XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Concentrated plasticity-based second order formulation for analysis of semi-rigid steel-concrete composite frames and beams with partial shear connection

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Abstract. The present work aims at the implementation and validation of a displacement-based two-dimensional numerical formulation including several sources of non-linearities in steel-concrete composite frames, such as: second order effects; plasticity; beam-to-column semi-rigid connections; and partial shear connection on beams. The finite element method is used together with the co-rotational approach in order to allow large displacements and rotations in the numerical model. The degradation of axial and flexural stiffness is determined exclusively at the nodal points of the finite element mesh, characterizing the concentrated plasticity. In cross sections, the Strain Compatibility Method (SCM) is used to capture the axial strains in the components of the section and also the slip in the steel-concrete interface. Sliding is considered by introducing a degree of freedom at the steel-concrete interface in the analysis of the cross section. In this way, the constitutive models of the materials and the shear connection elements are described by continuous functions. The semi-rigid connections are simulated by means of zero-length pseudo springs that are introduced at the finite elements ends, making them hybrid. To validate the proposed numerical formulation, the results obtained are compared with numerical and experimental data available in the literature. Since the model proposed here starts from the concentrated simulation of nonlinear effects, a study of the finite element mesh refinement is also carried out.

Keywords: Second-order effects, Semi-rigid connections, Partial shear connection, Concentrated plasticity, Steel-concrete composite frames

1 Introduction

In steel and steel-concrete composite structures analysis some factors, such as geometric imperfections, material nonlinearity, semi-rigid connections and partial interaction in composite beams can contribute to the reduction of the structural system bearing capacity.

The objective of the present work is to evaluate the non-linear behavior of steel-concrete composite elements considering the non-linear effects concentrated in the nodal points. For this, the refined plastic hinge method will be coupled to the strain compatibility method. Thus, the partial interaction will be simulated by introducing a local degree of freedom in the cross sectional analysis. Semi-rigid connections, on the other hand, will be addressed by introducing of zero-length pseudo-springs at the finite elements ends.

The numerical simulation of steel-concrete composite beams with partial shear connection considering non-linear effects concentrated in nodal points is approached in a few papers [1–3]. In all these works, the numerical simulation of the deformable connection is introduced in the numerical models similarly. The simulation is done using the considerations of the AISC LRFD [4] for composite beams design. Basically, the composite section moment of inertia is reduced by explicitly considering the degree of interaction provided by the shear connectors. Additionally, the consideration of the partial interaction in a concentrated way coupled to beam-to-column connections, in addition to the plasticity and geometric non-linearity, makes the present work comprehensive in the evaluation of the overall stability of steel concrete structural systems.

2 Finite element formulation

In the present work, the displacement-based formulation with concentrated plasticity in the nodal points is applied. In this case, the axial and flexural stiffness degradation occurs exclusively at the FE nodes.

It is important to highlight some considerations involving the finite element formulation used in this paper:

- All elements are initially straight, prismatic and the cross-section remains plane after deformation;
- the effects of global instability that may occur in three-dimensional problems (e.g., lateral and torsional buckling) are ignored considering a locking system out of plane;
- the effects of local instability are neglected, such effects as the buckling of the steel section plates, so the section can reach its full plastic rotation capacity;
- large displacements and rigid body rotations are allowed;
- the shear strains effects are ignored;
- yielding of the cross-section is governed by only normal stress;
- there is full interaction between concrete slab and steel reinforcement bars;
- there is no vertical separation of the elements in the interface (uplift);
- the friction between steel and concrete is neglected; and
- the connections behavior is defined exclusively by the bending moment.

2.1 Kinematics relations

Figure 1 shows the kinematics of the element and the displacements (translations and rotations) notations used below. If the structural element presents large displacements and/or large rotations, the global degrees of freedom contain the rigid motion and the deformational part. The co-rotational approach aims to separate these parts.

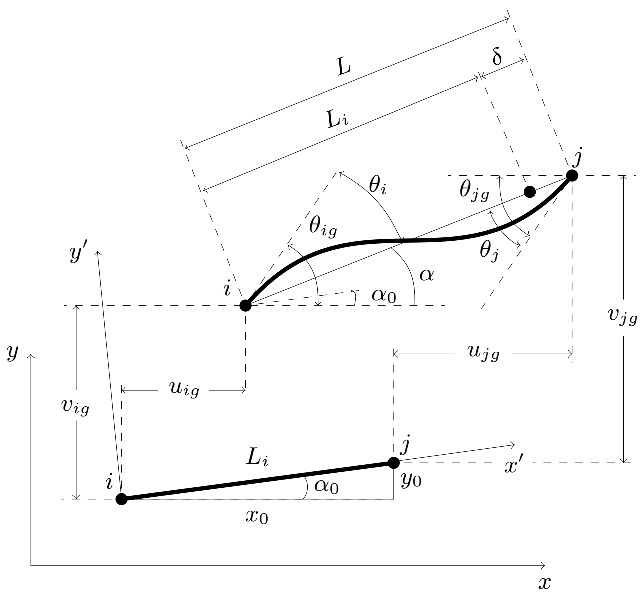

Figure 1. Displacements in global system coordinates

Chhang et al. [5] described the rigid body motion is defined by the global displacements (translations u_{ig} and v_{ig}, and rigid rotation $\alpha - \alpha_0$). It defines a local coordinate system (x', y') that moves continuously with the element. The local system is used to describe the deformational part of the motion.

The relation between global $(u_{ig}, v_{ig}, \theta_{ig}, u_{jg}, v_{jg}, \theta_{jg})$ and local $(\delta, \theta_i, \theta_j)$ degrees of freedom is obtained by a simple differentiation of the co-rotational displacements described in function of global displacements and can be seen in [6]. In a matrix form, this relation is expressed by:

$$\Delta \mathbf{u}_l = \mathbf{B} \Delta \mathbf{u}_g \tag{1}$$

where $\Delta \mathbf{u}_l$ and $\Delta \mathbf{u}_g$ are the incremental displacements in local and global systems, respectively, and the transformation matrix **B** is responsible to transform the global displacements in local responses and vice-versa [6].

2.2 Element formulation

The co-rotational approach is convenient for establishing the relationship between the local and global variables [7]. Starting from the Virtual Work Principle, it is possible to describe a relation between the forces in the two referential systems. By an analytical development the global stiffness matrix is given by:

$$\mathbf{K}_g = \frac{\Delta \mathbf{f}_g}{\Delta \mathbf{u}_g} = \mathbf{B}^T \mathbf{K}_l \mathbf{B} + \frac{\mathbf{z}\mathbf{z}^T}{L} N + \frac{1}{L^2}\left(\mathbf{r}\mathbf{z}^T + \mathbf{z}\mathbf{r}^T\right)(M_i + M_j) \tag{2}$$

where $\mathbf{K}_l$, N, M_i and M_j are the stiffness matrix and the forces in local system, respectively, and:

$$\mathbf{r} = \begin{bmatrix} -c & -s & 0 & c & s & 0 \end{bmatrix}^T \tag{3}$$

$$\mathbf{z} = \begin{bmatrix} s & -c & 0 & -s & c & 0 \end{bmatrix}^T \tag{4}$$

The stiffness matrix in the local system is obtained using consistent interpolation functions [8], eliminating locking problems. In addition, it is deduced based on Green's tensor and curvature via Euler-Bernoulli theory, as can be seen in Lemes [6].

2.3 Concentrated plasticity approach

In the model of the structural systems using corotational-FE, the beam-column finite element is used, defined by nodes i and j, as shown in Fig. 1. The inelastic flexure terms of the matrix $\mathbf{K}_l$ are obtained by a similar approach proposed by Ziemian and McGuire [9]. In order to avoid any numerical integration in calculating element stiffness matrices during the analysis, the flexure terms are calculated considering the moment-curvature relationship $(M \times \Phi)$ tangent varying linearly along the finite element length to the likely situation of a linear moment gradient [9]. Thus:

$$EI(x) = \left[\left(1 - \frac{x}{L}\right) EI_{T,i} + \frac{x}{L} EI_{T,j}\right] \tag{5}$$

where $EI_{t,i}$ and $EI_{t,j}$ are the tangent flexural stiffness, obtained as described in Subsection 3.2, in the nodal points i and j, respectively.

The reduced stiffness matrix (flexure terms), is defined using the second derivative of Hermite interpolation functions [10], described in **N**, that is:

$$\mathbf{k}^* = \int_0^L \mathbf{N}^T EI_T(x) \mathbf{N} dx \tag{6}$$

in which:

$$\mathbf{N}^T = \left[\frac{2}{L}\left(2 - \frac{3x}{L}\right) \quad \frac{2}{L}\left(1 - \frac{3x}{L}\right)\right] \tag{7}$$

3 Cross sectional analysis

3.1 Cross sectional degrees of freedom

Two situations are considered here: a bare steel section; and a steel-concrete composite beam with a linear degree of composite action. The analysis of bare steel section was presented in Lemes et al. [11]. In a steel-concrete composite beam section with partial shear connection, the strain field is discontinuous in the steel-concrete interface as showed in Figure 2. Thus, the linear equations that describes the cross sectional deformed shape, in slab ($\varepsilon_{i,slab}$) and steel section ($\varepsilon_{i,steel}$) are expressed as a function of the axial strain in plastic centroid (PC) of the slab, ε_c, and PC of the steel section, ε_s, respectively. That is:

$$\begin{aligned} \varepsilon_{i,l} &= \varepsilon_c + \Phi (y_i - d_{slab}) \\ \varepsilon_{i,p} &= \varepsilon_s + \Phi (y_i - d_{steel}) \end{aligned} \tag{8}$$

where d_{slab} and d_{steel} are the distances of the section PC to slab PC and steel section PC, respectively.

In the matrix notation that follows, ε_c, ε_s and Φ are three degrees of freedom of the steel-concrete composite beam section and are components of the strain vector **X**, described as:

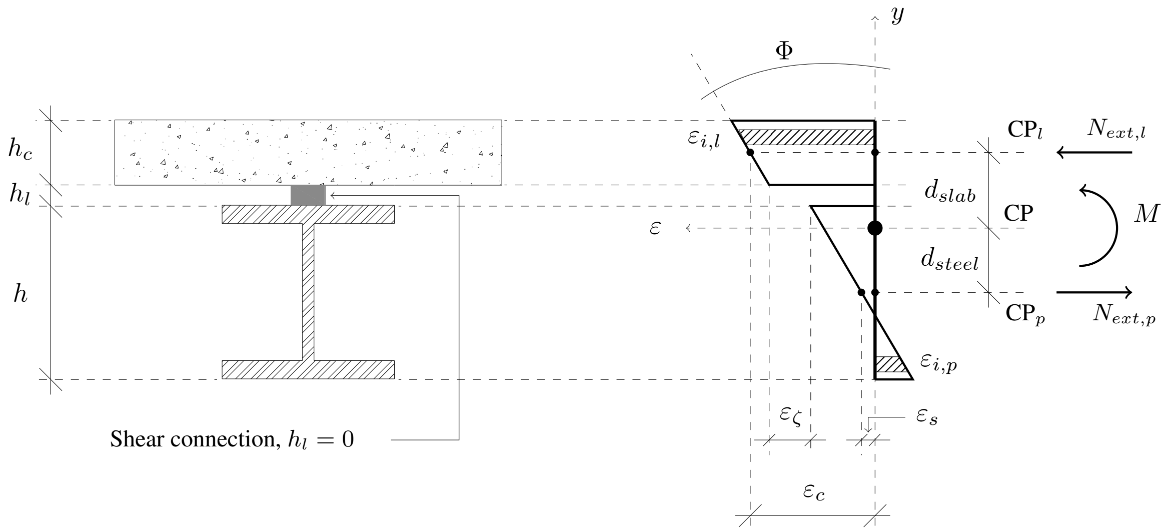

Figure 2. Descontinuous strain field

$$\mathbf{X}^T = [\varepsilon_c \quad \varepsilon_s \quad \Phi] \tag{9}$$

Exactly as done previously, the internal force vector for this case is expressed by the classical integration and discretized sums representing the reinforcing bars in concrete slab. Thus:

$$\mathbf{f}_{int} = \begin{Bmatrix} N_{int,slab} = \int_{A_l} \sigma\left[\varepsilon_l\left(\varepsilon_c, \Phi\right)\right] dA + \sum_{i=1}^{n_b} \sigma_i\left[\varepsilon_l\left(\varepsilon_c, \Phi\right)\right] A_{bi} \\ N_{int,steel} = \int_{A_a} \sigma\left[\varepsilon_p\left(\varepsilon_s, \Phi\right)\right] dA \\ M_{int} = \int_{A_l} \sigma\left[\varepsilon_l\left(\varepsilon_c, \Phi\right)\right] y dA + \int_{A_a} \sigma\left[\varepsilon_p\left(\varepsilon_s, \Phi\right)\right] y dA + \sum_{i=1}^{n_b} \sigma_i\left[\varepsilon_l\left(\varepsilon_c, \Phi\right)\right] y_i A_{bi} \end{Bmatrix} \tag{10}$$

with A_{ri} being the i^{th} reinforcing bar area and n_b is the number of reinforcing bars. The materials constitutive relationship can be seen in Lemes [6].

In the case of the structural element with partial shear connection, the external axial force is dismembered being part acting on the slab, $N_{ext,slab}$, and another part acting on the steel section, $N_{ext,steel}$. The quantification of the absorbed portions by each component of the cross section is valued considering the possibility of slipping at the steel-concrete interface. Thus, the element external forces, including the total axial force (N) and the external bending moment (M_{ext}), can be writing as:

$$\mathbf{f}_{ext} = \begin{Bmatrix} N_{ext,slab} \\ N_{ext,steel} \\ M_{ext} \end{Bmatrix} = \begin{Bmatrix} N_{ext,slab} \\ N - N_{ext,slab} \\ M_{ext} \end{Bmatrix} \tag{11}$$

The axial force absorbed by the concrete slab can be defined as a fraction of the portion that would be absorbed if there were full interaction between steel and concrete, N_{slab}^{full} [12]. The reduction factor is defined by $f\left(\gamma_{eff}\right)$. So the axial force on the slab considering deformable shear connection is:

$$N_{ext,slab} = f\left(\gamma_{eff}\right) N_{slab}^{full} \tag{12}$$

the function of degree of composite action, $f\left(\gamma_{eff}\right)$, is keeping constant during the analysis. Thus, it is possible to introduce de partial shear connection by a linear approach.

3.2 Moment-curvature relation and stiffness parameters

The cross sectional deformed shape is calculated by the equilibrium of the external, $\mathbf{f}_{ext}$, and internal, $\mathbf{f}_{int}$, forces that can be numerically expressed by the following nonlinear equation:

$$\mathbf{F}(\mathbf{X}) = \mathbf{f}_{ext} - \mathbf{f}_{int} \cong 0 \tag{13}$$

with **F** and **X** being the equilibrium force vector and strain vector, respectively. All of this parameters are dependent of the number of degrees of freedom of the section, as discussed in 3.1. Applying the expansion in Taylor series in Equation 13, the equilibrium deformed shape is can be easily found by a nonlinear solution procedure. Here, the Newton-Raphson method is used as follows:

$$\mathbf{X}^{k+1} = \mathbf{X}^k - \left[\frac{\partial \mathbf{F}(\mathbf{X})}{\partial \mathbf{X}}\right]^{-1} \mathbf{F}(\mathbf{X}^k) \tag{14}$$

In the set of equations form, the nonlinear incremental force-strain relationship, can be defined as follows:

$$\begin{Bmatrix} \Delta N_{slab} \\ \Delta N_{steel} \\ \Delta M \end{Bmatrix} = \begin{bmatrix} \dfrac{\partial N_{int,slab}}{\partial \varepsilon_c} & \dfrac{\partial N_{int,slab}}{\partial \varepsilon_s} & \dfrac{\partial N_{int,slab}}{\partial \Phi} \\ \dfrac{\partial N_{int,steel}}{\partial \varepsilon_c} & \dfrac{\partial N_{int,steel}}{\partial \varepsilon_s} & \dfrac{\partial N_{int,steel}}{\partial \Phi} \\ \dfrac{\partial M_{int}}{\partial \varepsilon_c} & \dfrac{\partial M_{int}}{\partial \varepsilon_s} & \dfrac{\partial M_{int}}{\partial \Phi} \end{bmatrix} \begin{Bmatrix} \Delta \varepsilon_c \\ \Delta \varepsilon_s \\ \Delta \Phi \end{Bmatrix} \tag{15}$$

In the steel-concrete composite beam section with partial shear connection, the axial stiffness is calculated by the sum of the slab (EA_{slab}) and steel section (EA_{steel}) axial stiffnesses, such as:

$$EA_T = EA_{slab} + EA_{steel} \tag{16}$$

being:

$$EA_{slab} = \left.\frac{\Delta N_{slab}}{\Delta \varepsilon_c}\right|_{\Delta M=0} \quad ; \quad EA_{steel} = \left.\frac{\Delta N_{steel}}{\Delta \varepsilon_s}\right|_{\Delta M=0} \tag{17}$$

The effective flexural stiffness of the section, EI_{eff}, is expressed as an explicitly dependent of the function of the degree of composite action [6, 12]. Thus:

$$EI_{eff} = \frac{EI_T^{null}}{1 - f\left(\gamma_{eff}\right)\left(\dfrac{EI_T^{full} - EI_T^{null}}{EI_T^{full}}\right)} \tag{18}$$

4 Semi-rigid connection

In this finite element, zero-length rotational pseudo-springs are present in the nodal points. These springs are responsible to simulate the connection nonlinear behavior via $M \times \phi_c$ through their rotational stiffness parameter, S_c.

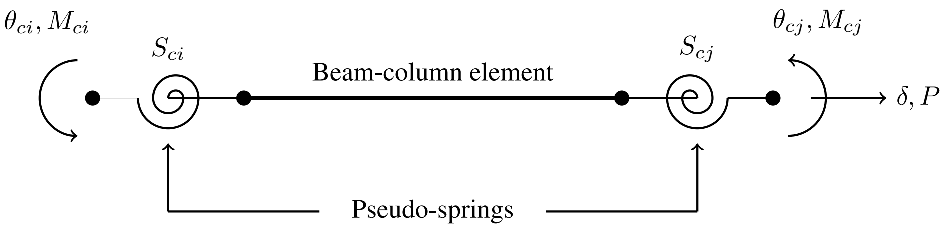

Figure 3. Co-rotational hybrid beam-column finite element

For the element shown in Fig. 3, the force-displacement relationship is expressed by [13]:

$$\begin{Bmatrix} \Delta P \\ \Delta M_{ci} \\ \Delta M_{cj} \end{Bmatrix} = \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & S_{ci} - \dfrac{S_{ci}^2\left(S_{cj} + k_{33}\right)}{\beta} & \dfrac{S_{ci}k_{23}S_{cj}}{\beta} \\ 0 & \dfrac{S_{cj}k_{32}S_{ci}}{\beta} & S_{cj} - \dfrac{S_{cj}^2\left(S_{ci} + k_{22}\right)}{\beta} \end{bmatrix} \begin{Bmatrix} \Delta \delta \\ \Delta \theta_{ci} \\ \Delta \theta_{cj} \end{Bmatrix} \tag{19}$$

with $\beta = (S_{ci} + k_{22})(s_{cj} + k_{33}) - k_{23}k_{32}$. ΔP, ΔM_{ci} and ΔM_{cj} are the axial force and bending moments increments; $\Delta\delta$, $\Delta\theta_{ci}$ and $\Delta\theta_{cj}$ are the axial displacement and nodal rotations increments, respectively.

In this paper, the connections stiffness are evaluated by a multi-linear model as presented in Lemes [6].

5 Numerical application

The six storey steel frame showed in Fig. 4 was initially studied by Vogel [**?**] considering both distributed and concentrated plasticity analysis. The geometry, cross-sections, loads and the used finite element mesh are represented in the same figure. All the loads are incremental. The structure has a global geometric imperfection equal to 1/450. Here, concrete slabs are connected to the beams. These slabs have 12 cm of height and width of 110 cm.

The steel section has yield stress equal to 235 MPa and a modulus of elasticity taken as 205 GPa. The study adopted an elastic-perfectly-plastic relationship without considering the material strain hardening. Concrete has compressive strength of 3 kN/cm^2. The beam-to-column connections are simulated by a constant stiffness as well as the partial shear connection.

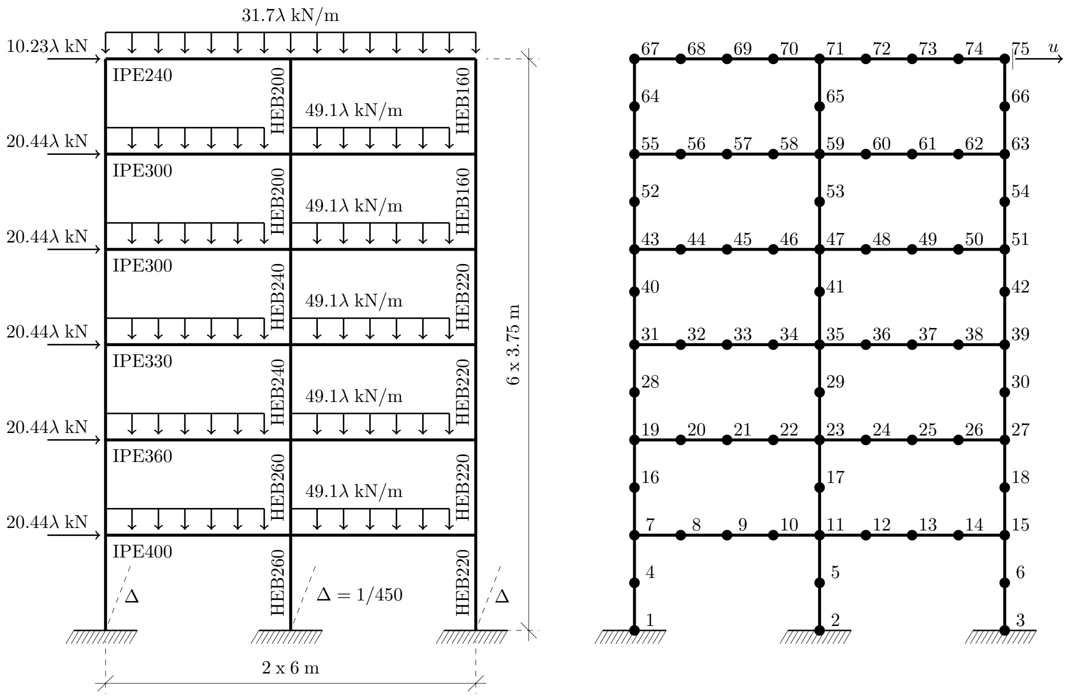

Figure 4. Six-storey frame: geometry, loads and adopted FE mesh

6 Conclusions

Acknowledgements

The authors would like to thank CAPES and CNPq (Federal Research Agencies), Fapemig (Minas Gerais State Research Agency), UFOP and UFLA for their support during the development of this work.

Authorship statement

The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

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