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A comprehensive model of fatigue for steel beams

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Abstract. Steel beams may be subjected to cyclic loads and consequently damaged by the fatigue phenomena. Concerning to identify and to monitor the crack initiation, the crack evolution and the subsequent failure of this structures, in this paper it is proposed a model that allows the study of the fatigue phenomena, either for ultralow and low cycles and high and ultrahigh cycles, considering both stationary and non-stationary fatigues. A new state variable for the identification of the beginning of the crack propagation is introduced for each plastic hinge, which is called the pre-damage variable. Its evolution law is based on Manson-Coffin law and also on the Basquin law. Fundamental concepts of the Lumped Damage Mechanics (LDM) are also included. It allows the consideration of the damage variable itself and its influence in the constitutive relationships. A new damage law is settled and takes into consideration effects from both different types of fatigues. Appropriately, the crack closure effect is used through the unilateral damage assumption. Finally, the accuracy of the model is its validation is done by a finite element analysis (FEA) to predict the experimental behavior of tests of steel beam-column that are available in the literature.

Keywords: ultralow and low cycle fatigue, high and ultrahigh cycle fatigue, lumped damage mechanics.

1 Introduction

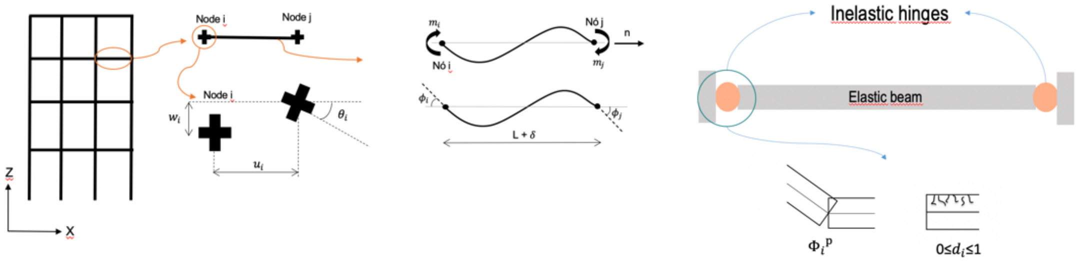
Understanding the fatigue mechanism is indispensable for the analysis of fatigue properties and of the technical conditions which affect fatigue life and fatigue crack growth. Fatigue prediction methods can only be evaluated by defining a crack initiation process followed by a crack growth period (Schijve [1]). Peerlings et al. [2] say that methods of design against fatigue traditionally focus on these two stages and there are damage-tolerant approaches that use analyses to define the number of loading cycles that the component can resist before the cracking initiation starts. Thus, willing to have a better idea of fatigue failure mechanism induced by repeated loading, both full-scale and small-scale fatigue tests were conducted in laboratories, which the results of those tests could be represented by the well-known Wohler S-N diagram, for example, as showed in Farahmand et al. [3]. According to the same authors, through these test analyses, it was possible to identify the existing types of fatigue and its particular behaviors: the ones from the low cycle fatigue (LCF) and the ones from the high cycle (HCF).

Once it is known that fatigue is one of the major causes of in-service failure throughout the engineering history (Homan [4]), an interest of the civil engineering community is to predict the failure under those circumstances. For that, damage mechanics has gradually become a systematic theory that tries to provide a better understanding of the mechanisms of fatigue failure by the introduction of damage variables which represent the deterioration of a material element (Xiao et al. [5]). In this sense, through the LDM, this paper presents a new general model that is capable to represent the structure behavior under LCF or HCF.

2 A comprehensive Model of Fatigue for Steel Beams

2.1 Notation

Consider a planar frame where each element is connected by two nodes and each node has three degrees of freedom (Fig. 1a). These values are put in the generalized displacement matrix $\{U\}^T = \{u_1, w_1, \theta_1, \dots, u_\mathrm{n}, w_\mathrm{n}, \theta_\mathrm{n}\}$. Intending to characterize the modification of the structure, the deformation matrix is defined $\{\Phi\}_b^T = \{\phi_i, \phi_j, \delta\}$ as well the generalized stresses matrix $\{\mathrm{M}\}_b^T = \{m_i, m_j, n\}$ (Fig. 1a), where ϕ_i and ϕ_j are the relative rotations; δ is the elongation of the chord; m_i and m_j are the bending moments; n is the axial force.



(a) (b)

Figure 1. (a) Planar frame and generalized displacements, generalized strains, and stresses; (b) Representation of each element.

The generalized displacements are related to the generalized deformation through the kinematic equation (eq. 1) where $[\mathrm{B}]_\mathrm{b}$ is the kinematic transformation matrix of each element (Flórez-López, Marante and Picón [6]).

$$\{\Phi\}_b = [B]_b\{U\};\, where\ [B]_b = \begin{bmatrix} 0 & \cdots & \frac{\sin\alpha_b}{L_b} & -\frac{\cos\alpha_b}{L_b} & 1 & \cdots & -\frac{\sin\alpha_b}{L_b} & \frac{\cos\alpha_b}{L_b} & 0 & \cdots \\ 0 & \cdots & \frac{\sin\alpha_b}{L_b} & -\frac{\cos\alpha_b}{L_b} & 0 & \cdots & -\frac{\sin\alpha_b}{L_b} & \frac{\cos\alpha_b}{L_b} & 1 & \cdots \\ 0 & \cdots & -\cos\alpha_b & -\sin\alpha_b & 0 & \cdots & \cos\alpha_b & \sin\alpha_b & 0 & \cdots \end{bmatrix} \tag{1}$$

The matrix of external forces is defined by the matrix $\{P\}^T = \{Ru_1, Rw_1, M_1, \dots, Ru_\mathrm{n}, Rw_\mathrm{n}, M_n\}$ where Ru_n are the horizontal forces; Rw_n are the vertical ones; and M_n are the external moments. The relationship between the external nodal forces and the generalized stresses is settled by the equilibrium equation, showed in eq. (2). This equation is only valid for cases where the inertial forces are neglected (Flórez-López, Marante and Picón [6]).

$$\sum [B]_b^T\{M\} = \{P\} \tag{2}$$

As can be seen in Fig. 1a, each frame element is assumed as an assemblage of an elastic beam with two inelastic hinges, where which one considers three sets of internal variables: the generalized plastic deformation matrix $\{\Phi^p\}_b^T = \{\phi_i{}^p, \phi_j{}^p, 0\}$, the pre-damage set $\{\boldsymbol{\Omega}\} = (\omega_i, \omega_j)$ and the damage set $\{\mathbf{D}\} = (d_i, d_j)$. Besides that, it is considered the accumulated values of the plastic rotations $\boldsymbol{P} = (p_i, p_j)$ and of the bending moments $\boldsymbol{B} = (b_i, b_j)$.

2.2 Constitutive laws

In the following formulations, once the main objective of the new model is to represent the provinient effects of the fatigue phenomenon, where it can be under low or high cycles, and under stationary or non-stationary cases, both effects of LCF and HCF are considered and the equations are generalized embracing these possible cases.

Basically, there are four important references that allowed the development of the presente model: Zhou et al. [7], which provides the experimental study and data of the behavior of beam column joints under ultra low and LCF; Bai et al. [8], where it is proposed a damage model for the ultra low cycle fatigue case; Bazán et al. [9] that generalizes the Paris Law for inelastic hinges; and finally, the work of Lemaitre and Chaboche [10], which allows the assumption of the hypothesis of damage accumulation. In this way, the deduction of every equation herein presented is better detailed and explained in the undergraduate thesis of the first author of this paper. Such thesis is currently in development.

Then, the set of the constitutive laws is defined as follows: first, the elasticity and the plasticity laws are shown by eqs. (3) and (4), respectively, where $F(D)$ is the flexibility matrix; d_H is the HCF damage value while d_L is the LCF one; L is the lenght of the element; EI is the bending stiffness; AE is the axial stiffness; M_y is the plastic moment; and c is the kinematic hardening constant. These last both parameters (M_y and c) can be computed as presented in Flórez-López, Marante and Picón [6]. Notice that the crack closure effect, representend by h and considered in Bai et. al [8] for LCF, is also taken into account is this model.

$$\{\Phi\} - \{\Phi^P\} = [F(D)]\langle M\rangle^+ + [F(hD)]\langle M\rangle^-; where\ [F(hD)] = \begin{bmatrix} \frac{L}{3EI(1-d_{H_i}-cd_{L_i})} & \frac{-L}{6EI} & 0 \\ \frac{-L}{6EI} & \frac{L}{3EI(1-d_{h_i}-hd_{L_i})} & 0 \\ 0 & 0 & \frac{L}{AE} \end{bmatrix} \quad (3)$$

$$f_i = \max\left(\frac{m_i}{1-d_{H_i}-cd_{L_i}} - h{\Phi_i}^P, \frac{-m_i}{1-d_{H_i}-cd_{L_i}} + c{\Phi_i}^P\right) - M_y \leq 0; \quad (4)$$

In order to characterize the crack appereance and evolution, respectively, a two-sept process is defined: the first starts with a phase of plastic strains accumulation and ends with the crack initiation itself; and the second one is where the crack propagation occurs.

Thus, for the first stage, it is proposed a pre-damage evolution law, which suits for both stationary and non-stationary tests, as shows eq. (5), where ω_L and ω_h are the pre-damage variable for LCF and HCF, respectively; ${\Phi_{cr}}^p$ is the critical plastic strain; M_{cr} is the critical moment; and β and γ are coffiecients from the well-knowns Manson-Coffin and Basquin law, respectively. For arriving in eq. (5), just as briefly mentioned previously, a set of equations and relationships for both LCF and HCF were taken into account.

$$\dot{\omega} = \frac{\partial\omega}{\partial N} = \dot{\omega}_L + \dot{\omega}_h; \Rightarrow \omega = N\left(\frac{p(N) - p(N/2)}{{\Phi_{cr}}^p}\right)^{-\beta} + N\left(\frac{b(N) - b(N/2)}{M_{cr}}\right)^{-\gamma}; \omega(N_{cr}) = 1 \quad (5)$$

Equations (6) and (7) show the evolution law for the values of the accumulated plastic rotations p and for the accumulated bending moments b, where $\Delta\Phi^p$ and Δm are the plastic rotation and the bending moment amplitude, respectively.

$$\dot{p} = \frac{dp}{dN} = |\dot{\Phi}^p|;\ p \cong 2\Delta\Phi^p N \quad (6)$$

$$\dot{b} = \frac{dm}{dN} = |\dot{m}|;\ b \cong 2\Delta m N \quad (7)$$

Note that when the number of cycles N is equal to the critical number of cycles N_{cr} the pre-damage variable assumes the value of 1. That indicates where the first stage finishes and consequently where the second one begins. In this way, the damage evolution law that is able to describe this process is given by eq. (8), where α, Q and r are parameters of the model. This relationship takes into account the hypothesis of the damage accumulation proposed by Lemaitre and Chaboche [10].

$$\dot{d} = \dot{d}_L + \dot{d}_H = -\frac{(1-d)}{\beta\alpha}\left(\frac{(1-d)^\alpha p(N) - p(N/2)}{{\Phi_{cr}}^p}\right)^{-\beta} + Q\left(\frac{g(N) - g(N-1)}{2}\right)^r \quad (8)$$

Added to that, it is considered that $dg = |dG|$, where G is the value of the energy release rate, that can be computed according to eq. (9), where m is the moment value.

$$G = \frac{L\langle m\rangle_+^2}{2EI(1-d_h)^2};\ \langle m\rangle_+ = \begin{cases} 0\ if\ m < 0 \\ m\ if\ m \geq 0 \end{cases} \quad (9)$$

2.3 Parameter computation

Willing to have a procedure that does not disregard the effect of the connections between the steel elements, it is proposed a computation method to obtain the values of ${\Phi_{cr}}^p$, β, M_{cr} and γ.

Consider two high cycle fatigue tests ($p = 0$) in a steel beam, which one with amplitudes ΔmA and ΔmB and let NA and NB represent the correspondent number of cycles to crack initiation. Then, solving a system of equations using a pre-damage evolution law for stationary fatigue for both tests, relations showed in eqs. (10) and (11) are obtained. The pre-damage evolution law for this particular case is also presented in the undergraduate thesis previously mentioned.

$$M_{cr} = \Delta mA \left(\frac{\Delta mA}{\Delta mB}\right)^{\left(-\frac{\ln(NA)}{\ln\left(\frac{1}{NB}\right)+\ln(NA)}\right)} \quad (10)$$

$$\gamma = \frac{ln\left(\frac{1}{NB}\right) + \ln(NA)}{\ln\left(\frac{\Delta mA}{\Delta mB}\right)} \quad (11)$$

Consider now two ultra-high cycle fatigue tests where NC and ND are the values to crack initiation. In order to obtain β and ${\Phi_{cr}}^{p}$, the least square technique is used pondering that the calculated values to crack initiation need to be as close as possible to the experimental ones.

A procedure for the updating of the constants Q, r, and α when low cycle fatigue tests are available is then proposed. Assume that the damage variation in a cycle can be neglected and that the amplitude of plastic rotation is constant. Even in such a case, the bending moment amplitude is not constant, then eq. (12) is defined.

$$\frac{\Delta m(N)}{1 - d(N)} - c\Delta\Phi^{p} - 2M_{y} = 0 \quad (12)$$

By integrating the damage evolution law for stationary cases (also shown in the undergraduate thesis mentioned above) and considering the relationship settled by eq. (12) one obtains eq. (13). However, there is no analytical solution for d_L. On the other hand, the problem is reduced to a single differential equation in $d_L(N)$ with the initial condition $d_L(N_{cr}) = 0$. This differential equation can be solved by different approximate or numerical methods. In Flórez-López, Marante and Picón [6] it is observed that the critical value of damage that corresponds to the sudden collapse of the hinge is around $d_c \cong 0.6$. Considering now different tests under different plastic amplitudes $\Delta\Phi_{F}^{p}$, $\Delta\Phi_{G}^{p}$, $\Delta\Phi_{H}^{p}$,..., where N_{cF}, N_{cG}, N_{cH},... are the numbers of cycles to the total collapse of these tests. The system of equations showed in eq. (14) allows the updating of Q, r and α.

$$d_{h} = \left(\frac{L\left(Q^{2}\Delta\Phi^{p2} + 4cM_{y}\Delta\Phi^{p} + 4M_{y}^{2}\right)}{6EI}\right)^{r} Q(N - N_{cr}) \quad (13)$$

$$\begin{aligned} d\left(\Delta\Phi_{F}^{p}, N_{cF}; Q, r, \alpha\right) &= 0.6; \\ d\left(\Delta\Phi_{G}^{p}, N_{cG}; Q, r, \alpha\right) &= 0.6; \\ d\left(\Delta\Phi_{H}^{p}, N_{cH}; Q, r, \alpha\right) &= 0.6; \end{aligned} \quad (14)$$

3 Numerical Simulation for a Stationary and a Non-Stationary test

The set of equations written in the previous sections define a conventional finite element that was included in the commercial software ABAQUS Standard via the UEL option, according to Abaqus Analysis User's Manual [11]. This FE was utilized to simulate the tests presented in Zhou et al. [7].

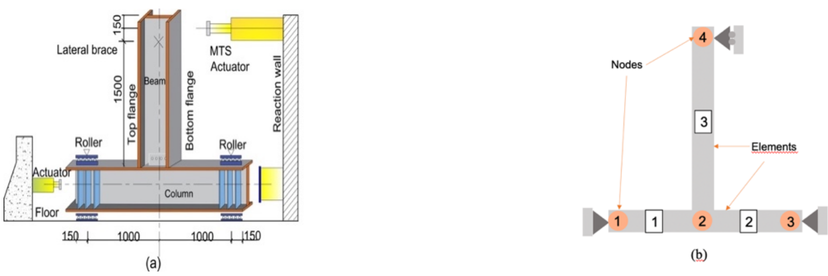


Figure 2. (a) Experimental set up of the tests from Zhou et al.; (b) FE mesh.

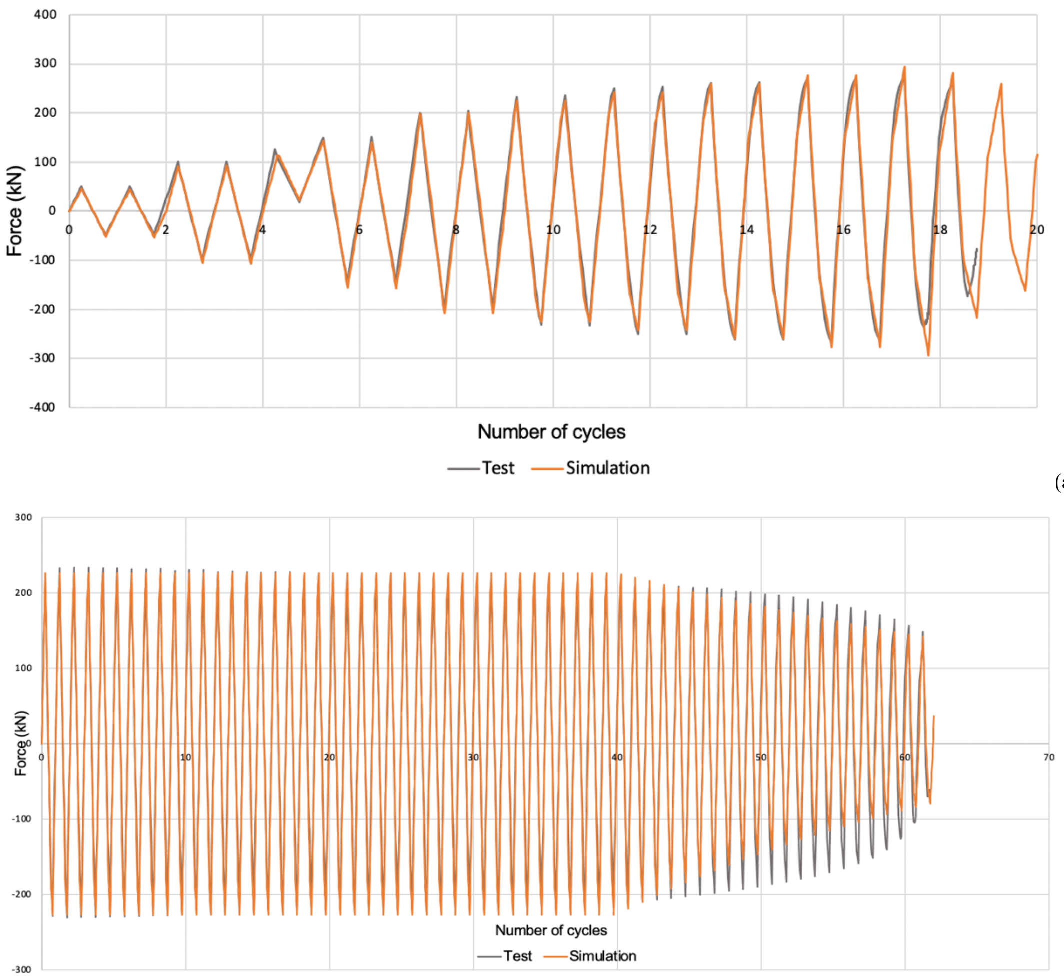
In Fig. 2a there is the experimental set-up of the tests and in Fig. 2b the FE mesh used in the simulations.

It is important to highlight that according to the experimental tests, it was assumed that there was no yielding in the column elements (1 and 2). Due to the assumption of the boundary conditions, there were plastic rotations and damage only at the hinge at node 2 in the beam element.

Besides that, for elements 1 and 2, $EI = 35000\text{kN}.m^2$; M_y =any arbitrary large number, once plasticity is not involved in these elements, while for element 3, $EI = 19000\text{kN}.m^2$ and $h = 0.3$ (according to Bai et. al [8]). Then, also for this element, through the methodological procedures that were presented in the previous section, the obtained values for the parameters were: $M_y = 307kN.m$, $C = 16000kN$, $M_{cr} = 1150kN.m$, $\gamma = -24$, $\beta = -1.782699$, ${\Phi_{cr}}^p = 0.03264731$, $\alpha = 0.45$, $Q = 0.013455$ and $r = -0.1$.

The numerical simulation of two tests reported in Zhou et. al [7] is presented: SP-2 and SP-9. The first one is relative to a LCF test (i.e, less than 20 cycles to fracture) while the second one is relative to a HCF. Added to that, SP-2 corresponds to a non-stationary test, i.e, the amplitude values are not constants, while SP-9 corresponds to a stationary test.

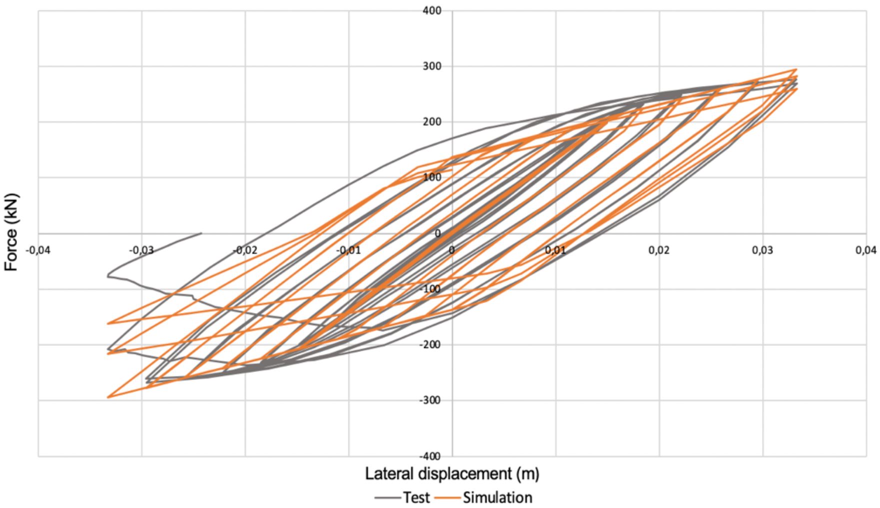
In Fig. 3a and 3b it is shown the comparison between the experimental and numerical values of force versus cycle for each one of them.



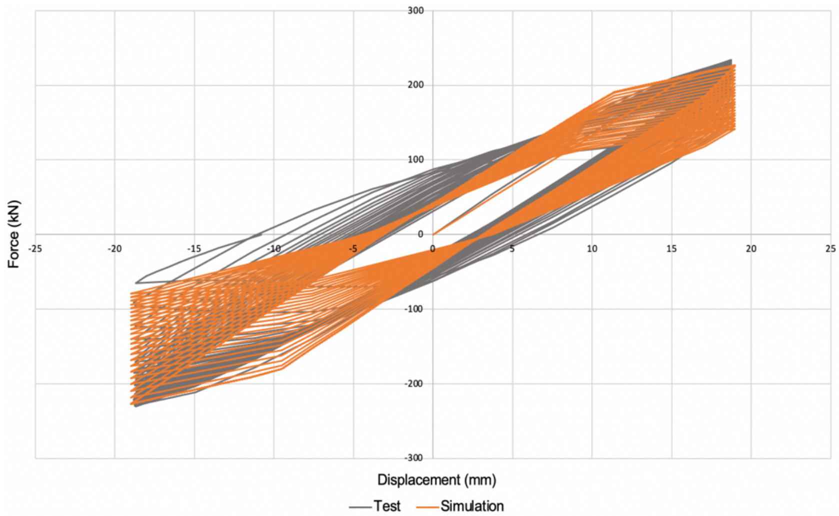
(b)

Figure 3. (a) Force vs Cycle for SP-2 test; (b) Force vs Cycle for SP-9 test.

The graph of force vs displacement at the top of the structure is shown for both tests in Fig. 4a and 4b while the damage evolution law is presented in Fig. 5a and 5b, where it can be analyzed that the damage begins only in a critical number of cycle $N_{cr} = 17.875$ for SP-2 and $N_{cr} = 40$ for SP-9.

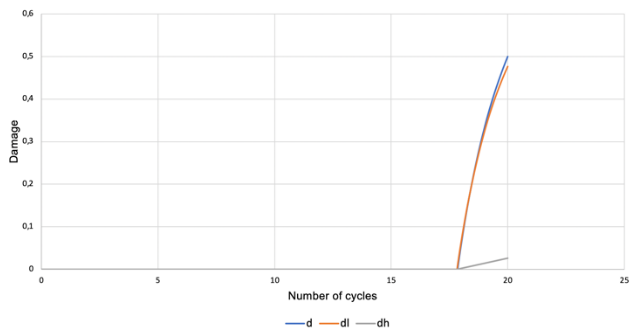


(a)

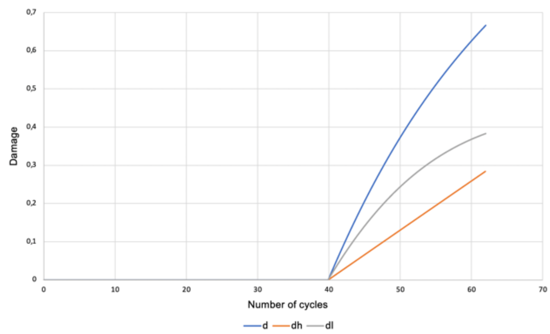


(b)

Figure 4. (a) Force vs Displacement for SP-2 test; (b) Force vs Displacement for SP-9 test.



(a)



(b)

Figure 5. (a) Damage evolution for SP-2 test; (b) Damage evolution for SP-9 test.

As can be seen, the updated values of the model parameters showed a satisfactory agreement between the experimental results and the numerical ones.

4 Conclusions

LCF is characterized by significant values of plastic strains under a relative short number of cycles to failure, while the HCF strains are in an elastic range and the number of cycles to failure is large. The damage model proposed here is a general model that can be used for both types of fatigue phenomena, and showed itself as an acceptable initial approach for this purpose.

Firstly, it is considered that the fatigue process in steel beams can be described as a two-steps phenomenon: a period of "incubation" in the interval $N < N_{cr}$ and a phase of crack propagation, where $N \geq N_{cr}$. The first stage is characterized by the introduction of the new pre-damage variable and the second one by the damage evolution law, where constants and non-constant amplitudes were taken into account. The consideration of both types of amplitudes enables to analyze the structure deterioration process.

This approach was possible due to the assumption of the most relevant effects in each type of fatigue phenomenon, such as the already mentioned values of the accumulated plasticity, bending moments, and also the ones from the energy release rate, once the effects from both possible types of fatigue needed to be translated into the model for the study of the damage propagation.

In light of this, along the cyclic loading applied, the crack closure effect, strength, and the stiffness degradation could be described and the mechanical behavior of the experimental tests could be represented in a good approach way throughout the kinematic, equilibrium and constitutive equations that were used in the numerical simulations.

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References

[1] J. Schijve, "Fatigue as a Phenomenon in the Material". Fatigue of Structures and Materials. Springer, Dordrecht, 2009.
[2] R. H. J. Peerlings, W. A. M. Brekelmans, R. de Borst and M. G. D. Geers, "Gradient-enhanced damage modelling of high-cycle fatigue". International Journal for Numerical Methods in Engineering, 2000.
[3] B. Farahmand, G. Bockrath and J. Glassco, "Conventional Fatigue (High-and Low-Cycle Fatigue)". Fatigue and Fracture Mechanics of High Risk Parts. Springer, Bonston, MA, 1997.
[4] J. Homan, "Some Basics About Fatigue". Fatec Engineering, 2020.
[5] Y-C Xiao, S. Li and Z. Gao, "A Continuum Damage Mechanics Model for High Cycle Fatigue". International Journal of Fatigue, 1998.
[6] J. Flórez-López, M. Marante and R. Picón, "Fracture and Damage Mechanics for Structural Engineering of Frames". [S.I.]: IGI Global, 2015.
[7] H. Zhou, Y. Wang, Y. Shi, J. Xiong and L. Yang, "Extremely low cycle fatigue prediction of steel beam-to-column connection by using a micro-mechanics based fracture model". Internation Journal of Fatigue, 2012.
[8] Y. Bai, M. Kurata and J. Flórez-López, "Macromodeling of Crack Damage in Steel Beams Subjected to Nonstationary Low Cycle Fatigue". J. Struct. Eng., 2016.
[9] J. A. V. Bazán, A. T. Beck and J. Flórez-López, "Random Fatigue of Plane Frames via Lumped Damage Mechanics". Engineering Structures, 2018.
[10] J. Lemaitre and J.-L Chaboche, "Mechanics of Solid Materials". New York: Cambridge University Press, 1990.
[11] Abaqus Analysis User's Manual, "1.1.22 UEL, User Subroutine to Define an Element".