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An elastoplastic model with damage based on the Thermodynamics of Frames

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Abstract. The representation of the mechanical behavior of reinforced concrete structures aims to describe the physical reality as accurately as possible, which for there are many available theories. Besides that, to go further in those researches of physical representation, there are postulated formulations based on the science of thermodynamics, which link the influence of mechanical behavior in the development of physical phenomena and vice versa, through a systematic procedure that allows the determination of thermodynamic forces associated with the state variables, which are able to represent these phenomena. Therefore, the model here presented is underpinned on thermodynamic models that describe physical phenomena of reinforced concrete structures and proposes a new damage law that allows the observation of the cracking evolution. A new cracking resistance function that describes the energy release rate during the damage evolution is established since this new damage law comes now from Gibbs Free Energy. Finally, in order to evaluate the accuracy of the new model, the results obtained by the proposed model are compared with those given by the Lumped Damage Mechanics theory, which presents itself as an efficient tool to describe the cracking appearance and evolution, for both analysis and computational implementation.

Keywords: thermodynamics of frames, reinforced concrete, cracking evolution, lumped damage mechanics.

1 Introduction

According to Soares et al. [1] , the reinforced concrete is one of the main structural and engineering materials used nowadays and belongs to a research field that has gained notoriety during the past century due to the fact that the engineers started to look for security and economy in the civil industry.

In order to have an acurrate description of the structural behavior there are proposed non linears theories, such as the plasticy theory, fracture mechanics and damage mechanics for example (Amorim [2]). Another theory avaliable is the Lumped Damage Mechanics (LDM), which considers the damage variable and plasticy coupled into plastic hinges (Flórez-López, Marante and Picón [3]).

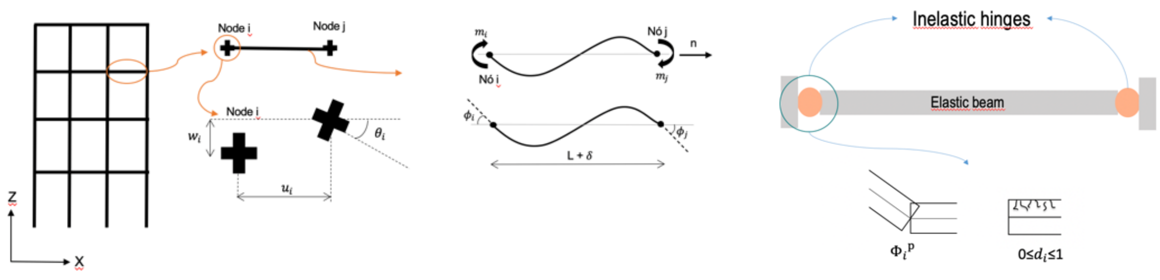
In addition, the mechanical behavior of the materials can be related to a thermodynamic behavior. While the object of study of the mechanics as a part of physics is the scientific comprehension of the bodies movements, the thermodynamics belongs to another field that investigates both transformation and exchange of energy (Haupt [4]). Therefore, in order to obtain a formalism it is possible to adopt the approach of the thermodynamic of irreversible process by introducing state variables, where a defined thermodynamic potential allows the definition of these state variables to the study phenomenon, which leads to the state laws (Lemaitre and Chaboche [5]).

In consequence, the model presented herein is based on thermodynamic formulations that are available in the literature, as the works done by Dahmer [6] and Brant [7], and it is capable of evaluating the physical phenomenon of cracking, by proposing a new damage law, which is provided directly from the Gibbs Free Energy.

2 Initial Concepts

2.1 Planar frame analysis

In a planar frame each node has three generalized displacements (Fig. 1a) which generates the matrix $\{\mathrm{U}\}^{\mathrm{T}} = \{\mathrm{u}_1, \mathrm{w}_1, \theta_1, \dots, \mathrm{u}_n, \mathrm{w}_n, \theta_n\}$. In order to characterize the modification of the structure, the deformation matrix $\{\Phi\}_b^{\mathrm{T}} = \{\phi_i, \phi_j, \delta\}$ is introduced as well the generalized stresses matrix $\{\mathrm{M}\}_b^{\mathrm{T}} = \{\mathrm{m}_i, \mathrm{m}_j, \mathrm{n}\}$, as also shows Fig. 1a.



(a) (b)

Figure 1. (a) Planar frame and generalized displacements; (b) Lumped damage model.

The matrix of external forces is defined by the matrix $\{\mathrm{P}\}^{\mathrm{T}} = \{\mathrm{Ru}_1, \mathrm{Rw}_1, \mathrm{M}_1, \dots, \mathrm{Ru}_n, \mathrm{Rw}_n, \mathrm{M}_n\}$ where Ru_n are the horizontal forces; Rw_n are the vertical ones and M_n are the external moments. Flórez-López, Marante and Picón [3] defined the kinematic equation that relates the member displacements $\{\mathrm{U}\}$ with the generalized deformations $\{\Phi\}$ through the consideration of the kinematic transformation matrix $[\mathrm{B}]_b$. The relationship between the displacements and deformations is showed in eq. (1). According to the same authors, the equilibrium equation in cases where the inertial forces are neglected is given by eq. (2).

$$\{\Phi\}_b = [B]_b\{U\}; where\ [B]_b = \begin{bmatrix} 0 & \cdots & \dfrac{\sin\alpha_b}{L_b} & -\dfrac{\cos\alpha_b}{L_b} & 1 & \cdots & -\dfrac{\sin\alpha_b}{L_b} & \dfrac{\cos\alpha_b}{L_b} & 0 & \cdots \\ 0 & \cdots & \dfrac{\sin\alpha_b}{L_b} & -\dfrac{\cos\alpha_b}{L_b} & 0 & \cdots & -\dfrac{\sin\alpha_b}{L_b} & \dfrac{\cos\alpha_b}{L_b} & 1 & \cdots \\ 0 & \cdots & -\cos\alpha_b & -\sin\alpha_b & 0 & \cdots & \cos\alpha_b & \sin\alpha_b & 0 & \cdots \end{bmatrix} \quad (1)$$

$$\sum[B]_b^T\{M\} = \{P\} \quad (2)$$

2.2 Lumped Damage Mechanics (LDM)

The (LDM) is based on concepts of fracture mechanics and classic damage, and allows to describe what can be observed in the physical reality through the quantification of the structural damage. It represents the mechanical behavior by taking into account the internal variable of damage $(D)_b = (d_i, d_j)$ lumped into the plastic hinges, which are called inelastic hinges, as can be seen in Fig. 1b (Flórez-López, Marante and Picón [3]).

2.3 Essential elements of Thermodynamics of Frames

Dahmer [6] and Brant [7] based their works in the main concepts of Thermodynamics of Solids proposed by Lemaitre and Chaboche [5] and wrote the fundamental concepts that structures and give form to Thermodynamics of Frames. Such concepts are, for example, the principal of virtual power; the first and second principles of the Thermodynamics of Frames; and the State Laws.

According to Mazars and Pijaudier-Cabot [8], one of the main advantages of the thermodynamic method is the thermodynamic potential that can be chose accordingly to the necessities of the study. In this sense, the authors opted for the Gibbs Free Energy (G_b) , which is a thermodynamic potential that can be written as a function of the generalized stresses M, of the absolute temperature T and also of the internal variables V_α $(\dot{G_b} = (M, T, V_\alpha))$. As presented in their works, the last term V_α could refer to the plastic rotation Φ^p or to the damage variable d, for example. In this sense, it could be obtained a relationship that allows to observe if the process is thermodynamically admissible or not (eq. 3), where the variable S_b is the entropy of the system.

$$\dot{G}_b - \{\dot{M}\}^T\{\Phi\} - S_b\dot{T}_b \geq 0 \tag{3}$$

From the knowledge of such mentioned postulations, both authors were able to arrive in the determination of the State Laws. Lemaitre and Chaboche [5] affirm that once the thermodynamic potential is defined it is possible to postulate these Laws, which permit the association between the thermodynamic forces to the internal variables. Deriving the function $\dot{G}_b = (M, T, V_\alpha)$ with respect to time and combining it with eq. (3), the expression showed in eq. (4) is obtained (Dahmer [6] and Brant [7]).

$$\underbrace{\{\dot{M}\}_b\left(\left\{\frac{\partial G_b}{\partial M}\right\}^T - \{\Phi\}_b^T\right)}_{\text{Elasticity}} + \underbrace{\dot{T}_b\left(\frac{\partial G_b}{\partial T_b} - S_b\right)}_{\text{Heat transfer}} + \underbrace{\left\{\frac{\partial G_b}{\partial V_\alpha}\right\}^T\{\dot{V}_\alpha\}_b}_{\text{Energy dissipation}} \geq 0 \tag{4}$$

When reversible process is considered, the rate of the internal variables assumes the value of zero $\left(\{\dot{V}_\alpha\}_b = 0\right)$, where there is no energy dissipation, and, a isothermal process is also considered, then $\dot{T}_b = 0$. So, the first State Law is established. Similarly, if a reversible process with only temperature change is taken into account, the second State Law is defined; and, for formality, the last Law is written to express the thermodynamic force A_α associated to the internal variables V_α. The set of related equations is then presented:

$$\left\{\frac{\partial G_b}{\partial M}\right\} = \{\Phi\}_b; \left(\frac{\partial G_b}{\partial T_b}\right) = S_b \; and \; \left\{\frac{\partial G_b}{\partial V_\alpha}\right\} = \{A_\alpha\} \tag{5}$$

3 Elastoplastic Model Based on the Thermodynamics of Frames

The proposed thermodynamic potential is a function of the generalized stresses, of the plastic rotations and of the damage variable, $G_b = G_b(\{M\}, \Phi^p, d)$, as follows:

$$G_b = \underbrace{\frac{1}{2}\{\mathrm{M}\}^T[F(D)]\{\mathrm{M}\}}_{\text{Complementary deformation energy}} + \underbrace{\{\mathrm{M}\}^T\{\Phi^p\}}_{\text{Plasticity}} - \underbrace{\frac{1}{2}\{\Phi^p\}^T[\mathrm{H(D)}]\{\Phi^p\}}_{\text{Plastic hardening}} + \underbrace{e^q Ei\big(q(d-1)\big) + c}_{\text{Crack resistance increase}} \tag{6}$$

The constants here presented are q, which depends of the characteristics of the element, and c, which is a constant of integration. The matrix $[H(D)]$ represents the kinematic hardening and the matrix $[F(D)]$ is the flexibility matrix, where both consider the damage variable, just as presented in Dahmer [6] and Brant [7]. However, in this model, differently from the thermodynamics formulations of the same authors, the energy release rate of a structural element is defined by:

$$y_d = \frac{F^0 m^2}{2(1-d)^2} + \frac{1}{2}h(\Phi^P)^2 = \frac{Lm^2}{6EI(1-d)^2} + \frac{1}{2}h(\Phi^P)^2 \tag{7}$$

Through the Griffith criterion it is determined that during the process of damage propagation, the values of the energy release rate must be equal to the values of the crack resistance. Then, the damage evolution must respect the equality showed in eq. (8), where, as can be seen, the crack resistance equation $R(d)$ for this model is different than the one used in LDM, as well as the one used by Dahmer [6] and Brant [7].

$$y_d = R(d) \; or \; \frac{Lm^2}{6EI(1-d)^2} + \frac{1}{2}h(\Phi^P)^2 - \frac{R_0 e^{qd}}{(1-d)} = 0 \tag{8}$$

Deriving the chosen potential (eq. 6) with respect to moments, just as defined by the State Laws, the elasticity law is obtained:

$$\left\{\frac{\partial \mathrm{G_b}}{\partial \mathrm{M}}\right\} = \{\Phi\} = [\mathrm{F(D)}]\{\mathrm{M}\} + \{\Phi^{\mathrm{P}}\} \tag{9}$$

Furthermore, deriving the same potential with respect to the internal variable of plastic rotations and also with respect to the damage variable, eqs. (10) and (11) are obtained, respectively.

$$\left\{\frac{\partial G_b}{\partial \Phi^p}\right\} = \{A^p\} = \{\mathrm{M}\} - [\mathrm{H(D)}]\{\Phi^p\} \tag{10}$$

$$\left\{\frac{\partial G_b}{\partial d}\right\} = \{A^d\} = \begin{cases} \dfrac{Lm_i{}^2}{6EI(1-d_i)^2} - \dfrac{R_0 e^{qd}}{(1-d)} + \dfrac{1}{2}h(\Phi_i{}^p)^2 \\ \dfrac{Lm_j{}^2}{6EI(1-d_j)^2} - \dfrac{R_0 e^{qd}}{(1-d)} + \dfrac{1}{2}h(\Phi_j{}^p)^2 \end{cases} \tag{11}$$

As a result, the first relationship shows the thermodynamic force $\{A^p\}$ related to the plastic rotations, and the second one the driving force $\{A^d\}$ related to the damage variable. Then, the yield function is expressed by the eq. (12), where $k0$ is another parameter of the model.

$$\begin{gathered} f^p = |A^p| - (1-d)k_0 \leq 0 \; or \\ f^p = |m - (1-d)h\Phi^P| - (1-d)k0 \leq 0 \end{gathered} \tag{12}$$

The damage evolution law (eq. 13) is written in agreement with the Griffith criterion, once the values of energy release rate are equal to the crack resistance in case that there is damage propagation.

$$\begin{cases} \dot{d} = 0 \; se \; y_d < R \\ y_d = R \; se \; \dot{d} > 0 \end{cases} \; or \; \begin{cases} \dot{d} = 0 \; se \; \dfrac{1}{2}\dfrac{Lm^2}{3EI(1-d)^2} - \dfrac{R_0 e^{qd}}{(1-d)} + \dfrac{1}{2}h(\Phi^P)^2 < 0 \\ \dfrac{1}{2}\dfrac{Lm^2}{3EI(1-d)^2} - \dfrac{R_0 e^{qd}}{(1-d)} + \dfrac{1}{2}h(\Phi^P)^2 = 0 \; se \; \dot{d} > 0 \end{cases} \tag{13}$$

The computation of the parameters R_0, q, h e k_0, as well as the determination of the values of plastic damage (d_p) and ultimate damage (d_u) can be defined by a system of nonlinear equations that are better detailed and explained in the undergraduate thesis of the first author of this paper. Such thesis is currently in development.

4 Example

In order to validate the purposed model, two different simulations were made: one considering the (LDM) and the other considering the model presented in the previous section, which is based on the Thermodynamics of Frames (TFM). The input data for both simulations is justified and compared in Tab. 1, where the Updating technique, that is showed in Kim and Park [9] and Chen and Metwally [10], was used willing to decrease the incertitude and to adjust the parameters so that the simulations coincide in the most reliable way with the observed experimental properties.

Table 1. Comparison between the input values used of each simulation.

	LDM		TFM	
Property	Beams	Columns	Beams	Columns
E (Y. Modulus, tn/m²)	7.92889x10^6	4.586667x10^6	8,106667x10^6	8,54016x10^6
Mcr (Critical Moment, tn.m)	25.316	15.2177	25.316	15.2177
Mp (Plastic Moment, tn.m)	48.6192	39.44	58.6192	40.44
Mu (Ultimate Moment, tn.m)	75.005	72.6192	80.005	78.6192
Φ_{p_u} (Ultimate Plastic Rotation)	0275	0.336	0.275	0.336

It is important to highlight that the TFM has a different damage evolution law as well as a different set of equations that allows the computation of the involved parameters. For the LDM, these parameters values were obtained according to Flórez-López, Marante and Picón [3]. Table 2 shows the calculated values for both models, where d_{p} is the plastic damage value and d_{u} is the ultimate damage value.

The schematic representation of the elements and nodes that compose the analyzed structure is presented in Fig. 2: it was considered that in the first level the structure was being pushed (positive direction of x-axis), while in the second one it was being pulled (negative direction of x-axis). Then, simultaneously, the displacement value on node 2 was increased while on node 3 it was decreased. In this way, it could be observed that the damage first appeared on the columns (which bending stiffness is lower than the beams one) in the following sequence: hinges 22, 45, 12, 23, 44, 55, 11, 56, 33 and finally 34.

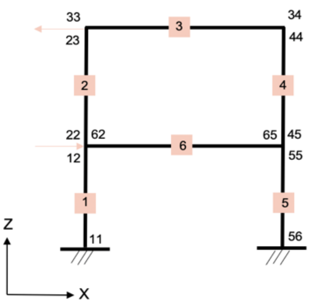


Figure 2. Structure representation, with its respective nodes and elements.

Table 2. Parameters R_0, q, h e k_0 for each simulation, as well as d_p and d_u

	LDM		TFM	
Parameter	Beams	Columns	Beams	Columns
R_0 (tn.m)	0.5747x10^{-1}	0.2393x10^{-1}	0.5621x10^{-1}	0.2221x10^{-1}
q	-1.349676028	-1.472740264	6.826481453	7.7438465727
h (tn.m)	661.6001508	536.5416752	1386.798644	1349.4547724
k_0 (tn.m)	56.18754316	43.88873470	83.51040128	57.631887965
d_p	0.1346978842	0.101363931	0.298066852	0.2983061599
d_u	0.6200193222	0.627640854	0.806391483	0.8461622635

Figure 3a and 3b shows the damage evolution for each hinge considering LDM and TFM, respectively. It can be noticed that the TFM gives bigger values of damage: while in LDM $d_p = 0.11$ and $d_u \cong 0.62$, in TFM $d_p = 0.29$ and $d_u \cong 0.82$.

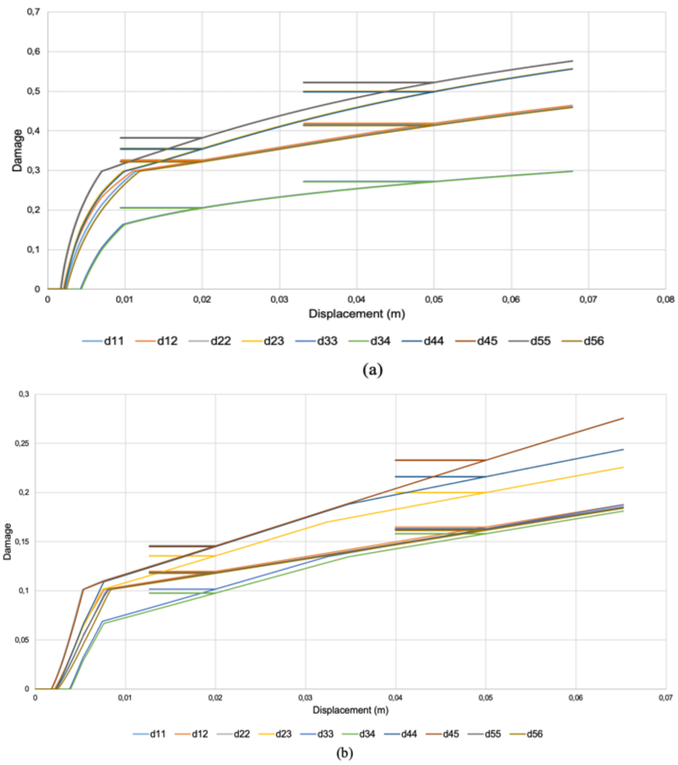


Figure 3. (a) Relationship between Damage and Displacement for (a) TFM and (b) LDM.

For both levels (1 and 2) it was possible to establish a relationship between force and displacement, quantifying the bending stiffness loss of the structure due to the crack growth in each unloading process. Figure 4a and 4b shows these phenomenon for TFM and LDM, respectively.

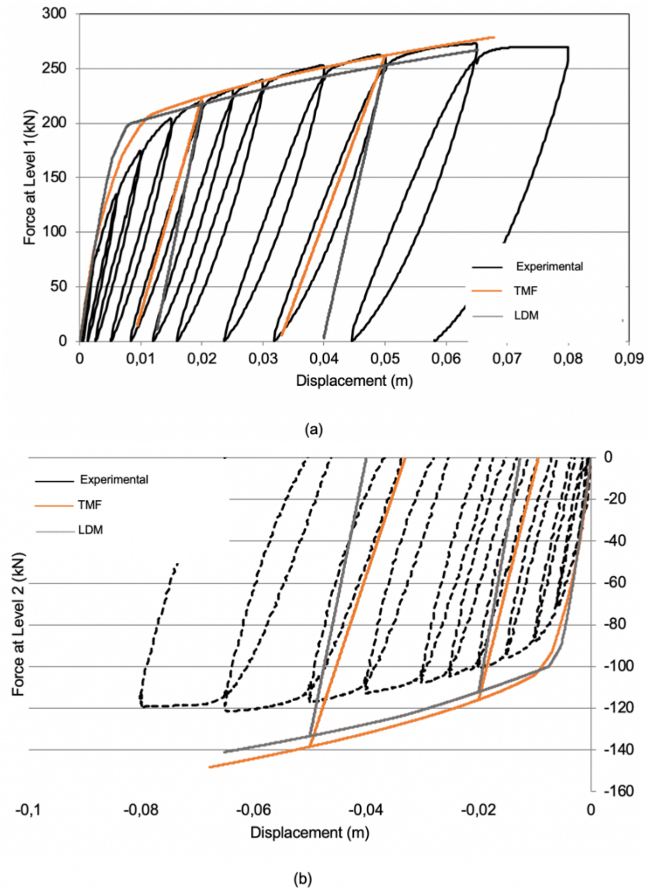


Figure 4. (a) Relationship between Plastic Rotation and Displacement for (a) TFM and (b) LDM.

5 Conclusions

The present model defined a new damage evolution law that derivates from the fundamental relationships from the science of Thermodynamics and added to that the model has an exchange of information with an existing model for cracking evaluation, which is the LDM. In other words, even though the new proposition comes from a different path than LDM, which is based in the concept of complementary potential energy, it still considers the same kinematic relationships as well as some constitutive equations (elasticity and plasticity laws).

Due to the consideration of the Gibbs Potential, new relationships to the computation of the parameters were required. The Updating technique, that was utilized in order to improve the modeling, presented itself as a suited form for obtaining these values.

Once the new damage evolution law was taken into account, the model required different considerations than the ones from LDM, as the energy release rate along the damage evolution and consequently a resistance function that could allow the Griffith criterion to be respected. The purposed resistance function showed itself as a satisfactory alternative, once the simulation results for damage propagation and the bending stiffness penalization were similar to the experimental results.

It is necessary to point up that the range for plastic and ultimate damage in TMF is relative different from LDM. However, through the relationships of force and displacement it could be noticed that this difference exists due to the fact that the models have descriptions of the behavior of the damage variable that are distinct from each other, but even in this way, the TMF properly reproduces the stiffness loss due to the displacement increments that the structure is subjected to.

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