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STRUCTURAL OPTIMIZATION OF DOMES STRUCTURES CONSIDERING AESTHETICAL ASPECTS

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Abstract. Large-scale domes are commonly used in cultural buildings like stadiums, sports gymnasiums, hangars, and so on. Given a large number of people that interact with the environment, architectural aspects are desired in the conception of the structure. This paper deals with sizing structural optimization problems concerning the minimization of the masses of domes considering axial forces, displacements, and frequencies as the constraints of the problems. It is very common to use tubular cross-sectional areas of the members in the structural configuration. In this sense, it may be desirable that the bars should have the same outer diameters for visual comfort as well as aesthetical aspects. For instance, the designer may set just one outer diameter as the design variable. Thus limiting the maximum number of different thicknesses, also as design variables, to be used in the optimized structural configuration. The analyzes of the domes involve bending and torsion moments. Also, it can be attractive to use a reduced number of distinct cross-sectional areas minimizing costs of fabrication, transportation, storing, checking, welding, and so on. As a result, it is expected a labor-saving when the structure is welded, checked, and so on.

Keywords: Structural optimization, Differential Evolution, Steel space frames, Cardinality constraints

1 INTRODUCTION

Large-scale domes are commonly used in big cultural buildings like stadiums, sports gymnasiums, hangars, and so on. Given a large number of people that interact with the environment, architectural aspects are desired in the conception of the structure, most structures of this type are designed with Circular Hollow Sections (CHS) bars due to their high capacity of absorbing axial stresses as well as a good response to flexural bucking effect. The idea of designing such structure with all members having the same outer diameter is very convenient, not only considering aesthetical aspects, but it also is a cost and labor-saving in joints assembly and welding matters.

This paper deals with sizing structural optimization of dome structures concerning weight minimization. Axial forces, vertical displacements, and the first natural frequency of vibration are the constraints of the problem. The design variables are the outer diameter and the thickness of the hollow cross-section of the bars. The task of finding the most economical solution for this type of structure is not trivial. A Differential Evolution (DE) algorithm is adopted as the optimization algorithm, and it is coupled to an Adaptive Penalty Method (APM) [1] to handle the constraints violations. Also, an automatic member grouping with cardinatily constraints is applied to study different solutions where a maximum number of distinct cross-sectional areas are going to be assigned.

The paper is organized as following: Section 2 provides an overview of recent related works, Section 3 presents the formulation of the optimization problem discussed in this paper, Section 4 provides a brief explanation of the automatic member grouping by using cardinality constraints, Section 5 presents the basic concepts of DE and APM, Numerical Experiments and the Analysis of Results are described in Sections 6 and 7, respectively, and finally the Conclusions and Extension are reported in Section 8.

2 RELATED WORK

Recent researches and advances in metaheuristic algorithms have been proving its robustness in solving structural optimization problems, with the most various formulations and constraints. In 2011 Kaveh and Talatahari [2] studied structural optimization of geodesic domes using a charged system search. Later, in 2014, Kaveh and Javadi [3] worked with shape and size optimization of trusses with multiple natural frequencies of vibrations as constraints using a hybrid method. In this work, a Particle Swarm strategy and a Ray Optimizer was enhanced by a Harmony Search algorithm. In 2016, Kaveh et al. [4] proposed an optimal design of dome structures with natural frequencies of vibration as constraints . Recently, in 2017, Kaveh and BolandGerami [5] studied structural optimization for large-scale space steel frames by using cascade enhanced colliding body optimization with stresses and geometric constraints.

In 2018, Artar et al. [6] presented an optimization process using MATLAB-SAP2000 Open Application Programming Interface (OAPI) to minimize the weight of space steel frames with semi-rigid connections using Genetic and Harmony Search algorithms.

A truss optimization problem with multiple natural frequencies of vibrations as constraints and automatic member grouping was investigated by Carvalho et al. (2017) [7]. In this work, the members of large-scale domes are grouped in the optimized solutions by using the cardinality constraint with a special encoding proposed by Barbosa and Lemonge in 2004 [8]. An automatic member grouping is studied by Lemonge et al. [9] for the optimal design of steel-framed structures.

The use of DE can be found in Talatahari's work in 2015 [10], where an Eagle Strategy is used to optimize framed structures. In 2019, Vargas et al. [11] studied structural multi-objective optimization problems with the application of the DE coupled with the APM, introduced by Barbosa and Lemonge in 2002 [1]. Recently, in 2019, Resende et al. [12] proposed a design optimization of 3D steel frameworks under constraints of natural frequencies of vibration and horizontal displacements due to wind load, by using the DE and the APM.

Maheri et al. [13] presented an enhanced imperialist competitive algorithm for optimum design of skeletal structures in 2017. These authors presented in 2018 [14] an enhanced Honey Bee Mating optimization algorithm for the design of side sway steel frames. Hasançebi in 2017 [15], studied the cost-efficiency analysis of tall buildings in steel frameworks employing a parallel evolution strategy.

3 FORMULATION OF THE OPTIMIZATION PROBLEM

This optimization problem deals with weight minimization of a 120-bar dome structure depicted in Figure 1 where the members are originally grouped in a standard module as shown in Figure 2. The nodal coordinates, the vertical load and the maximum allowable displacement of each node are presented in Tab.1. The dome is modeled as a 3D steel frame, under constraints of axial forces, nodal displacements and natural frequencies of vibration. The dome is subjected to nodal loads in the gravity direction. The objective is to find a set of hollow cross-sectional areas Eq.(1) defined by the outer diameter(D) and different thicknesses(t) used to minimize the weight of the whole structure (w). Where N means the number of members (Eq (2)), $\rho = 7850kg/m^3$ is the specific mass of the steel, A_i and Li are the cross-sectional area and the length of the *i-th* member respectively. The search space refers to continuous design variables concerning the outer diameter (D) and the thickness (t) of the hollow section, within the upper and lower bounds as defined in Eqs. (3 and 4), respectively. A generic circular hollow section (CHS) is illustrated in Fig.(3) and its geometric properties are calculated as described in Eqs.((5), (6) and (7)), where A is the cross-sectional area, I_x and I_y are the moment of inertia about the principal axis, and I_o is the polar moment of inertia which for circular sections is equal to the torsional constant.

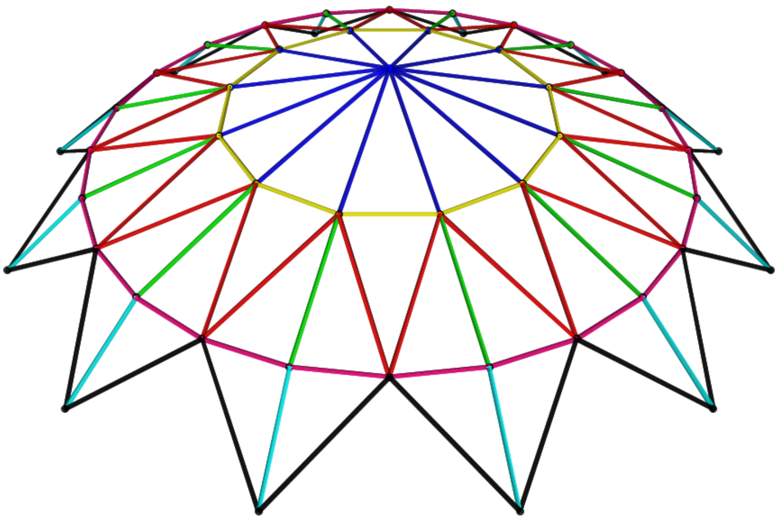

Figure 1. The 120-bar dome.

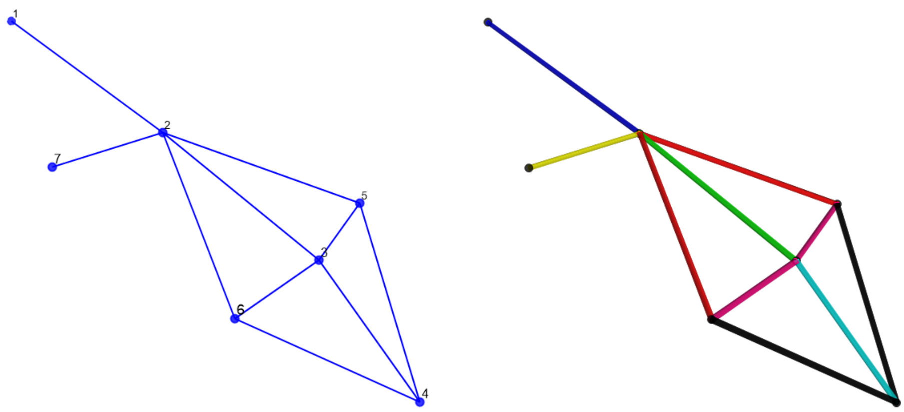

Figure 2. The standard module and the original member grouping

Table 1. Nodal coordinates, loads and allowable displacements

Node	x_1 (m)	x_2 (m)	x_3 (m)	Vertical load (kN)	Allowable displacement (mm)
1	0	0	7	120	17.5
2	6.941	0	5	60	17.5
3	12.5	0	3	20	12.5
4	15.89	0	0	0	0
5	12.0741	3.2352	3	20	12.5
6	12.0741	-3.2352	3	20	12.5
7	6.0111	-3.4705	5	60	17.5

$$\boldsymbol{x} = \{A_1, A_2, ..., A_N\} \tag{1}$$

$$w(\boldsymbol{x}) = \sum_{i=1}^{N} \rho A_i Li \tag{2}$$

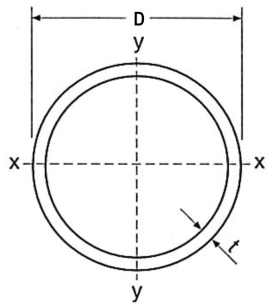

Figure 3. Generic Circular Hollow Section - CHS

$$60mm \leq D \leq 350mm \tag{3}$$

$$3.5mm \leq t \leq 25mm \tag{4}$$

$$A = \frac{\pi}{4}[D^2 - (D - 2t)^2] \tag{5}$$

$$I_x = I_y = \frac{\pi}{64}[D^4 - (D - 2t)^4] \tag{6}$$

$$I_o = \frac{\pi}{32}[D^4 - (D - 2t)^4] \tag{7}$$

The structure is subjected to maximum vertical displacements and the first natural frequency of vibration as constraints (Eqs.(8) and (9)). The $\bar{\delta}$ is the maximum allowable vertical displacement, δ is the nodal displacement, $\bar{f}_1 = 7Hz$ is the minimum allowable natural frequency of vibration and f_1 is the first natural frequency of vibration. The natural frequencies are obtained by the evaluation of the eigenvalues of the matrix $[(f_{nf}^2 \times M) + K]$ [16], in which M and K are, respectively, the mass and stiffness matrices, f_{nf} are the corresponding eigenvectors (structure vibration modes) to nf eigenvalues (natural frequencies of vibration of the structure).

$$\frac{\delta}{\bar{\delta}} - 1 \leq 0 \tag{8}$$

$$1 - \frac{f_1}{\bar{f}_1} \leq 0 \tag{9}$$

The members will be subjected only to axial forces. Since the loads are applied only at the nodes, the major stress effects in the members are from tension and compression forces. Bending and shearing effects are not taking into account in this sizing optimization problem due to their little contribution in this case. Thus, the axial force constraint is written by the Eq.(10), where P_r is the required axial strength and P_c is the available axial strength.

$$\frac{P_r}{P_c} - 1 \leq 0 \tag{10}$$

The axial strength will be calculated differently for members subjected to tension ($P_r > 0$) and compression ($P_r < 0$), as it is shown in Eq.(11)

$$P_c = \begin{cases} \dfrac{f_y A}{\gamma_{a1}} & if \quad P_r \geq 0 \\ \\ \dfrac{\chi f_y A}{\gamma_{a1}} & if \quad P_r < 0 \end{cases} \tag{11}$$

Where $f_y = 245MPa$ is the specified minimum yield stress of the material, A is the cross-sectional area of the member, and $\gamma_{a1} = 1,10$ is a safety factor for limit state of yielding according to Brazilian standards [17]. For members subjected to compression, a reduction factor χ, which considers the effect of geometric imperfection and eccentricity of the applying load, must be used for flexural buckling limit state. To determine χ, a reduced slenderness ratio (λ_o) must be calculated as it is shown in Eq.(12). In this expression, P_{cr} is the Euler elastic buckling load defined by Eq.(13), in which I is the inertia about the minor axis, $E = 200GPa$ is the steel elasticity modulus, and Lb is the effective buckling length.

$$\lambda_o = \sqrt{\frac{f_y A}{P_{cr}}} \tag{12}$$

$$P_{cr} = \frac{\pi^2 EI}{Lb^2} \tag{13}$$

The reduction factor χ is defined by Eq.(14), in which if $\lambda_o \leq 1.5$ it suggests an inelastic buckling state and if $\lambda_o > 1.5$ an elastic buckling (Fig. (4)).

$$\chi = \begin{cases} 0,658^{\lambda_o^2} & if \quad \lambda_o \leq 1,5 \\ \\ \dfrac{0,877}{\lambda_o^2} & if \quad \lambda_o > 1,5 \end{cases} \tag{14}$$

4 AUTOMATIC MEMBER GROUPING AND CARDINALITY CONSTRAINTS

In a real structure design, it is attractive that the designer can choose the maximum number of different cross-sectional areas to be assigned in the optimized structure. It can be lead to several advantages concerning: profiles fabrication and joints assembly cost, symmetrical and architectural aspects, labor-savings in material, checking, etc. The procedure consists in linking the variables to a maximum number of groups previously defined by the designer. For this reason, it must be considered additional constraints to the optimization problem regarding the maximum number of different cross-sectional areas allowable

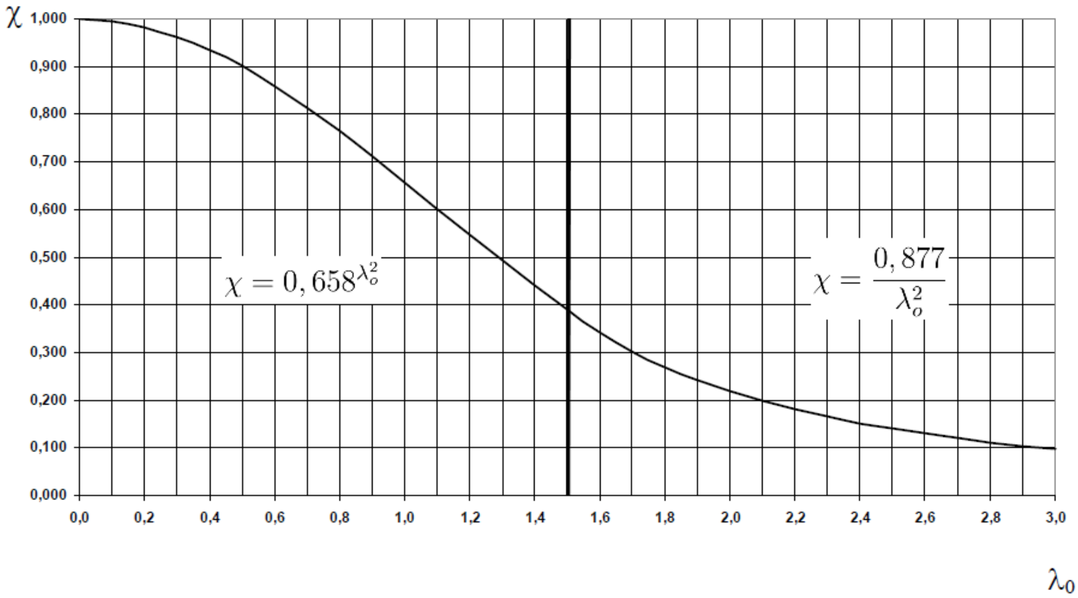

Figure 4. Reduction factor variation with slenderness [17]

to be used (m). Which, of course, must not be higher than the number of members $(m \leq N)$. The additional cardinality constraint requires that no more than m different cross-sectional areas should be used as Eq(15) proposed by Barbosa and Lemonge [8].

Figure 5 shows an example of a candidate vector with a cardinality constraint $(m = 2)$ for the automatic member grouping. The structure previous linked in seven different groups of members are detailed in Fig. 2. The values correspondent of the seven groups will vary continuously between 1 and the cardinality number m, and then it will be rounded to become in an integer. As the outer diameter must be the same for the whole structure, this number points to the corresponding thickness, defining the cross-sectional area for that group. In other words, D is the design variable concerning the outer diameter, t_1 and t_2 are the thickness (at most two different thickness, i.e., $m = 2$). The rest of the design variables are the seven cross-sectional (at most two distinct values, i.e., $m = 2$) to be assigned to the members of the standard module, and, consequently for the whole structure.

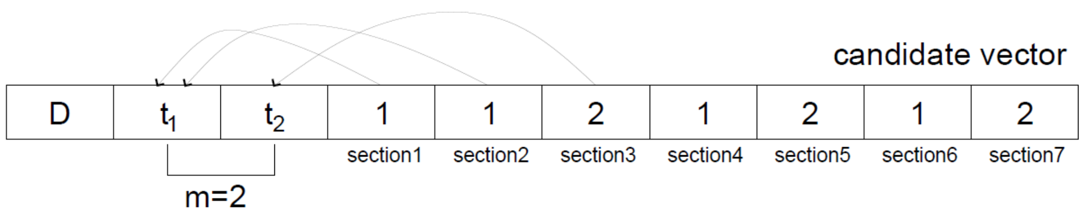

Figure 5. Generic candidate vector with cardinality constraint $(m = 2)$

$$A_i \in C_m = \{S_1, S_2, ..., S_m\} \qquad i = 1, 2, ..., N \tag{15}$$

Where the cross sectional areas S_j, $j = 1, 2, ..., m$ are unknown but belong to a larger $(M > m)$ given set $S = \{A_1, A_2, ..., A_M\}$.

5 DIFFERENTIAL EVOLUTION ALGORITHM (DE) AND ADAPTIVE PENALTY METHOD (APM)

The search algorithm used to solve the structural optimization problem is the Differential Evolution introduced by Storn and Price([18]) in 1995. It consists of an evolution of a candidate vector population in the search space. Both lower and upper bounds must be defined, before initializing the population, for each variable and then a pseudo-random candidate vector population is generated in the search space. After that, the evolution of the vector is governed by Eq(16).

$$\mathrm{v}_{i,g} \leftarrow \mathrm{x}_{r0,g} + F \times (\mathrm{x}_{r1,g} - \mathrm{x}_{r2,g}) \tag{16}$$

The vector $\mathrm{x}_{r0,g}$ is named base vector. It is a randomly chosen vector that must be different from the target vector $\mathrm{v}_{i,g}$. The difference vectors $\mathrm{x}_{r1,g}$ and $\mathrm{x}_{r2,g}$ are also randomly determined with an exception of being different from both the base and target vector. The scale factor $F \in (0, 1)$, is a scalar that controls the rate of population evolution. Figure (6) illustrates how the population evolve in DE.

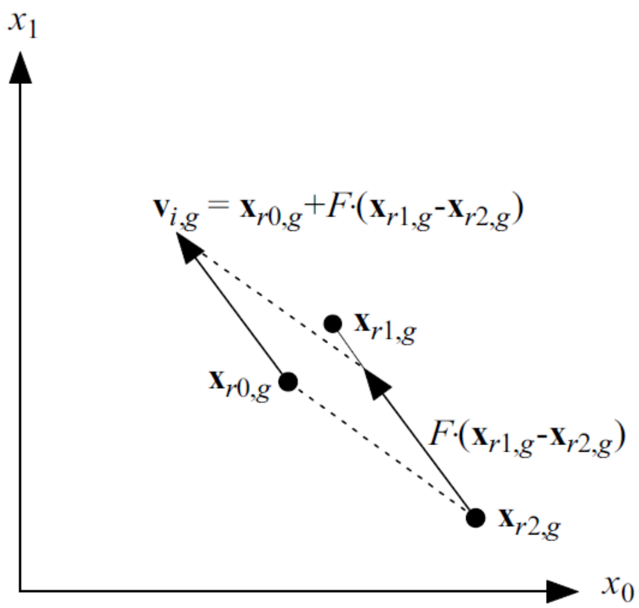

Figure 6. Illustration on how the the vectors are combined in the DE algorithm. [19]

In the DE algorithm mutation and crossover operators are considered.There is a pre-determined probability of crossover (Cr) as well as a probability of mutation between the new and the old individual. The pseudo-code of DE, in C language, is described in Fig(7).

To handle the constraints, an Adaptive Penalty Method (APM) proposed by Barbosa and Lemonge [1] is adopted. The APM adapts the value of the penalty coefficients of each constraint by using information collected from the population, such as the mean objective function and the level of violation of each restriction. The fitness function is defined as Eq.(17)

$$W(\boldsymbol{x}) = \begin{cases} w(\boldsymbol{x}) & if\ \ \boldsymbol{x}\ \ isfeasible \\ \\ \bar{w}(\boldsymbol{x}) + \sum_{jj}^{nc} k_{jj} v_{jj}(\boldsymbol{x}) & otherwise \end{cases} \tag{17}$$

Where $w(\boldsymbol{x})$ is the objective function of a candidate vector without penalization, and $\bar{w}(\boldsymbol{x})$ is defined by Eq.(18).

```
// initialize...

do // generate a trial population
{
   for (i=0; i<Np; i++)  // r0!=r1!=r2!=i
   {
      do r0=floor(rand(0,1)*Np); while (r0==i);
      do r1=floor(rand(0,1)*Np); while (r1==r0 or r1==i);
      do r2=floor(rand(0,1)*Np); while (r2==r1 or r2==r0 or r2==i);
      jrand=floor(D*rand(0,1));

      for (j=0; j<D; j++)  // generate a trial vector
      {
         if (rand(0,1)<=Cr or j==jrand)
         {
            uj,i=xj,r0+F*(xj,r1-xj,r2);  //check for out-of-bounds ?
         }
         else
         {
            uj,i=xj,i;
         }
      }
   }

   // select the next generation

   for (i=0; i<Np; i++)
   {
      if ( f(ui)<=f(xi) ) xi=ui;
   }
} while (termination criterion not met);
```

Figure 7. Pseudo-code of DE [19]

$$\bar{w}(\boldsymbol{x}) \begin{cases} w(\boldsymbol{x}) & if \quad w(\boldsymbol{x}) > \langle w(\boldsymbol{x}) \rangle \\ \\ \langle w(\boldsymbol{x}) \rangle & if \quad w(\boldsymbol{x}) \leq \langle w(\boldsymbol{x}) \rangle \end{cases} \tag{18}$$

Where $\langle w(\boldsymbol{x}) \rangle$ is the mean value of the objective function of the current population of candidate vectors. The penalty parameter k_{jj} is defined in Eq(19).

$$k_{jj} = |\langle w(\boldsymbol{x}) \rangle| \frac{\langle v_{jj}(\boldsymbol{x}) \rangle}{\sum_{ll=1}^{nc} [v_{ll}(\boldsymbol{x})]^2} \tag{19}$$

The variable $\langle v_{jj}(\boldsymbol{x}) \rangle$ is the violation of the jj-th constraint averaged over the current population considering only infeasible individuals. The complete formulation of the APM can be found in [].

6 NUMERICAL EXPERIMENTS

The numerical experiments analyzed in this paper refers to a dome structure of 120 members and 49 joints. The dome is generated by 12 sections of 10 members and 7 joints (the standard module depicted in Fig. 2, which is repeated every 30 degrees around the central vertical line. A preliminary member grouping is defined based on the symmetry of the structure where the members are divided into 7 different groups listed as following: one(blue), two(green), three(cyano), four(black), five(magenta), six(red) and seven(yellow), as shown in Fig. 2 as well as the whole structure (Fig. 1). In this numerical example, the structure is subjected to vertical loads in gravity direction.

Figure 8 shows the vertical load acting on the whole structure as well as axial forces of the members where the red color refers to compression and the blue to tension. Since it concerns a statically indeterminate structure, the value of internal forces depends on the cross-sectional areas and the stiffness matrix.

Five experiments are discussed in this section. Four of them setting cardinality constraints with $m = 1$, $m = 2$, $m = 3$ and $m = 4$ and one with no cardinality constraint (no.c.c.). The best solution of each experiment is the lightest structure found after ten independent runs, each one of them with 50 candidate vectors evolved in 200 generations.

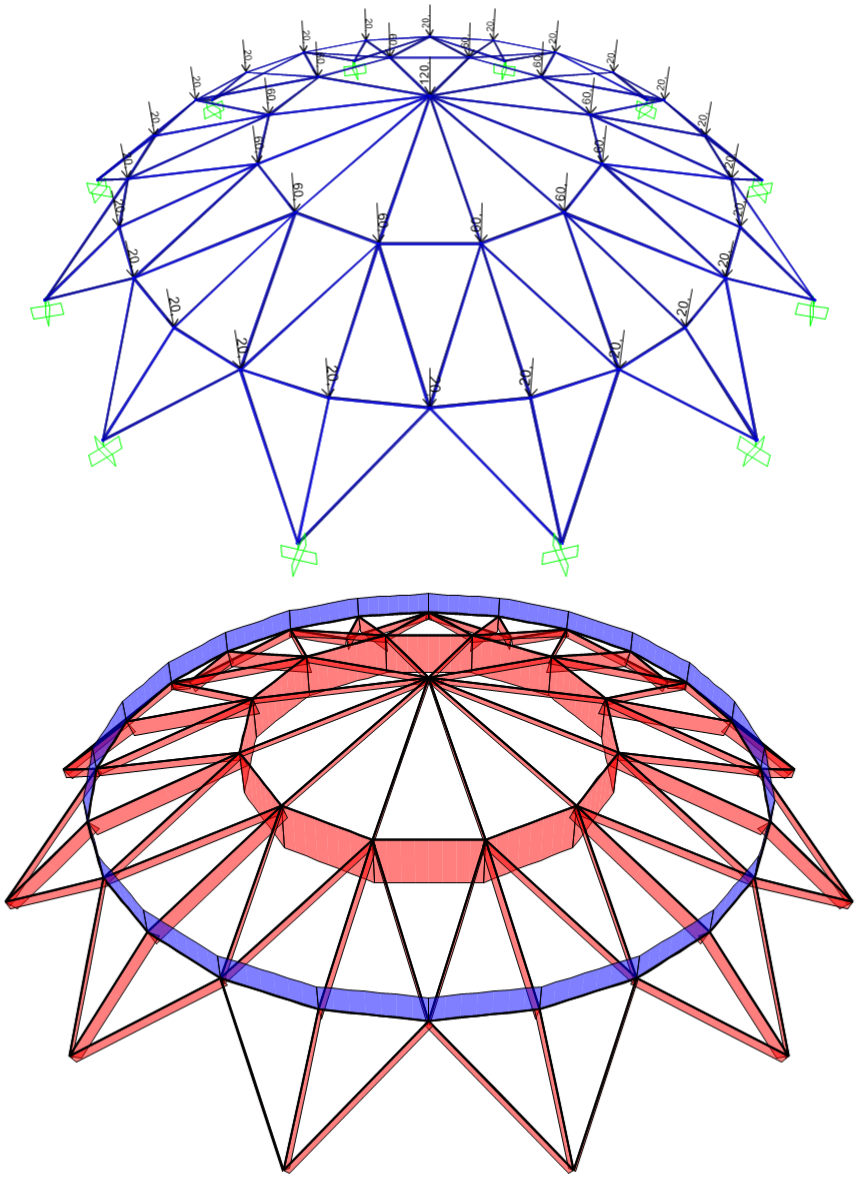

Figure 8. Vertical loads (upper) and axial force (down).

7 ANALYSIS OF RESULTS

Table 2 shows the optimized sizing design variables D, t_i (i = 1,7) and the final weight w for $m = 1, 2, 3$ and 4 as well as the case where no cardinality constraints (no.c.c) was set.

As expected, the best weight w decreased when the cardinality constraint was increased from $m = 1$ (Fig. 9) to $m = 2$ (Fig. 10) and $m = 3$ (Fig. 11). However, when the cardinality $m = 4$ (Fig. 12) is set, the best solution found is exactly the same of $m = 3$, indicating that a possible optimal solution of the problem was reached with only three distinct cross-sectional areas.

One can observe that the experiment with no cardinality constraint (Fig. 13), which could use seven different cross-sectional areas, according to original member grouping, reached a worse solution than the experiments with cardinality constraints ($m = 2$, $m = 3$, $m = 4$), which shows that the members were automatically linked in a more efficient way. Setting $m = 3$ or $m = 4$ the final weight (7212 kg, using only 3 or 4 cross-sectional areas), was 0.97% of the final weight (7404 kg, using 7 cross-sectional areas), found when no cardinality constraints was set.

Figure 14 shows the trade-off curves with the weight of the best solution varying with the cardinality constraint. Figures 9, 10, 11, 12 and 13 show the best solutions for $m = 1$, $m = 2$, $m = 3$, $m = 4$ and no.c.c., respectively.

Table 2. Sizing design variables D, t_i (i = 1,7) and the final weight w for $m = 1, 2, 3$ and 4 as well as no cardinality constraints (no.c.c).

m	D (mm)	t_1 (Blue)	t_2 (Red)	t_3 (Green)	t_4 (Black)	t_5 (Cyano)	t_6 (Magenta)	t_7 (Yellow)	w (kg)
1	155.5	3.5	-	-	-	-	-	-	8223
2	85.3	7.3	3.5	-	-	-	-	-	7319
3	87.6	6.5	3.5	9.7	-	-	-	-	7212
4	87.6	9.7	6.5	6.5	3.5	-	-	-	7212
no.c.c.	91.4	3.5	10.1	3.5	6.9	6.3	5.0	4.1	7407

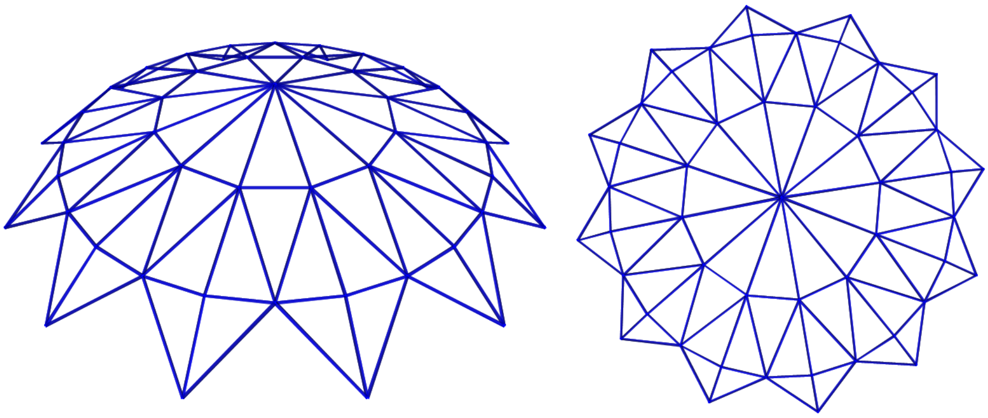

Figure 9. The best result for $m = 1$

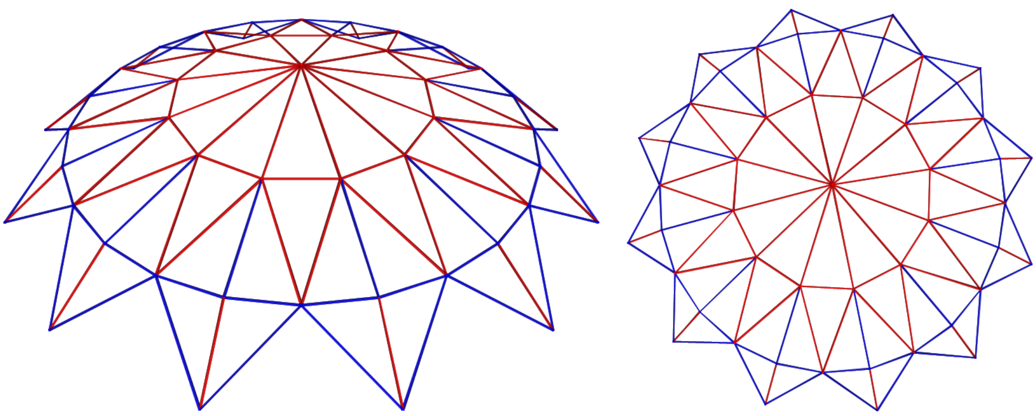

Figure 10. The best result for $m = 2$.

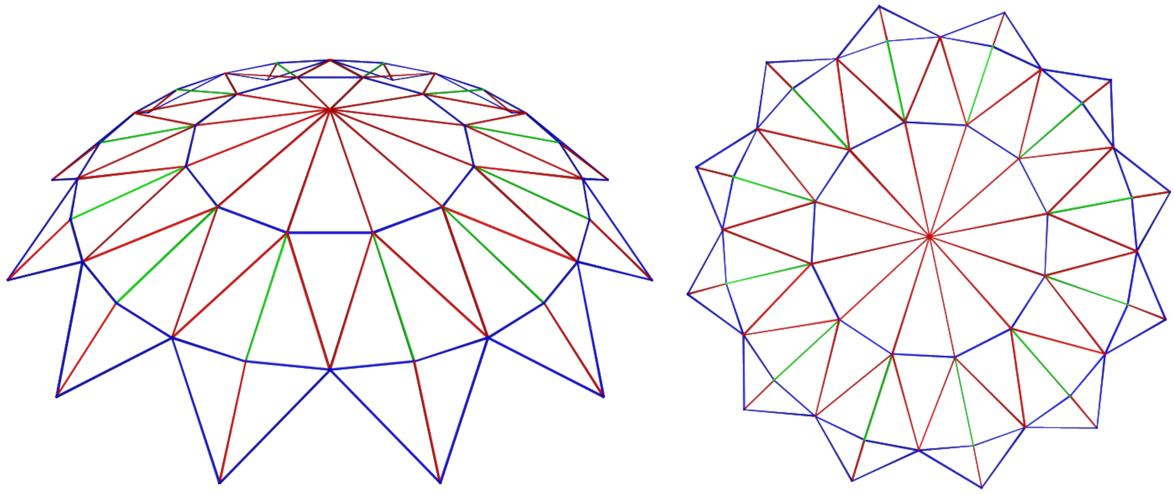

Figure 11. The best result for $m = 3$.

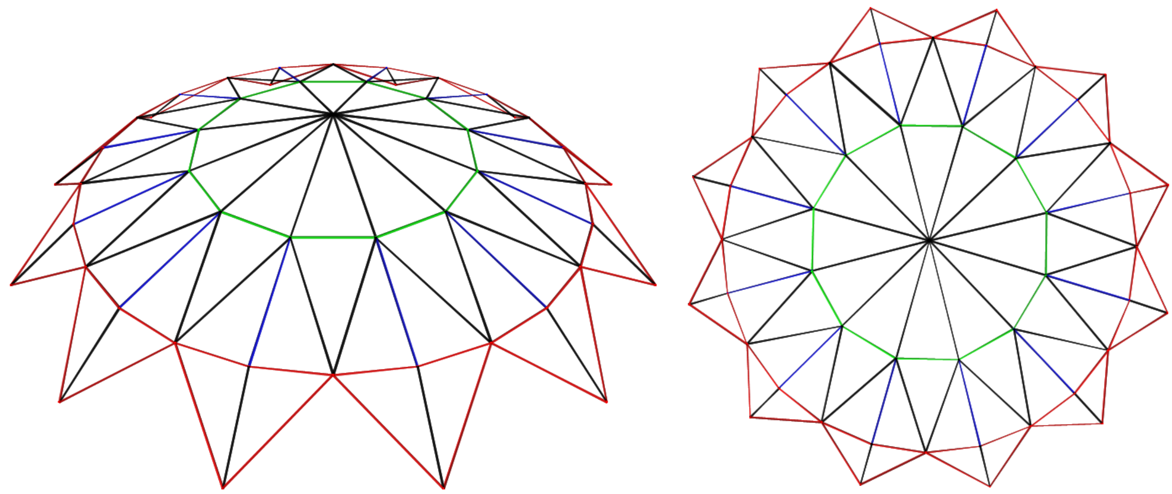

Figure 12. The best result for $m = 4$.

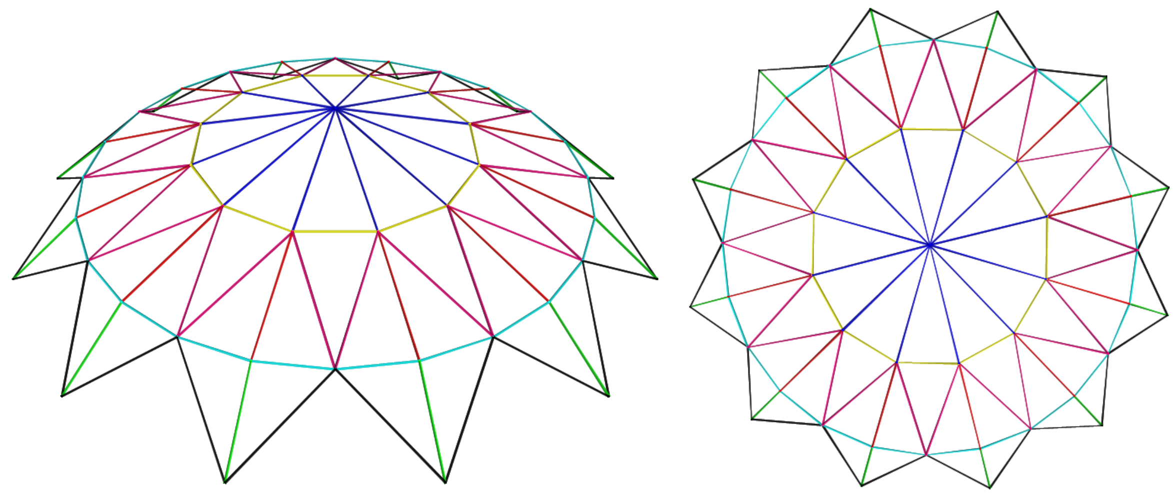

Figure 13. The best solution for no cardinality constraint.

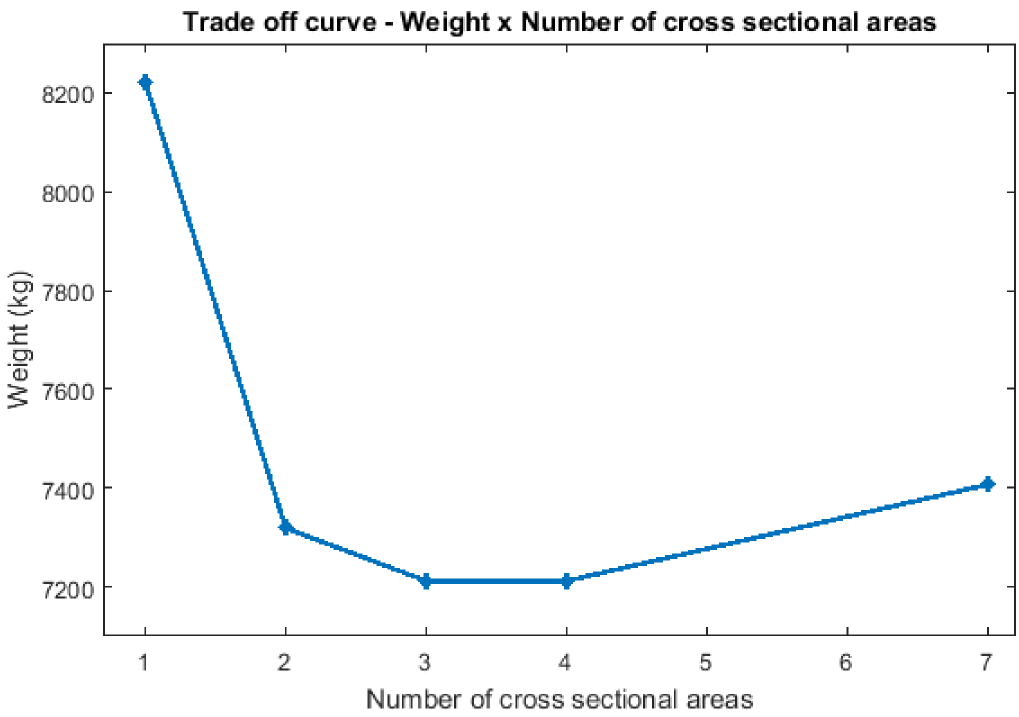

Figure 14. Trade off curve concerning the final weight and the different cross-sectional areas.

8 CONCLUSIONS AND EXTENSIONS

The study conducted in this paper focused on minimization of 3D steel dome modeled as a framed structure under constraints of axial forces, displacements, and natural frequencies of vibration. In the problems addressed here, aesthetical aspects of the structure are taken into consideration, in which all the members are designed in circular hollow sections and must have the same outer diameter changing only its thickness.

The numerical experiments analyzed here provided very interesting results. The application of an automatic member grouping with cardinality constraint led to better solutions with a limited number of different cross-sectional areas, showing different and counter-intuitive ways of linking the members to achieve better results.

It is important to highlight that the search mechanism found a lighter structure for the problem constrained to four different cross sectional areas than for the problem with no cardinality constraint. It is not possible to assume that there is a chance to find a lighter structure with no cardinality constraint. The complexity of the problem leads to an onerous search which can not ensure a better solution. That is one reason why the cardinality constraint comes as an useful device in order to simplify the problem by reducing the design variables number.

The optimization study of dome structures considering aesthetical aspects can be of great value before the real structure design conception, providing the designer information about the member grouping, outer diameter, and thickness that satisfy the constraints.

As extensions and future works, the approaches can be extended to larger structures and the sizing design variables taken from commercial tables of profiles.

Acknowledgments

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Remark

The codes used to solve the optimization problems presented in this chapter are written in $Matlab^{®}$ language and the final results, as well as the figures, are checked by the $SAP - 2000^{®}$.

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