of melting a filament of material in a heated nozzle extruder and depositing on a build platform. By using a 3-axis motion system, this nozzle is moved in the XY plane printing a layer of the prototype. The build platform is moved down one step (known as the slice thickness) in the Z direction when the current layer is finished, and the cycle is repeated for the next layer until the complete model is built-up, Pham and Gault [3] and Dawoud, Taha and Ebeid [4]. According to Baumers et al. [5] and Vashishtha et al. [6], 3D printing reduces production costs by reducing material waste and shortens manufacture times for complex prototype designs that would otherwise require expensive moulds and long fabrication periods. Additionally, 3D printing has the potential for use in small fieldable fabrication units and the ability to fabricate changed designs without retooling, Oztan et al. [7].

On the other hand, the range of available 3D printable materials is constantly expanding and unlocking advanced applications. Thus, the using of 3D printers to manufacture industrial parts is increasing. One of the most used materials as an engineering plastic for 3D printing is the ABS (Acrylonitrile Butadiene Styrene) material. ABS is an amorphous copolymer that contains 15–30% acrylonitrile, 5–30% butadiene, and 40–60% styrene, Bruder [8]. In the ABS the butadiene part is uniformly distributed over the acrylonitrile-styrene matrix. ABS was introduced in the market in 1948. Figure 1 shows the monomers that make up the ABS material.



Acrylonitrile





Butadiene Styrene

Figure 1. Monomers that make up the ABS.

ABS possesses excellent properties, which include good dimensional stability under stress, thermal resistance, lightweight, easy formability, reflectivity, toughness, easy processing ability, chemical resistance, and cheapness. ABS prepared by injection moulding generally presents superior mechanical properties in all the conducted tests if compared to those of 3D printing, for example, Dawoud, Taha and Ebeid [4] show that there is a difference in tensile strength between injection moulded (37.7 MPa) and FDmodelled (34.3 MPa) specimens, the latter being the highest tensile strength for printed samples observed at −P45 [raster angle (even layers) = +45°; Raster angle (odd layers) = -45°; Gap = 5 mm]. According to these authors, this is related to the nature of the injection moulding process, which results in higher material compaction as well as the enhancement of crystalline structure, therefore enhancing mechanical strength.

Especially the correct adoption of material elastic properties is central to secure and consistent design. The study of the behavior of nonhomogeneous solids printed using 3D printers requires the determination of their effective properties via experimental methods, computational methods, analytical methods, or a combination of them. However, the methodologies that allow the researchers to characterize the mechanical properties of the parts after the impression without damage them are scarce. In this way, this research presents the calibration of elastic properties by means of a hybrid numerical-experimental methodology that objective the automatic updating of a finite element model regarding the experimental natural frequency for predicting the response of elastic solids containing a distribution of heterogeneities, which, here, consists of a uniform and periodic distribution of circular cylindrical holes.

The rest of the paper is organized as follows. In Section 2, it is presented the statement of the problem for the linear elastic orthotropic medium with finite dimensions and defined the optimization problem in the determination of the elastic constants. In Section 3, it is considered the acoustic test for the elastic medium and used the natural frequencies obtained from this acoustic test in search of the solution of the problem, presented in Section 2. Section 4 presents a brief description of the finite element model updating employed. Section 5 presents a theoretical modal analysis to establish the natural frequencies and vibrations modes related to the sample, which permit experimental modal analysis presented in Section 6. From Sections 5 and 6, it was possible to determine the elastic properties via FEM (Finite Element Model) updating and compare the frequencies obtained by the calibrated model and by the acoustic test, which are presented in Section 7. In Section 8, it is presented the conclusions of this work.

2. Problem description

This paper proposes the determination of the elastic properties of an ABS sample by finite element model updating. The sample contains a periodic square distribution of cylindrical circular holes manufactured by a FDM 3D printer. Figure 2 illustrates the evaluated sample.

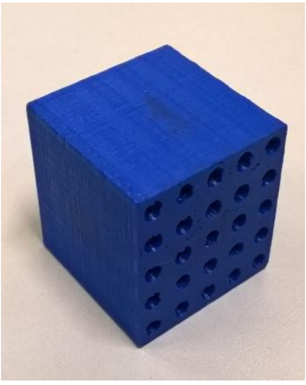


Figure 2. ABS Sample of 30x30x30 mm³

The finite element model updating was performed in order to determine the elastic constants of material based on the dynamic response obtained by the acoustic test carried out on the sample. The orthotropic material was considered. Thus, the decision variables of the optimization problem were the 9 independent elastic constants of the material as follows: D1111, D2222, D3333, D2323, D1313, D1212, D1122, D1133, and D2233, which relate stresses and strains according to Eq. (1).

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \\ \sigma_{33} \\ \sigma_{12} \\ \sigma_{13} \\ \sigma_{23} \end{Bmatrix} = \begin{bmatrix} D_{1111} & D_{1122} & D_{1133} & 0 & 0 & 0 \\ D_{2211} & D_{2222} & D_{2233} & 0 & 0 & 0 \\ D_{3311} & D_{3322} & D_{3333} & 0 & 0 & 0 \\ 0 & 0 & 0 & D_{1212} & 0 & 0 \\ 0 & 0 & 0 & 0 & D_{1313} & 0 \\ 0 & 0 & 0 & 0 & 0 & D_{2323} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \\ \varepsilon_{33} \\ \gamma_{12} \\ \gamma_{13} \\ \gamma_{23} \end{Bmatrix} \quad (1)$$

The optimization problem was defined by means of an objective function based on the natural frequencies obtained numerically ($f_i^{Numerical}$) and experimentally ($f_i^{Experimental}$), as shown in Eq. (2).

$$\begin{aligned} &\underset{D_{ikjl}}{\text{minimize}}\left(\max_{i=1...n}\left|\frac{f_i^{Numerical}-f_i^{Experimental}}{f_i^{Numerical}}\right|\cdot 100\%\right)\\ &\text{Subject to:}\\ &2.50\ GPa \le D_{ikjl} \le 4.70\ GPa,\ i=k=j=l,\ \text{i=1,2,3.}\\ &0.58\ GPa \le D_{ikjl} \le 1.08\ GPa,\ i=j, k=l,\ i\neq k,\ i=1,2,3,\ k=1,2,3.\\ &1.35\ GPa \le D_{ikjl} \le 2.50\ GPa,\ i=k,\ j=l,\ i\neq j,\ i=1,2,3,\ j=1,2,3. \end{aligned} \tag{2}$$

3. Acoustic Test

The equilibrium of a certain sample with an arbitrary number of degrees of freedom satisfy the Eq. (3):

$$[\mathrm{M}]\ddot{u}+[\mathrm{C}]\dot{u}+[\mathrm{K}]u=p(t), \tag{3}$$

where [M]ü is the inertia therm; [C]u̇ is the damping; [K]u is the elastic term; and [M], [C] and [K] are the mass, damping and stiffness matrix, respectively.

At this stage, two simplified assumptions are adopted: (i) the applied impulse present a short time of excitation, and then $p(t)=0$; (ii) the test is under free vibration without damping (i.e. [C] = 0). In this paper, a problem of undamped free vibration is considered, according to

$$[\mathrm{M}]\ddot{u}+[\mathrm{K}]u=0. \tag{4}$$

By a simple substitution of $u=\varphi_{ij}\sin(\omega_{ij}\,t)$ could be observed that u is the solution of Eq. (4), for all time steps. Therefore, this system presents a non-zero and undetermined solution that yields

$$\left([\mathrm{K}]-\omega^2[\mathrm{M}]\right)\varphi=0. \tag{5}$$

The Equation (5) is called of a generalized eigenvalues and eigenvectors problem (or modal equation), in which the eigenvalues ω_{ij} are the natural frequencies of the system and eigenvectors (φ_{ij}) are the modes of vibration. Therefore, it is well established that the resonance phenomenon is linked to the stiffness matrix [K] and mass matrix [M].

Draw upon the relation of [K], [M] and ω_{ij}, the Resonance Impact Method (RIM) can be defined as a refined experimental techniques used to obtain the material elastic properties through natural frequencies using a short duration impact. When microphones are used in this characterization, this technique is called acoustic test. In recent years, the RIM methodology has been used with success to determine the material properties [9–12]. Herrera et al. [9] used RIM to characterize ceramic foams under high temperatures. Carrasco et al. [10] tested 29 Brazilian wood samples, obtaining the material dynamic behavior. Gidrão [12] analyzed 15 concrete mixtures and obtained the dynamic elastic properties of these material.

Nevertheless, technical codes as ASTM E1876-01 [13] are based on studies of Pickett [14], where is present the relation between elastic properties and natural frequency, only for isotropic cylindrical and prismatic samples. Therefore, there is a limitation to the use of Pickett's equations to predict the response of elastic solids containing a distribution of heterogeneities with a uniform and periodic distribution of circular cylindrical holes. In this article, a more precise methodology is employed: a determination of elastic properties using a reverse modal problem with finite element model updating.

4. Finite element model updating

The methodology applied in this paper uses the finite element model updating technique, in which the decision variables are calibrated by an optimization method coupled to the finite element model. Based on experimental results, an objective function is developed, and after the optimization method is applied in order to minimize/maximize this function. This technique is currently widely used in solving various problems, such as damage detection, structural health monitoring and determination of mechanical properties [15-17].

In this work, the optimization method chosen was the Genetic Algorithm (GA). GA is an evolution-based stochastic optimization method proposed by Holland [18] in 1975. GA is inspired by the natural selection theory, in which individuals more adapted to the environment have a better survival chance. This method simulates the evolutionary process in order to obtain the best solutions for a given problem. Nature-inspired algorithms are termed meta-heuristics, i.e., it mimic nature to solve optimization problems Binitha [19].

GA differs from traditional methods because it is stochastic, i.e., there is a certain randomness in the process that allows several points in space to be explored, increasing the chances of being found optimal globally. Other important aspects of GA are: it works with a population of individuals, uses cost or reward information, and uses probabilistic transitions Goldberg [20].

The first algorithm step is the random generation of a population of individuals (also called chromosomes), where each one represents a possible problem solution. Each chromosome is a vector composed by genes that describe the decision values of the problem variables.

An objective function (cost function) is defined to evaluate the possible solution. Then a fitness function is constructed based on the objective function to scale the individuals [21]. Higher fitness individuals get higher grades, increasing their chance to stay in the next generation. After the fitness calculation, basically three operations are applied: selection, crossover and mutation.

In the selection are chosen individuals who will participate in the next generation. In general, the selection methods preferably choose individuals with higher grades. However, other individuals also have a probability of being selected in order to maintain the diversity of the population. There are several methods, in this work the roulette method with elitism was applied.

In the roulette method each individual receives a portion of roulette wheel proportional to their fitness. Then the roulette wheel is rotated a certain number of times (equal to the population size) and individuals are selected. Elitism is a strategy that maintains the best individuals in the next generation without undergoing any modification, avoiding that high fitness structures are discarded.

After the selection, the crossover with a probability given by the crossover rate occurs. This is the main genetic operator, so the crossover rate is usually high. The crossover consists of exchanging information (genes) between individuals, called parents, generating new individuals, called children. The intention is that the children inherit the best characteristics of each parent and be more fit, generating more fittest individuals with each generation.

The last operation is the mutation, in which randomly chosen genes are modified in order to guarantee the population diversity. The mutation occurs with a probability given by the mutation rate, which is much smaller than the crossover rate. A high mutation rate can make the process too random, so the children will hardly resemble their parents. A small mutation rate can generate stagnation in the objective function value.

After the genetic operators application, the finite element model is processed and the objective function values are calculated for each individual of the new generation. Then, there is the stopping criterion, this can be set on the maximum generations number, the maximum/minimum objective function value or the maximum given parameter error, for example. When the criterion is met, the calibration process is terminated and there is a solution that is the result of a process based on natural selection. Figure 3 illustrates a scheme about the methodology used in this work.

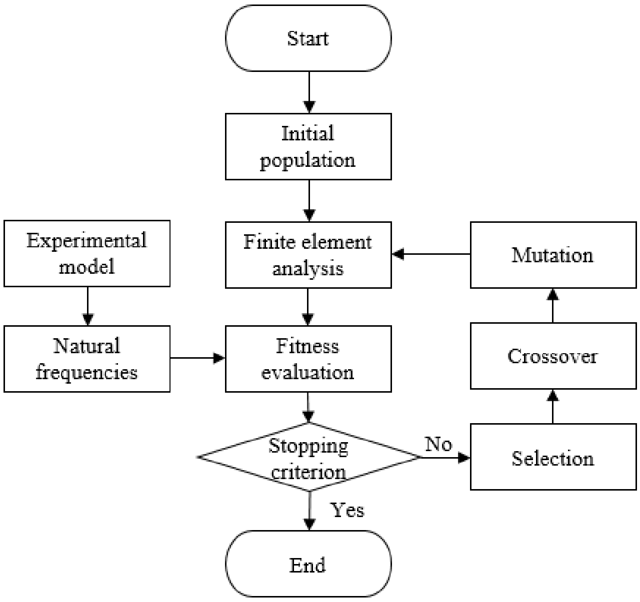
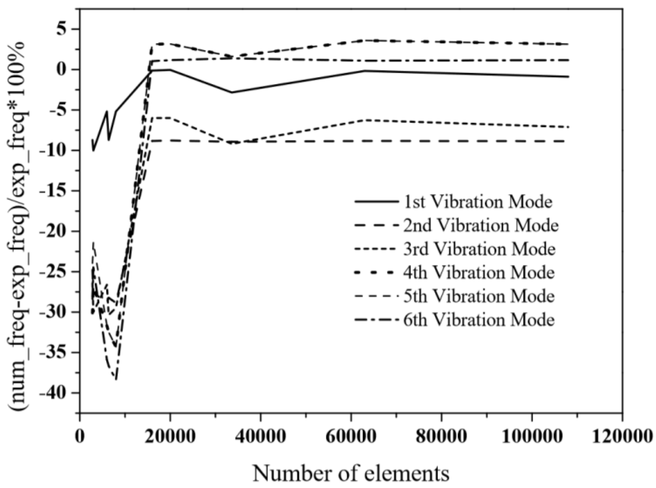


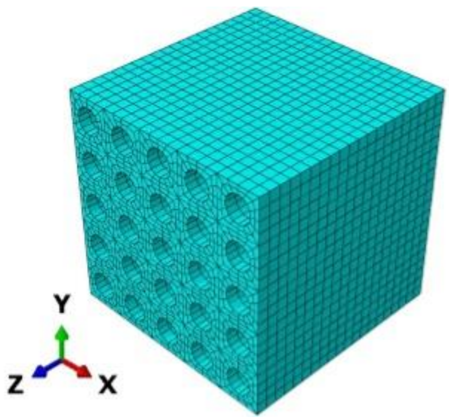
Figure 3. Methodology applied in the numerical model calibration

5. Numerical Model: Theoretical modal analysis

Initially, it is necessary determinate the order of each vibration mode. In this way, the model shown in Fig. 4.b was subjected to modal analysis (Eq. (5)) in ABAQUS finite element software [22]. The free-free boundary condition was used to represent the acoustic test conditions. For that, a *Step* was defined using linear perturbation procedure (frequency), in which the first six natural frequencies of the model were requested. Lanczos solver was used to obtain eigenvectors and eigenvalues. The mesh used, after the mesh convergence test is presented in Fig. 4.a, it was consisted of 16000 elements of type C3D8R, which is an eight-node brick with reduced integration and linear geometric order. Figure 5 illustrates these modes of vibrations and their natural frequencies.



(a)



(b)

Figure 4. Numerical Model (a) Mesh convergence test (b) Finite Element Model

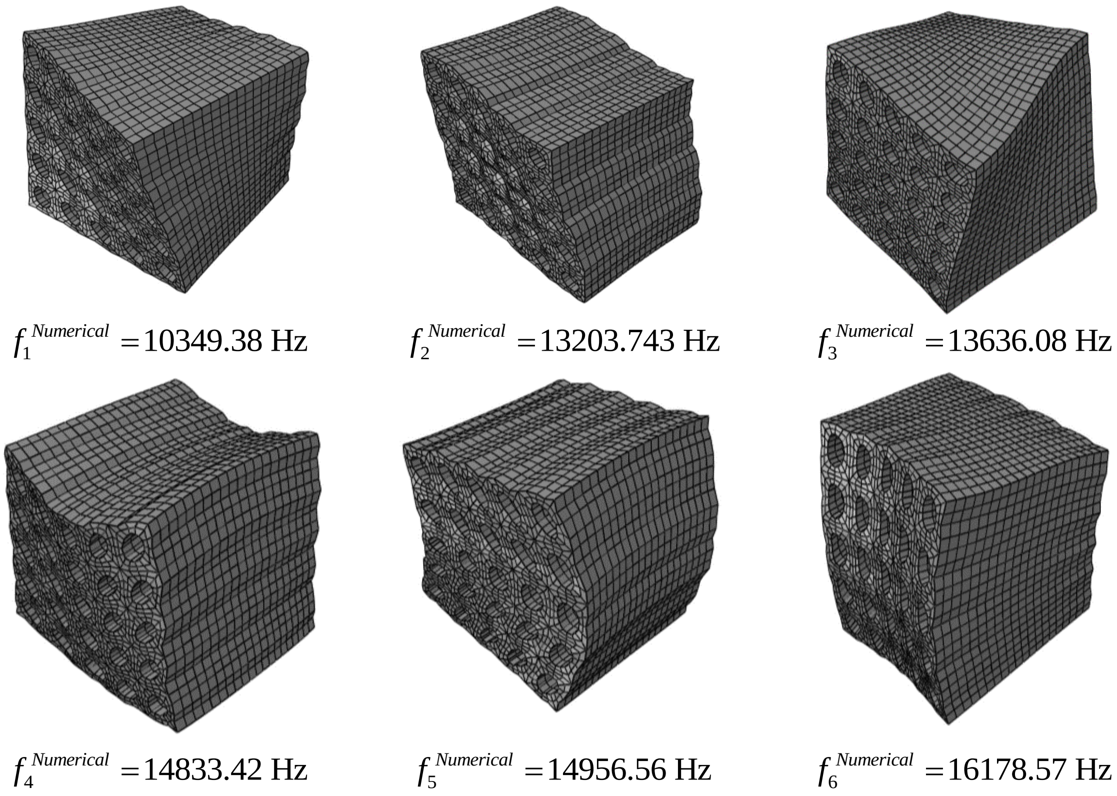
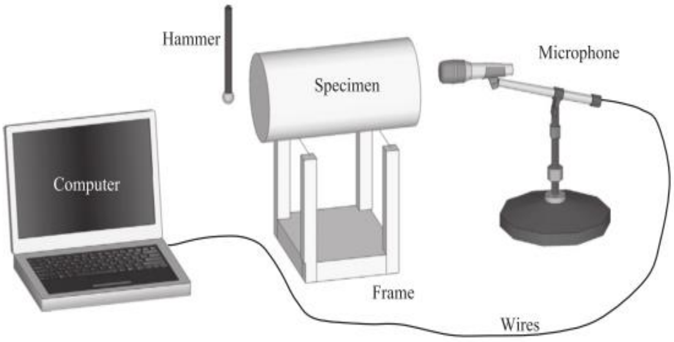


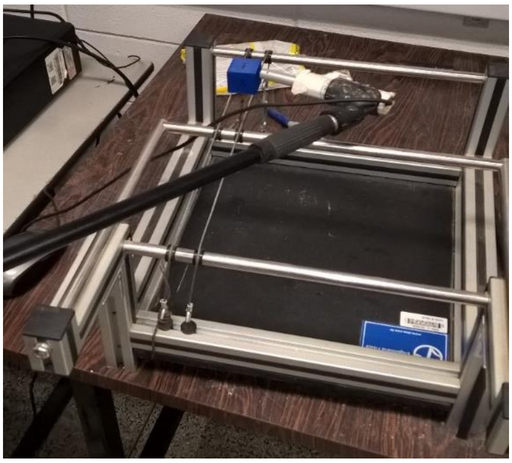
Figure 5. Vibration modes

6. Experimental Modal analysis

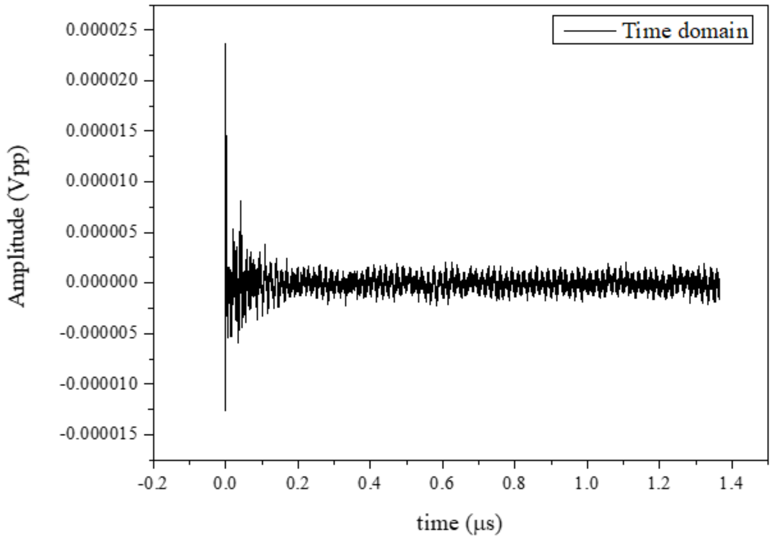
Once the order of natural frequencies and its vibrations modes were well stabilished, acoustic tests were employed to find the natural frequencies of the sample and correlate to the materials elastic properties through FEM updating. The test consisted in positioning the sample on steel wires attached to a metal frame (see Fig. 6.a and 6.b) and an impact was manually applied with a hammer on each specimen face (i.e., 1, 2 or 3), while a microphone captured the sound radiated by the specimen's surface in another position. Fig. 6.c shows a typical time vs. amplitude signal in ABS-sample tested obtained by hammer impact with eccentric excitation. After excitation, an onboard sound card of a regular notebook captured the acoustic signal at a 96 kHz acquisition rate. The first 1024-point block was selected, multiplied by a Hanning window and then zero-padded for the obtaining of an 8192-point vector. A Fast Fourier Transform was applied for the detection of natural frequencies peaks. It was possible identified the first six specimens vibration modes, according to Fig. 6.d presents a typical signal that is determined through a Fast Fourier Transform (FFT).



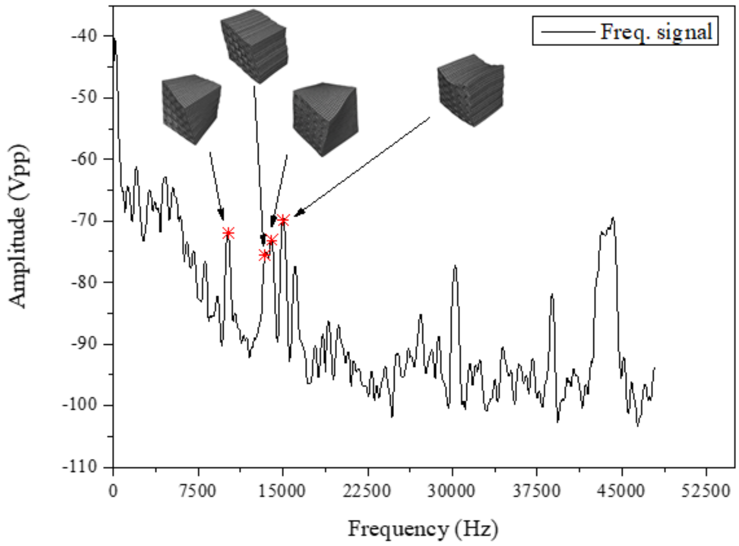
(a)



(b)



(c)



(d)

Figure 6. Acoustic test (a) test setup [23] (b) sample (c) time vs. amplitude signal (d) frequency vs. amplitude signal

The frequencies obtained by the experimental study are presented in Table 1.

Table 1. Frequencies obtained experimentally

Vibration Mode	1st	2nd	3rd	4th	5th	6th
Frequency (Hz)	10114.58	13484.375	13945.19	14976.56	14976.56	15967.63

7. Results

With the frequencies obtained by the acoustic test (Table 1), it was possible to solve the optimization problem, Eq. (2). The finite element model has been updated through genetic algorithms implemented in Python language. The parameters adopted in GA were a population with 25 individuals, crossover rate of 0.9, mutation rate of 0.1 and a selection by the roulette method with elitism. The stopping criterion was defined as the maximum number of generations in 200. The evolution of genetic algorithm is shown in Fig. 7.

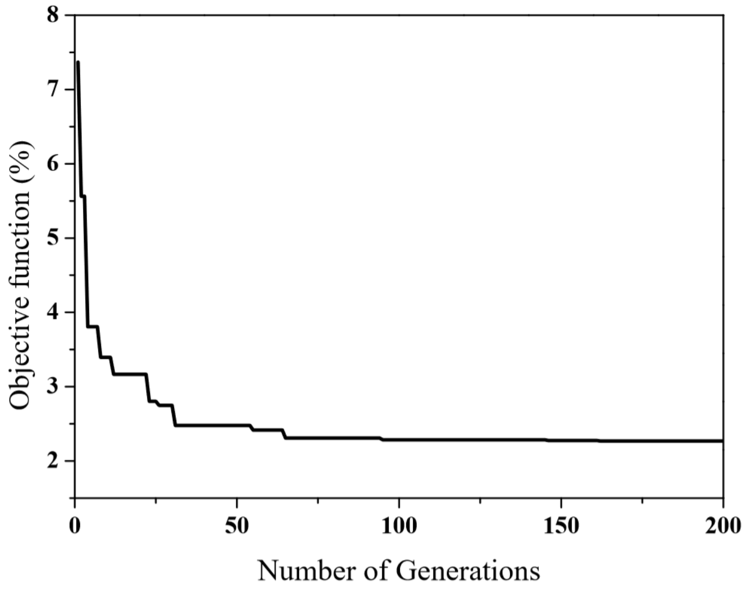


Figure 7. Evolution of the genetic algorithm

The elastic constants obtained by the calibration are shown in Table 2.

Table 2. Elastic constants determined by calibration

Constant	Calibrated value
D1111 (GPa)	3.25
D2222 (GPa)	2.9
D3333 (GPa)	4.7
D2323 (GPa)	1.01
D1313 (GPa)	1.06
D1212 (GPa)	0.91
D1122 (GPa)	1.35
D1133 (GPa)	2.45
D2233 (GPa)	2.05

The Table 3 illustrates the natural frequencies obtained by the finite element model calibrated and by the acoustic test. The error calculated between models results is less than 3%.

Table 3. Comparison between numerical and experimental results

Experimental frequency (Hz)	10114.58	13484.375	13945.19	14976.56	14976.56	15967.63
Numerical frequency (Hz)	10349.38	13203.743	13636.08	14833.42	14956.56	16178.57
Error (%)	2.27	2.13	2.27	0.97	0.13	1.30

The vibration models corresponding to the frequencies shown in Table 3 are presented in Fig. 8.

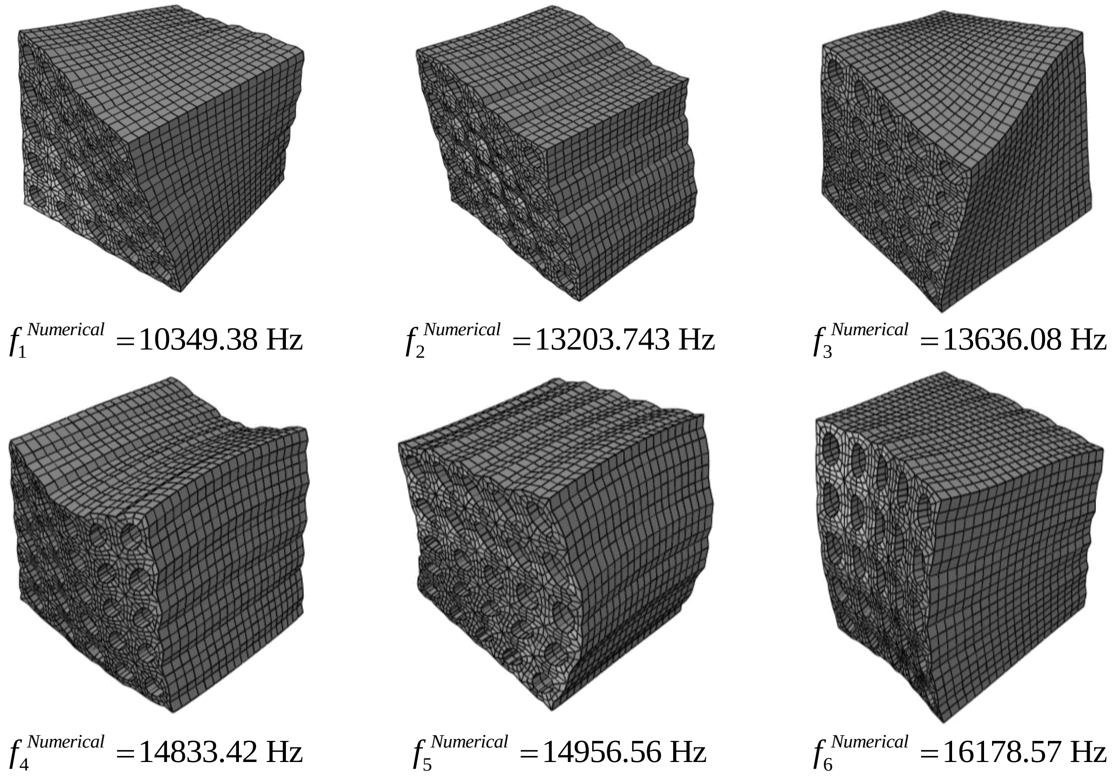


Figure 8. Vibration modes after calibration

8. Conclusions

In this paper we have shown some results concerning the calibration of elastic properties by means of a hybrid numerical-experimental methodology that objective the automatic updating of a finite element model regarding the experimental natural frequency of linear elastic solids. The natural frequencies obtained via finite element model calibrated differ from the natural frequencies obtained via acoustic test by relative errors less than 3%.

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