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REGULARIZED SOLUTION FOR STRUCTURE HEALTH MONITORING

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Abstract. Structural damage is a key concern for several areas in civil engineering sector (buildings, bridges), mechanical engineering, aerospace sector (aircrafts, rockets, satellites), automotive sector, and many other Therefore, the structure health monitoring is an important topic research and operation. The structural damage identification can be addressed as an inverse vibration problem. This inverse problem can be formulated as a generalized least square problem, where the stiffness matrix must be identified in order to have a best matching between measurements and the mathematical model added to a regularization operator. We solve the forward problem using finite element method, and an entropic regularization is also applied to the cost function. The optimization problem is solved by using a hybrid method, combining a stochastic metaheuristic with a local searching method. The Multi-Particle Collision Algorithm (MPCA) is the metaheuristic technique and the Hooke-Jeeves (HJ) direct search method complete the hybrid optimizer. The proposed methodology is applied to different study cases showing good results. A cantilever beam is the testing structure for the developed approach.

Keywords: Damage identification, regularized inverse solution, hybrid optimization approach, Multi-Particle Collision Algorithm (MPCA), Hooke-Jeeves (HJ) method.

1 Introduction

The health monitoring for structure is a critical issue for many activities, such as aerospace engineering. Indeed, if a damage is not detected and not repaired, the structure could collapse, implying serious consequences, with severe human and material losses. Structural damage identification is an important branch of the vibration inverse problem. Inverse problems belong to a class of ill-posed problems.

The Russian mathematician Andrei Nikolaevich Tikhonov is cited as the first scientist to formulate a general technique to compute an inverse solution with his *regularization theory* [1]. The regularization technique is based on minimization of a functional. We follow similar scheme, and the damage identification is formulated as an optimization problem, where the objective function has two parts. One part evaluates the agreement between the mathematical model and the measurements, and another term of the objective function is the regularization operator.

The forward problem is a matrix differential equation of second order evoluting with time from the second law of the Newtonian mechanics. The structure is discretized using finite elements, where mass, damping, and stiffness matrices are structure characteristics. The response to an external forcing is associated with structure elements, where the stiffness deviation of one or more elements is interpreted as a structural damage. Therefore, damage identification problem is linked to find the system stiffness matrix. So, this inverse solution is obtained by solving the optimization model containing two parts, as mentioned in the last paragraph.

The structural health monitoring, by damage detection, is recognized as a hard problem, in particular dealing with a system with high degrees-of-freedom (DOF). One difficulty is because the objective function can have many local minima. Another mathematical difficulty happens when the search space drops on a hyperplane, where gradient-based methods fail. One approach for calculating an inverse solution is applying metaheuristics without use of gradient information. A hybrid optimization method is employed here, for avoiding be trapped into local minimum, and a local searching method for speeding-up the convergence. The hybrid approach combines the Multi-Particle Collision Algorithm (MPCA) metaheuristic [2], with the Hooke-Jeeves (HJ) direct search method [3] as a second stage to identify the stiffness matrix. This hybrid optimization was already used for mass-spring problem stiffness identification problem [4]. An entropic regularization is also applied, and the regularization parameter is determine by numerical experimentation.

The Cantilever beam structure will be analized here, where the initial conditions for the strucutre are assumed to be at rest. Synthetic observations are considered for testing the inverse methodology.

2 Determining Structural Damage by an Inverse Solution

The direct problem is expressed in a matrix form, describing a system with many degrees-of-freedom (DOF). The second order and non-homogeneous system of ordinary differential equation represents the vibration problem:

$$\mathbf{M}\frac{d^2\mathbf{u}(t)}{dt^2} + \mathbf{C}\frac{d\mathbf{u}(t)}{dt} + \mathbf{K}\mathbf{u}(t) = \mathbf{F} \tag{1}$$

where $\mathbf{M},\ \mathbf{C},\ \mathbf{K}$ are matrices of mass, damping, and stiffness, respectively; $\mathbf{F}$ is the forcing term; and $\mathbf{u}(t)$, $d\mathbf{u}(t)/d$t, $d^2\mathbf{u}(t)/dt^2$ represents the displacements, velocity, and acceleration, respectively. The initial conditions are given by:

$$\begin{cases} \mathbf{u}(0) = \mathbf{u}_0 \\ d\mathbf{u}(0) = d\mathbf{u}_0/dt \end{cases}$$

It is hard to calculate an analytical solution for arbitrary values of $\mathbf{K},\ \mathbf{C},\ \mathbf{M}$, and $\mathbf{F}$. Therefore, a numerical solution for this forward problem is obtained using the Newmark method [5].

The inverse problem for damage determination is expressed as an optimization problem. The optimal inverse solution is found by minimizing the functional below:

$$J(\mathbf{K}) = \sum_{i=1}^{N_p} \Big[\mathbf{u}^{\text{Exp}}(t_i) - \mathbf{u}^{\text{Mod}}(t_i, \mathbf{K})\Big]^2 + \alpha\, \Omega[\mathbf{K}] \tag{2}$$

where $\mathbf{u}^{\text{Exp}}$ and $\mathbf{u}^{\text{Mod}}$ are the experimental and computed displacements at time t, respectively, with N_p meaning the number of measuring points, α is the regularization parameter, and $\Omega[.]$ is the regularization operator. The parameter α is determined by numerical experimentation following the Morozov's discrepancy principle [6].

The regularization process is based on the maximum principle of entropy, proposed as an inference criterion by Jaynes [7] on basis on the Shannon's theory of information [8]. The entropy regularization will be looking for the smoothest candidate solution. This regularization has been used in many application such as astronomy [9], tomography [10], geophysics [11], and heat transfer [12].

The discrete entropy of vector $\mathbf{r}$ is given by

$$S = \sum_{q=1}^{Q} s_q \, \log(s_q) \tag{3}$$

where $s_q = r_q/(\sum_{q=1}^{Q} r_q)$. The maximum of entropy S is reached when s_q belongs to an uniform distribution, implying $S_{\max} = \log Q$, and the minimum value for the entropy is verified when s_q is associated to the Dirac delta distribution.

2.1 Solving the Optimization Problem

The solution of the optimization problem, i.e. the inverse problem, the global optimization algorithm (MPCA) looks for *good* candidate solutions in the search space. If the stopping criteria is reached, the algorithm is interrupted. In the sequence, the local optimization algorithm (HJ) is activated, intensifying the searching process. The best solution found by HJ will be the solution for the inverse problem.

Multi-Particle Collision Algorithm

The Multi-Particle Collision Algorithm (MPCA) is a method based on the traveling of particles (neutrons) inside of the nuclear reactor [2]. Two main phenomena are identified during the neutron traveling: absorption and scattering. A set of particles (candidate solutions) are randomly generated. Three principal functions in the algorithm control all the process: perturbation, exploration, and scattering. Particles are perturbed, and depending on their fitness, they are absorbed or scattered to other region of the space search.

Particles in the whole population behave cooperatively, i.e., the best particle overall is over-copied for all other particles in the set, through a blackboard strategy. The *UpdateBlackboard* procedure is applied each a number of function evaluations $NFE_{\text{blackboard}}$. As stopping criterion, a maximum number of function evaluations NFE_{MPCA} is defined. Here, we are going to employ a recent version of the MPCA, where a new strategy to select new candidate solutions is adopted. The new strategy is called Rotation-Based Sampling (RBS) [4]. The new mechanism is based on Opposition-Based Learning (OBL), where a candidate solution is proposed and an opposite candidate (considering the distance of the candidate solution up to the center of the search space) is also evaluated. A generalization of the opposite procedure is considered taking into account the circle with radius being the distance between the candidate solution and the center of the search space, and a new candidate solution can be evaluate selecting a point on the circle.

Hooke-Jeeves Method

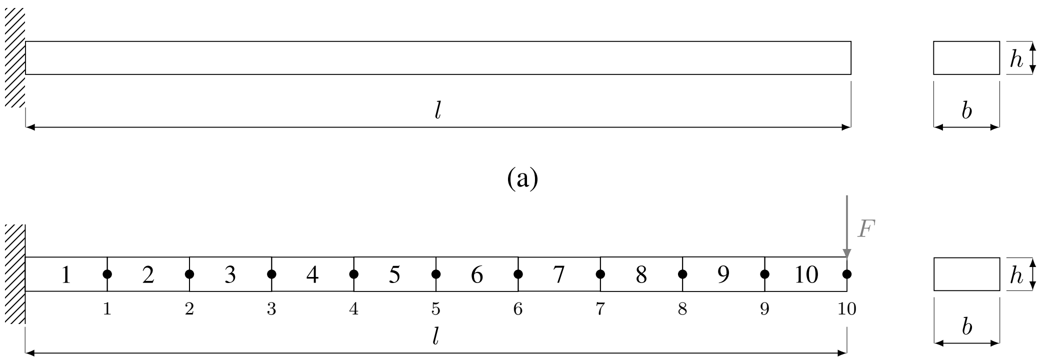
The direct search method of Hooke-Jeeves (HJ) [3] consists of the repeatedly application of exploratory movements about a base point which, if successful, is followed by pattern moves. In a D-

dimensional problem, a candidate solution is denoted as a vector $\mathbf{s}$ of length D. The exploratory movement consists adding one column of the search directions matrix $\mathbf{V}$, scaled by a step size $\mathbf{h}$, to the solution $\mathbf{s}$. This process is made over all the dimensions of the problem. A new solution is accepted if it is better than the previous $\mathbf{s}$. If the exploratory movement was successful, it will return an improved solution $\mathbf{s}^m$.

The pattern move $\mathbf{s}^{m*}$ is computed adding a search direction $\mathbf{s}^m - \mathbf{s}^c$ to $\mathbf{s}^m - \mathbf{s}^{n*}$. If $\mathbf{s}^{n*}$ is better than $\mathbf{s}^m$ then it will replace the latter, else $\mathbf{s}^m$ will become the new $\mathbf{s}^c$. If no improvement is found for $\mathbf{s}^c$, the step size $\mathbf{h}$ is reduced in ν times. As stopping criteria, a minimum step size ($\mathbf{h}_{\min}$) and a maximum number of function evaluations (NFE_{HJ}) are defined.

3 Damage Identification in a Cantilever Beam

The cantilevered beam shown in Figure 1a is modeled with ten beam finite elements – see Figure 1b. It is clamped at the left end, and each aluminum beam element, with $\rho = 2700\ kg/m^3$ and $E = 70\ GPa$, has a constant rectangular cross section area with $b = 15 \times 10^{-3}\ m$ and $h = 6 \times 10^{-3}\ m$, a total length $l = 0.43\ m$, and a inertial moment $I = 3.375 \times 10^{-11}\ m^4$. The damping matrix is assumed proportional to the undamaged stiffness matrix $\mathbf{C} = 10^{-3}\mathbf{K}$. An external varying force $\mathbf{F}(t) = 5.0 \times 2.0 \sin(\pi t)\ N$ is applied to the tenth element – see Figure 1b, in the free extreme of the beam, as shown in Figure 2. Initial conditions for displacement and velocity are assumed equal to zero: $\mathbf{u}(0) = 0,\ \dot{\mathbf{u}}(0) = 0$.



(a)

(b)

Figure 1. Cantilever Beam structure (a), and beam model with 20-DOF (b).

Strain-gages are sensors for measuring structural displacements, while rotations could be measured by rotation rate sensors or gyroscopes [13]. Here, synthetic observations were taken from the nodes of the structure, by executing the forward model. For the experiments, the numerical simulation was performed assuming $t_f = 2s$, with a time step $\Delta t = 4 \times 10^{-3} s$.

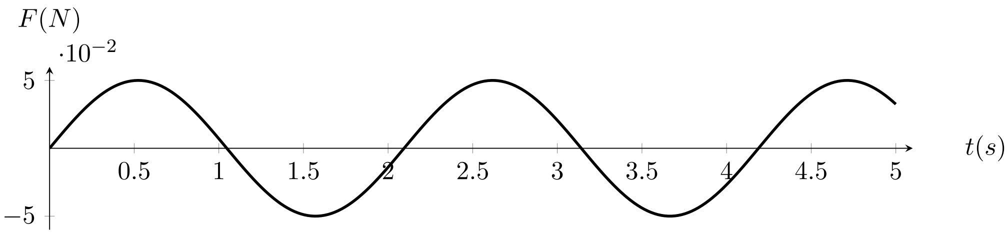


Figure 2. Load $F(t)$ applied on the cantilever beam.

Four numerical experiments are performed: noiseless data, white Gaussian noise data with zero mean, and three levels of noise: $\sigma^2 = 0.02$, $\sigma^2 = 0.05$, and $\sigma^2 = 0.10$ – only the last two cases are shown in this paper. The synthetic measurements are computed from additive noise:

$$\mathbf{u}^\delta(t) = \mathbf{u}(t)\left[1 + \delta(t)\,\sigma^2\right] \tag{4}$$

where $\mathbf{u}(t)$ is the displacement calculated by the forward model, and $\delta(t)$ is a random number with Gaussian distribution $\mathcal{N}(0, \sigma^2)$. Figure 3 shows the dynamic response for the displacement in the nodes 1, 5, and 10 of the system with (red color) and without (blue color) damages.

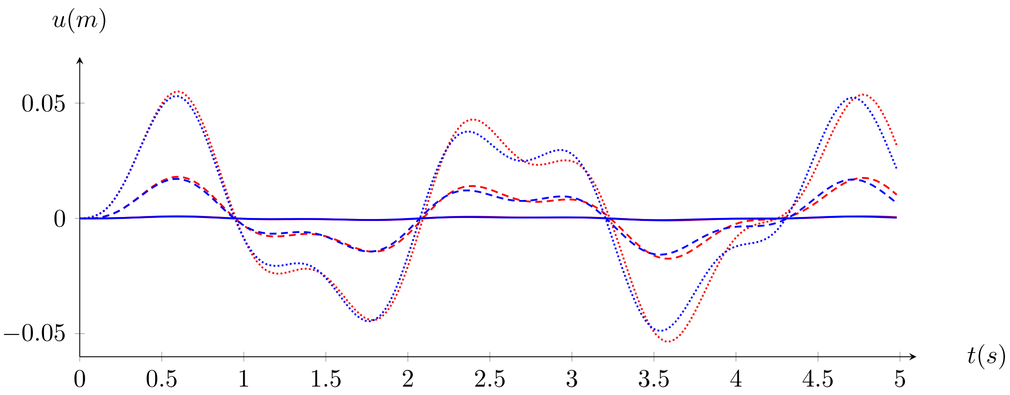


Figure 3. Dynamic responses for node 1 (solid line), node 5 (dashed line), and node 10 (dot-dashed line) of the structure with damages (red), and without damages (blue).

Different scenarios for the damage configuration were considered:

1. Measurement points on all discrete elements, with only one damaged element (node-1).
2. Measurement points on two elements for displacement (nodes 2 and 10) and on two elements for rotation (nodes 5 and 10), with only one damaged element (node-1).
3. Measurement points on all discrete elements, with several damaged elements (nodes 2, 4, 6, 9, 10).
4. Measurement points on two elements for displacement (nodes 2 and 10) and on two elements for rotation (nodes 5 and 10), with several damaged elements (nodes 2, 4, 6, 9, 10).

For noiseless observation data, the regularization is not necessary. The reconstructions for the scenarios 1, 2, 3 were perfect. For the scenario 4, the methodology identify the damages, but indicates false damages on the 9-th and 10-th elements.

The stiffness reconstruction for scenario-1 with noisy data can be visualized in Figure 4. The method can identify the location and the intensity of damage. The effect of regularization is clear: reconstruction **without regularization** is marked with **blue color**, while the **regularized solution** is shown on **red color** – see Figure 4. There are false damages from the inversion without regularization. The regularized reconstruction is almost perfect – only on 2-th element is indicate a smaller intensity damage.

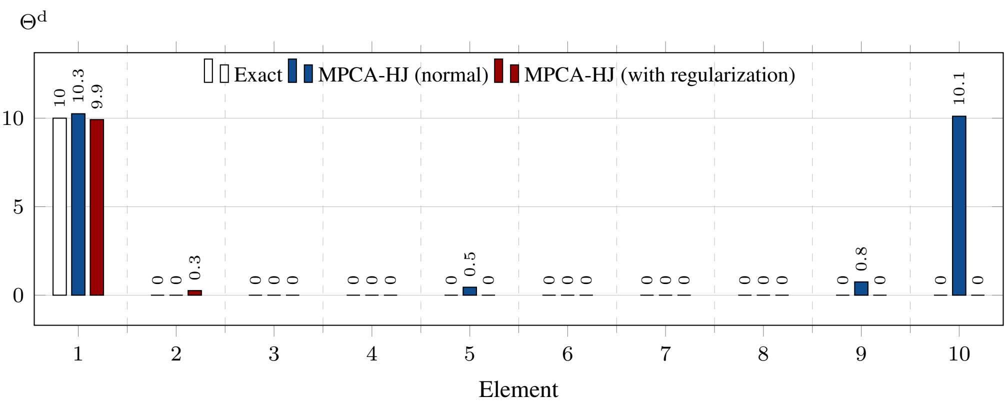


Figure 4. Results for the MPCA-HJ using entropy regularization with measurements on all discrete points, with only one damaged element.

For scenario-4, the damage configuration was assumed with 10% stiffness reduction on the 2nd element, 20% on the 4th, 30% on the 6th, 5% on the 9th element, and 10% on the 10th element. The remaining elements are assumed as undamaged. Measurements were taken from some degrees of freedom: displacements from node 2 and node 10, and rotation from node 5 and node 10. Therefore, four time-series with 500 points are stored. For the analysis for each time-series, an average of 50 executions of the inverse solution was calculated.

All damaged elements for the scenario 4 were identified – see Figure 5 – on nodes 2, 4, 6, 9, and 10. Hybrid optimizers with MPCA and other two different versions of MPCA (CBMPCA – Center-based Sampling – and RBMPCA – Rotation-based Learning) were effective, with no significant different performance among them. However, the MPCA obtained best answer to recover the stiffness for no damaged elements.

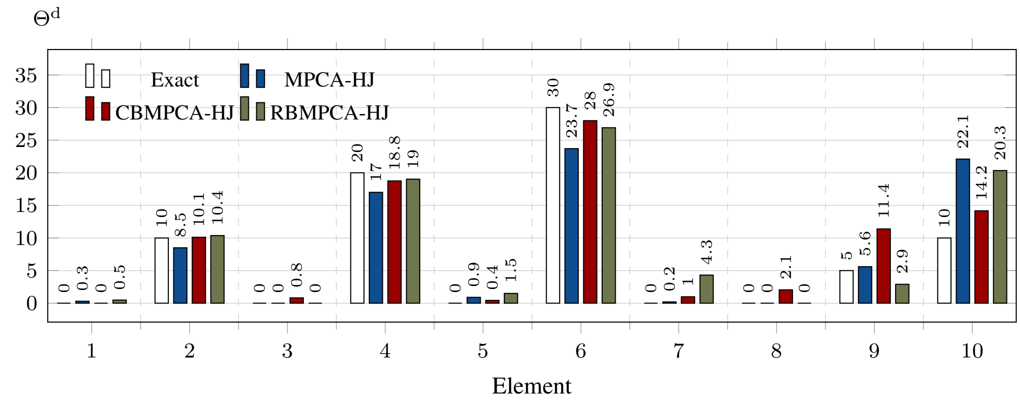


Figure 5. Results for the MPCA-HJ, CBMPCA-HJ, and RBMPCA-HJ using four measurement points, with damaged elements on nodes 2, 4, 6, 9, 10.

4 Final Remarks

The structural damage identification was formulated as an optimization problem, with the cost function described by the best agreement between the measurements and the mathematical searching for a smooth solution. The forward model was solved by using the finite element method and the Newmark method employed for time integration. The hybrid methodology, combining a metaheuristic – MPCA and two version of it – and the local searching method – the Hooke-Jeeves approach –, was an effective strategy to compute the optimal solution.

Four scenarios with different levels of noise were designed for testing the inversion methodology. The experiments showed the relevance to apply the regularization to achieve a more precise damage identification – see Figure 4. Scenario-4 presented a harder challenge, and good inverse results were also obtained for this scenario.

Finally, it is important to mention that true positions of damages were identified, considering all tested scenarios with different levels of noise, including noisy data with $\sigma^2 = 0.10$ – the highest noise level considered in our numerical experiments.

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