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A numerical methodology for the cross-sections analysis of steel-concrete composite beams with partial shear connection

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Abstract. The present work aims at the implementation and validation of a numerical formulation based on the Strain Compatibility Method (SCM) for the calculus of the cross-section's resistance capacity of steel-concrete composite beams with partial shear connection. It is developed, here, a strategy for capturing the longitudinal deformations at all cross-section's points. Thus, the section's discretization, isolating the steel profile from the concrete slab, is necessary. In this context, the classic SCM is modified by the insertion of one explicit degree of freedom at the steel-concrete interface, corresponding to a discontinuity on the deformations field, allowing the longitudinal sliding between the steel profile and the concrete slab. Therefore, the axial force dismemberment is done, on which a part is absorbed by the profile and the other part by the slab. By equilibrium, the difference between the slab's and profile's forces generates a shear force at the connection and, using the Ollgaard's model, the longitudinal sliding at the contact point, is found. Assuming the plane cross-section theory, a single curvature is assigned to both constituents of the section. Thereby, it is done the construction of the moment-curvature relationship using the standard Newton-Raphson method combined with continuation strategies aiming to capture the hardening and softening of the materials over the loading historic of the section. However, in order to implement the analysis, exclusively, in the cross-section, a fixed degree of freedom is assigned to it. For the validation of the proposed numerical formulation, the obtained results are confronted with numerical data available in the literature.

Keywords: Partial shear connection, Steel-concrete composite beams, SCM, Moment-curvature relationship

1 Introduction

In addition to the geometric, material and beam-to-column connection non-linearities, the steel-concrete composite structural elements may also presents partial shear connection. According to Lemes [1], partial interaction is understood as non-linearity in the shear connection at the interface between materials, which is treated as deformable. In this sense, the degree of composite action becomes an important property since it can define how rigid this connection will be.

If there is a slip at the steel-concrete interface, the interaction is defined as partial. For composite beams, this condition is characterized by the non-monolithic behavior of the element. This reduces the strength and stiffness of the structural element, but the reduced number of connectors provides savings.

Among the methods of simulating the effect of partial interaction, there are those that considered interface elements [2, 3], and those that consider partial shear connection within the formulation of a frame element Battini et al. [4], Chiorean and Buru [5]. For the second described case, it is necessary to have a procedure for analyzing the fibers of the elements or of the cross section at a nodal point.

This study aims to develop a numerical formulation for the evaluation of cross sections of steel-concrete composite beams with partial interaction. Thus, the strain compatibility method based on Euler Bernoulli's theory and presented in Lemes et al. [6] is modified by introducing an additional degree of freedom in the cross section describing the sliding in the steel-concrete interface. Thus, it will be possible to analyze the behavior of a cross section at a specific point in a structural element, allowing structural analysis by concentrating the effect of partial interaction at the nodes.

2 Cross sectional analysis

The Strain Compatibility Method (SCM) is a Euler-Bernoulli-based approach for the evaluation of compact cross-sections. When under external loads, a structure will gradually deform until it reaches equilibrium. Once the internal forces equal the external forces, the deformation stops. This deformation, at the cross-section level, is studied by SCM [6].

To make an accurate analysis of cross-sectional nonlinear behavior under external loads, a correct description of the materials behavior is required. The steel section material is described by a trilinear constitutive model being possible to consider the material strain hardening effect. The residual stress models are disregarding in this study. The concrete and reinforcing bars are considered using the uniaxial behavior described in Lemes et al. [6].

2.1 Cross sectional degrees of freedom

Two situations are considered here: a bare steel section; and a steel-concrete composite beam with a linear degree of composite action. In both situations, to describe the strain distribution, the cross-section discretization in fibers, shown in Figure 1, is very efficient [6, 7]. It is done to capture the axial strain, ε, in the plastic centroid (PC) of each layer, and then (through the material constitutive relationships) to obtain the respective stresses, σ_i.

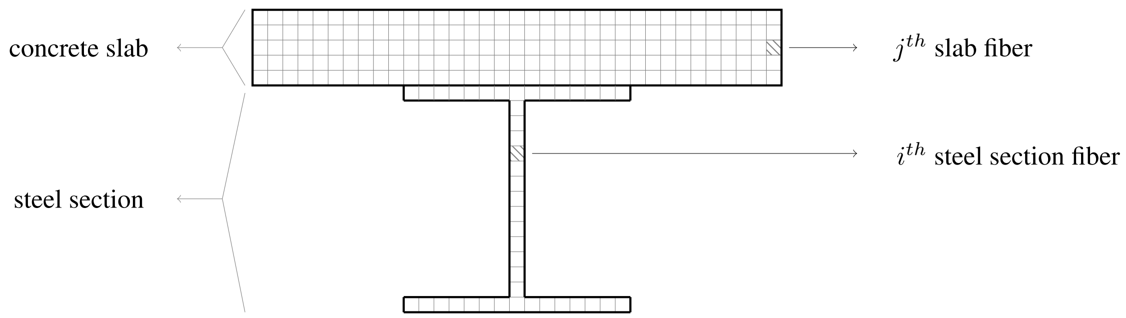


Figure 1. Cross section discretization

In a steel-concrete composite beam section with partial shear connection, the strain field is discontinuous in the steel-concrete interface as showed in Figure 2. Thus, the linear equations that describe the cross sectional deformed shape, in slab ($\varepsilon_{i,slab}$) and steel section ($\varepsilon_{i,steel}$) are expressed as a function of the axial strain in PC of the slab, ε_c, and PC of the steel section, ε_s, respectively. That is:

$$\begin{aligned} \varepsilon_{i,l} &= \varepsilon_c + \Phi\left(y_i - d_{slab}\right) \\ \varepsilon_{i,p} &= \varepsilon_s + \Phi\left(y_i - d_{steel}\right) \end{aligned} \tag{1}$$

where d_{slab} and d_{steel} are the distances of the section PC to slab PC and steel section PC, respectively.

In the matrix notation that follows, ε_c, ε_s and Φ are three degrees of freedom of the steel-concrete composite beam section and are components of the strain vector **X**, described as:

$$\mathbf{X}^T = [\varepsilon_c \quad \varepsilon_s \quad \Phi] \tag{2}$$

Exactly as done previously, the internal force vector for this case is expressed by the classical integration and discretized sums representing the reinforcing bars in concrete slab. Thus:

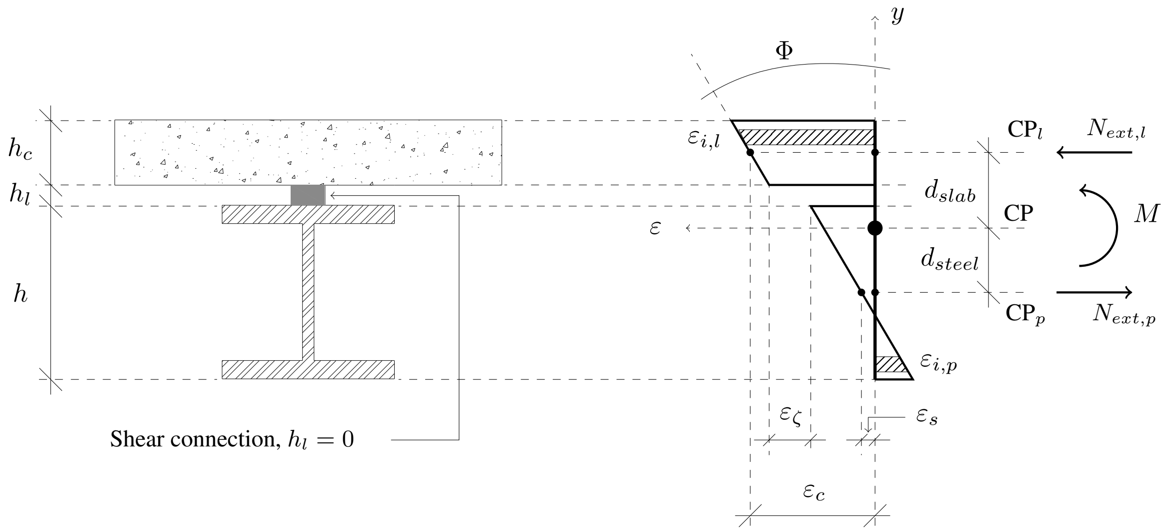


Figure 2. Descontinuous strain field

$$\mathbf{f}_{int} = \begin{cases} N_{int,slab} = \int_{A_l} \sigma\left[\varepsilon_l\left(\varepsilon_c, \Phi\right)\right] dA + \sum_{i=1}^{n_b} \sigma_i \left[\varepsilon_l\left(\varepsilon_c, \Phi\right)\right] A_{ri} \\ N_{int,steel} = \int_{A_a} \sigma\left[\varepsilon_p\left(\varepsilon_s, \Phi\right)\right] dA \\ M_{int} = \int_{A_l} \sigma\left[\varepsilon_l\left(\varepsilon_c, \Phi\right)\right] y dA + \int_{A_a} \sigma\left[\varepsilon_p\left(\varepsilon_s, \Phi\right)\right] y dA + \sum_{i=1}^{n_b} \sigma_i \left[\varepsilon_l\left(\varepsilon_c, \Phi\right)\right] y_i A_{ri} \end{cases} \tag{3}$$

with A_{ri} being the i^{th} reinforcing bar area and n_b is the number of reinforcing bars. The materials constitutive relationship can be seen in Lemes [1].

In the case of the structural element with partial shear connection, the external axial force is dismembered being part acting on the slab, $N_{ext,slab}$, and another part acting on the steel section, $N_{ext,steel}$. The quantification of the absorbed portions by each component of the cross section is valued considering the possibility of slipping at the steel-concrete interface. Thus, the element external forces, including the total axial force (N) and the external bending moment (M_{ext}), can be writing as:

$$\mathbf{f}_{ext} = \begin{Bmatrix} N_{ext,slab} \\ N_{ext,steel} \\ M_{ext} \end{Bmatrix} = \begin{Bmatrix} N_{ext,slab} \\ N - N_{ext,slab} \\ M_{ext} \end{Bmatrix} \tag{4}$$

The axial force absorbed by the concrete slab can be defined as a fraction of the portion that would be absorbed if there were full interaction between steel and concrete, N_{slab}^{full} [5]. The reduction factor is defined by $f\left(\gamma\right)$, described as a function of the degree of composite action, γ. So the axial force on the slab considering deformable shear connection is:

$$N_{ext,slab} = f\left(\gamma\right) N_{slab}^{full} \tag{5}$$

The function of degree of composite action, $f\left(\gamma\right)$, is a constant value defined as a input data.

2.2 Moment-curvature relation

In describing the strain distribution, the cross section discretization in the layers section, shown in Fig. 1 is very efficient. It is done to capture the axial strain, ε, in the center of each fiber, and then (through the material constitutive relations) to obtain the respective stresses. Thus, the axial strain in i^{th} layer can be obtained as discussed in Subsection 2.1 of this paper.

The cross sectional deformed shape is calculated by the equilibrium of the external, $\mathbf{f}_{ext}$, and internal, $\mathbf{f}_{int}$, forces that can be numerically expressed by the following nonlinear equation:

$$\mathbf{F}(\mathbf{X}) = \mathbf{f}_{ext} - \mathbf{f}_{int} \cong 0 \tag{6}$$

with **F** and **X** being the equilibrium force vector and strain vector, respectively. All of this parameters are dependent of the number of degrees of freedom of the section, as discussed in 2.1. Applying the expansion in Taylor series in Equation 6, results in the following set of nonlinear equations:

$$\mathbf{F}(\mathbf{X}) = \mathbf{F'}(\mathbf{X})\Delta\mathbf{X} \tag{7}$$

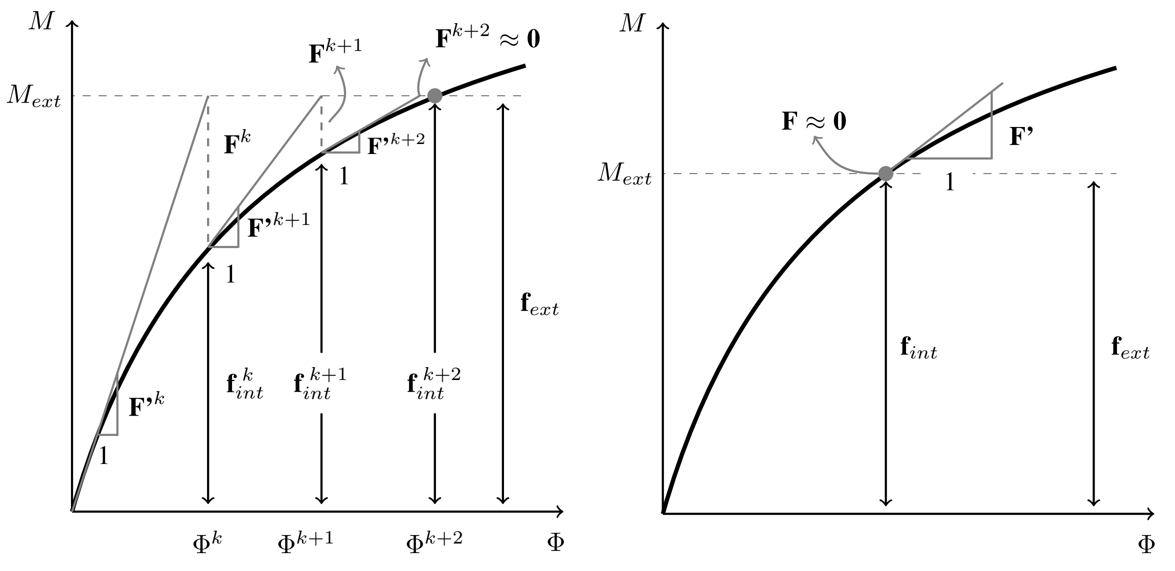
where **F'** is the Jacobian matrix of the nonlinear problem, that is:

$$\mathbf{F'}(\mathbf{X}) = -\frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}} \tag{8}$$

Although it is efficient to start the process with **X** = **0**, convergence is achieved only in the first iteration if external forces are null. Thus, for the next iteration $(k+1)$, the strain vector is calculated by the Newton-Raphson method as:

$$\mathbf{X}^{k+1} = \mathbf{X}^{k} - \left[\frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}}\right]^{-1} \mathbf{F}(\mathbf{X}^{k}) \tag{9}$$

The iterative process described in this section for a given external forces is illustrated in Figure 3.



(a) Iterative process

(b) Equilibrium point

Figure 3. Moment-curvature relationship

2.3 Generalized stiffness parameters

In the set of equations form, the Equation 7, can be defined as follows:

$$\begin{Bmatrix} \Delta N_{slab} \\ \Delta N_{steel} \\ \Delta M \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial N_{int,slab}}{\partial \varepsilon_c} & \dfrac{\partial N_{int,slab}}{\partial \varepsilon_s} & \dfrac{\partial N_{int,slab}}{\partial \Phi} \\ \dfrac{\partial N_{int,steel}}{\partial \varepsilon_c} & \dfrac{\partial N_{int,steel}}{\partial \varepsilon_s} & \dfrac{\partial N_{int,steel}}{\partial \Phi} \\ \dfrac{\partial M_{int}}{\partial \varepsilon_c} & \dfrac{\partial M_{int}}{\partial \varepsilon_s} & \dfrac{\partial M_{int}}{\partial \Phi} \end{bmatrix} \begin{Bmatrix} \Delta \varepsilon_c \\ \Delta \varepsilon_s \\ \Delta \Phi \end{Bmatrix} \tag{10}$$

In the steel-concrete composite beam section with partial shear connection, the axial stiffness is calculated by the sum of the slab (EA_{slab}) and steel section (EA_{steel}) axial stiffnesses, such as:

$$EA_T = EA_{slab} + EA_{steel} \tag{11}$$

being:

$$EA_{slab} = \left.\frac{\Delta N_{slab}}{\Delta \varepsilon_c}\right|_{\Delta M=0} \quad ; \quad EA_{steel} = \left.\frac{\Delta N_{steel}}{\Delta \varepsilon_s}\right|_{\Delta M=0} \tag{12}$$

The effective flexural stiffness of the section, EI_{eff}, is expressed as an explicitly dependent of the function of the degree of composite action [1, 5]. Thus:

$$EI_{eff} = \frac{EI_T^{null}}{1 - f(\gamma)\left(\dfrac{EI_T^{full} - EI_T^{null}}{EI_T^{full}}\right)} \tag{13}$$

2.4 Bending moment capacity

The ultimate bending moment capacity is obtained before the structural analysis (out of incremental-iterative cycle, discussed in Lemes [1] and Lemes et al. [7]. This strategy is adopted to reduce the execution time of the numerical simulations. Thus, the procedure described in Subsection 2.2 is made for each increment of bending moment until it singularizes the Jacobian matrix (Eq. 3). Herein, the incremental strategy is given by [8]:

$$M_{j+1} = M_j + \Phi EI \tag{14}$$

in which the index j refers to the previous increment, Φ is a constant curvature increment, EI is the cross-section flexural stiffness.

3 Numerical formulation arrangement

A brief flowchart for the solution of the partial shear connection in steel-concrete composite beams is presented in Table 1. This strategy is applied to obtain the cross section bearing capacity and also to measure the structural element stiffness with deformable connection.

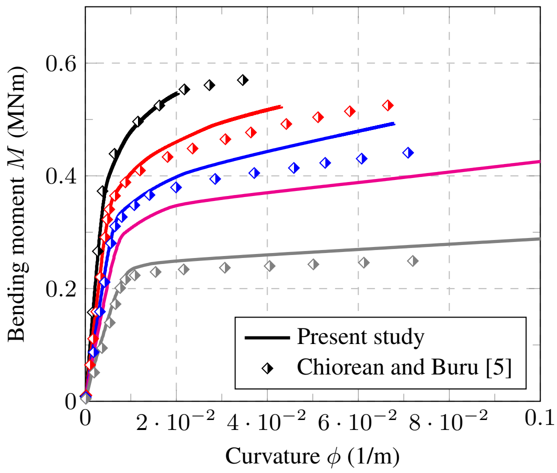
Table 1. Numerical strategy to obtain the stiffness of element with partial shear connection

1.	Consider $f(\gamma)$, M_{ext}, $N = 0$, section and material data as known
2.	Analyse the cross section with full interaction to obtain ΔN_l^{tot}
3.	Calculate the axial force in concrete slab, $N_{ext,l}$ (Equation 5)
4.	Assemble the external forces vector (Equation 4)
5.	Initialize: $\mathbf{X} = \mathbf{0}$
6.	**for** k←1, $nmax$ **do**
7.	Determine ε_l and ε_p (Equation 1)
8.	Assemble $\mathbf{f}_{int}$ (Equation 3)
9.	Calculate $\mathbf{F}(\mathbf{X})$ (Equation 6)
10.	**if** $\|\|\mathbf{F}\|\| \div \|\|\mathbf{f}_{ext}\|\| \leq Tol$ **then**
11.	Stop the iterative process and go to line 20
12.	**end if**
13.	Assemble the tangent constitutive section matrix **F'** (Equation 10)
14.	Check the **F'** singularity
15.	**if F'** is singular **then**
16.	Extrapolated ultimate bearing capacity
17.	Stop the process and go to 20
18.	**end if**
19.	Update the strain vector **X** (Equation 9)
20.	**end do**
21.	Calculate the axial end flexural stiffness (Equations 11 and 13)

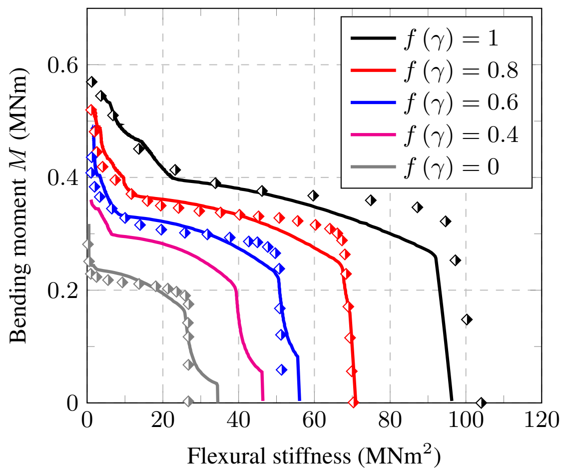
4 Numerical Application

One of tested composite beam sections presented in Chapman and Balakrishnan [9] is studied here, Fig. 4. The material data, as well as constitutive relationship, limit stress and strains and Young's modulus can be seen in Lemes [1]. ChioreanBuru2017 highlighted the yield stress of the steel section plates are different. To adequate this problem to the input of this research program, that made the analysis considering only f_y for a section, the plastic stress distribution in the steel cross section is made and the full yield bending moment is analytically calculated. Thus, the equivalent yield stress is defined by the simple relation between the full yield bending moment and the plastic section modulus. The residual stress model is applied considering the requirements of ECCS [10].

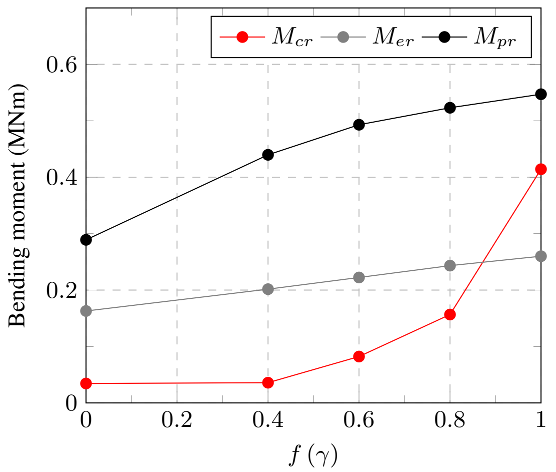
The results of the moment-curvature relation, moment-flexural stiffness and limit bending moments for various funtions of degree of composite action are illustrated in the Figure 4.



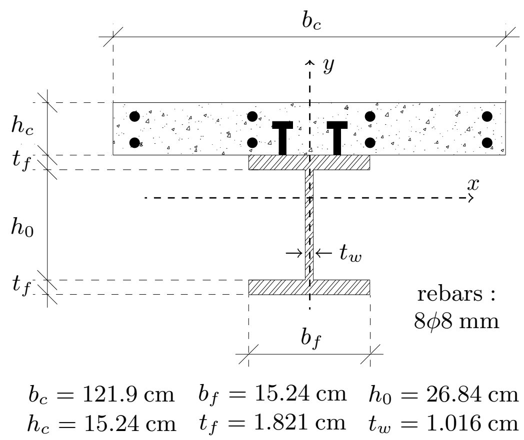
(a) $M \times \phi$ relation



(b) $M \times EI_{eff}$ relation



(c) Critical bending moments × Degree of composite action



(d) Cross section

Figure 4. Composite beam analysis

It is possible to observe that the results obtained in the present work are similar to those presented by Chiorean and Buru [5]. Of the curves presented, the reference does not present the result considering $f(\gamma) = 0.4$, however this value is maintained because it is the minimum interaction degree prescribed in the Brazilian standard [11]. Of the main differences between the proposed formulation and the Chiorean and Buru [5] result, we can highlight the use of residual stresses, the stop criterion of the non-linear process and the approach of the yield stresses (used here homogenously in the cross section).

It is interesting to highlight the drop in the bearing capacity and the flexural stiffness due to the reduction in the degree of interaction between the steel and concrete elements. For the beam with full interaction, practically twice the bearing capacity is compared to the section with null interaction. In Figure 4(c) highlights the variation of the limit moments (M_{pr} - ultimate moment, M_{er} - yield moment, M_{cr} - cracking moment). M_{er} showed practically linear variation due to the degree of composite action, and the behavior of the cracking moment was non-linear. This is due to the tensile stresses on the slab, which for full interaction are very small and as the degree of interaction reduces, the independence of the slab deformation increases, generating greater tensile stresses and the effect of cracking becomes more prevalent.

5 Conclusions

This paper presents a cross section nonlinear formulation for analysis of steel-concrete composite beam with partial shear connection. The formulation starts from the strain compatibility method with the explicit introduction of a degree of freedom for the simulation of the shear connection as deformable. Thus, three degrees of freedom are defined in the section: slab axial strain, steel section axial strain and curvature.

An example of a composite beam section was simulated considering the performance of a sagging bending moment. The results were consistent with those presented in the literature [5]. Some visible differences in the results can be justified by the considerations between the present work and the reference. In this case, the residual stresses, considered here, stand out; yield strength on the flange and web of the steel section; and the non-linear process stop criterion.

It is possible to affirm that the presented formulation has satisfactory results and can be applied in complete structural analyzes, either extracting the resistant bending moment or the axial and flexural stiffnesses.

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Authorship statement

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

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