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GBT-Based buckling analysis of cold-formed steel purlin-sheeting systems using restrained deformation modes

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Abstract. In Generalized Beam Theory (GBT), which incorporates genuine plate theory concepts, the cross-section displacement field is expressed as a linear combination of deformation modes, making it possible to write the equilibrium equations in a very convenient format. In practical applications, thin-walled steel members are often continuously braced along their lengths - e.g., cold-formed steel purlins restrained by sheeting. In order to incorporate such bracing in a GBT analysis, two approaches can be followed: (i) incorporate the restraints only at the member analysis stage, which means that the deformation modes are obtained for the unrestrained cross-section (conventional approach) or (ii) incorporate the restraints at the cross-section analysis stage, which means that the deformation modes already take them into account (restrained cross-section analysis approach - proposed in this work).This paper reports the results of an investigation on improving the GBT buckling analysis of continuously braced cold-formed steel purlins by including the displacement/rotation restraints at the cross-section analysis stage, i.e., during the determination of the cross-section deformation modes and calculation of the associated modal properties - the end product of this procedure are "restrained deformation modes". After describing, the procedures involved in the determination of such restrained deformation modes, the paper illustrates the advantages of their use in the GBT-based buckling analysis of cold-formed steel purlins restrained by sheeting. For validation and assessment purposes, some GBT-based results are compared with values provided by the codes GBTUL2.0 (conventional GBT approach) and Finite Strip Method (FSM) analyses. It is still worth noting that restrained cross-section analysis makes it possible to obtain semi-analytical solutions to determine critical buckling loadings in purlins restrained by sheeting - such solutions are based on one or two restrained deformation modes.

Keywords: Cold-formed Steel purlins; Buckling Analysis; Generalized Beam Theory (GBT).

1 Introduction

The Generalized Beam Theory (GBT) is a theory that incorporates genuine plate theory concepts (thus accounting for cross-section in-plane and out-of-plane deformations), the cross-section displacement field is expressed as a linear combination of deformation modes, making it possible to write the equilibrium equations and boundary conditions in a unique and very convenient format. In practical applications, thin-walled purlins are often continuously restrained along their lengths. In order to incorporate such (elastic) restraints in a GBT analysis, two approaches can be followed: (i) incorporate the restraints in the cross-section analysis, taking them into account at the deformation mode determination stage Schardt [1], Jiang and Davies [2] and Jiang et al. [3] adopted this approach for global and very specific distortional deformations or (ii) include the restraints only in the member analysis, as constraint equations, which means that the deformation modes are calculated without considering the restraints Camotim et al. [4], Basaglia et al. [5] and Bebiano et al. [6], authors of the code GBTUL2.0, adopted this approach for arbitrary deformation patterns, which amounts to combining the conventional deformation modes at the member analysis stage. This last approach requires larger deformation mode sets, and it may be argued that it somewhat "clouds" the structural interpretation of the results. The most successful end product of this intense research activity are design procedures based on the Direct Strength Method (DSM). Since the member's ultimate strength can be accurately predicted solely on the basis of its elastic critical buckling and yield loads/moments, the Direct Strength Method (DSM) is an efficient alternative to the more traditional "effective width method." Following the universal acceptance of the DSM approach to design cold-formed steel members, a DSM-based design procedure for a restrained member has already been included in the latest edition of the North American specification [7].

The aim of this work to present and illustrate the implementation and application of a novel GBT formulation for the buckling analysis of continuously braced thin-walled purlins, based on an improved cross-section analysis procedure that incorporates elastic restraints, whose approach to calculate the deformation modes corresponds to an enhancement of the standard GBT process proposed by [1,2,3]. The end product of this procedure is "restrained deformation modes" and, therefore, is capable of providing accurate distortional, lateral distortion and/or global critical buckling results with only a few restrained deformation modes. For validation and assessment purposes, some of these results are compared with values yielded by the codes CUFSM [8] and GBTUL2.0 [6].

2 GBT Formulation

The modeling of the restraint provided by the sheeting to the purlin involves continuous translational (acting tangential to the cross-section wall) and rotational elastic springs, which restrains the member transverse displacements and rotations. Figure 1 illustrates the case of a purlin-sheeting assembly: to simulate the restraint provided by sheeting, continuous translational (with K_{TG} tangential stiffnesses to the wall), and rotational elastic springs (with K_R stiffness) are continuously attached to the flange mid-points along the whole purlin length.

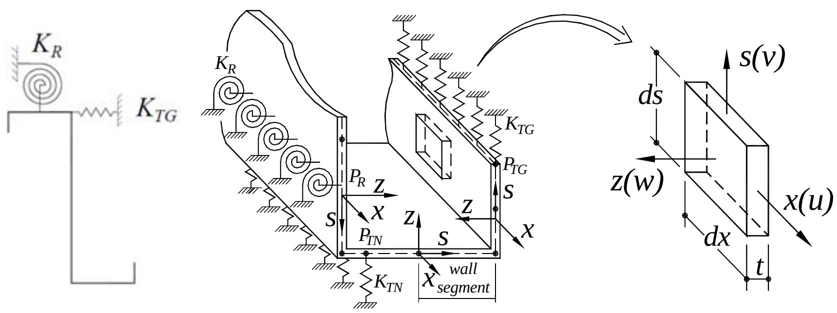

Figure 1. Continuous elastics restraints on the cross-section and local axes orientation.

In order to obtain a displacement field representation compatible with the classical beam theory, each displacement component u(x,s), v(x,s) and w(x,s) at any given point of the cross-section mid-line must be expressed as a combination of orthogonal functions. Therefore, one has

$$u(x,s) = u_i(s)\varphi_{1,x}(x), \qquad v(x,s) = v_i(s)\varphi_i(x), \qquad w(x,s) = u_i(s)\varphi_i(x) \tag{1}$$

where $(.)_{,x}\equiv d(.)/dx$, the summation convention applies to subscript i, functions $u_i(s)$, $v_i(s)$, $w_i(s)$ are midline "displacement profiles" and $\varphi_k(x)$ is a dimensionless amplitude function defined along the member length – information on the derivation of these expressions can be found in Silvestre and Camotim [9].

The equations providing the member buckling behaviour, taking into account the presence of elastic constraints (springs), are obtained by imposing the stationarity of the total potential energy functional

$$V = \frac{1}{2}\int_{\Omega} \sigma_{ij}\varepsilon_{ij}d\Omega + \frac{1}{2}\int_{L} K\Delta_r^2 dx \qquad .(2)$$

In the close vicinity of the member fundamental equilibrium path (adjacent equilibrium) (i) Ω is the member volume (n walls), (ii) σ_{ij} and ε_{ij} are the second Piola-Kirchhoff stress and Green-Lagrange strain tensors, respectively, both comprising pre-buckling and bifurcated components, (iii) L is the member length, (iv) K is the stiffness of a continuous (along with a longitudinal axis r) spring and (v) Δ are spring generalized displacements (translation or rotation) and (vi) the summation convention applies to subscripts i and j. Then, the equilibrium equations defining the member buckling eigenvalue problem are obtained by (i) linearizing the first variation of the total potential energy functional, at the fundamental equilibrium path, and (ii) discarding the pre-buckling strains, thus yielding

$$\delta V = \int_L \left(\delta\varphi_{,xx}^T C\varphi_{,xx} + \delta\varphi_{,x}^T D^I \varphi_{,x} + \delta\varphi_{,xx}^T D^{II}\varphi + \delta\varphi^T D^{II}\varphi_{,xx} + \delta\varphi^T B\varphi + \lambda W_j^0 \delta\varphi_{,x}^T X_j \varphi_{,x}\right) dx = 0. \quad (3)$$

where, (i) λ is the load parameter, (ii) φ is a vector containing the corresponding amplitude functions, (iii) W_j^0 is a vector whose components are the normal stress resultants profiles related to uniform internal forces and bending moments j, (v) C, D^I, D^{II}, and B are cross-section stiffness matrices and (vi) X_j is a geometric stiffness matrix stemming from the pre-buckling longitudinal normal stresses. In members with a uniform wall thickness (the ones dealt with here), the various matrix components C_{ik}, D^I_{ik}, D^{II}_{ik}, B_{ij} and X_{jik} are given by the expressions

$$C_{ik} = Et\int_b u_i u_k ds + \frac{Et^3}{12(1-\upsilon^2)}\int_b w_i w_k ds \quad (4)$$

$$B_{ik} = \frac{Et^3}{12(1-\upsilon^2)}\int_b w_{i,ss} w_{k,ss} ds + [F_V^T K_{TG} F_V + F_\theta^T K_R F_\theta] \quad (5)$$

$$D_{ik}^I = \frac{G}{3}\int_b w_{i,s} w_{k,s} ds \quad D_{ik}^{II} = \frac{\upsilon Et^3}{12(1-\upsilon^2)}\int_b w_i w_{k,ss} ds \quad (6)$$

$$X_{jik} = \frac{Et}{C_{jj}}\int_b u_j (v_i v_j + w_i w_j) ds \quad (7)$$

$$W_j = C_{jj}\varphi_{j,xx}^0 \quad (8)$$

where (i) E, ν and G are the material Young's modulus, Poisson's ratio and shear modulus, (ii) b is the cross-section mid-line length and (iii) $u_j\varphi^0_{j,xx}$ is the pre-buckling axial displacement field. The B matrix terms inside the brackets are associated with the elastic strain energy provided by the translational and rotational springs (one of the novelties of the present formulation), and their terms are defined as (i) F_v is the matrix of transverse membrane displacements v_r of each wall segment, (ii) F_θ is a matrix of rigid-body rotations of each wall segment obtaining as the difference of their end nodes divided by their length and (iii) K_{TG}, K_R and are diagonal matrices of translational and rotational spring stiffnesses.

Before the diagonalization process, the matrices are completely full, which implies in strongly coupled deformation modes, that do not have any mechanical interpretation. In order to uncouple those modes, the diagonalization process must be performed. The entire process can be accessed in Bebiano et al. [6] – this process has a particularity as the number of null eigenvalues.

After performing the cross-section analysis, one obtains the member GBT system of adjacent equilibrium equations, which (i) is expressed in modal form as

$$\bar{C}_{ik}\varphi_{k,xxxx} - \bar{D}_{ik}\varphi_{k,xx} + \bar{B}_{ik}\varphi_k - \lambda\bar{X}_{jik}\left(\bar{W}_j^0\bar{\varphi}_{k,x}\right)_{,x} = 0 \quad .(9)$$

The methods that have already been employed to solve the GBT-based eigenvalue problem are fairly standard in structural analysis. At this point, it should be mentioned that the GBT-based buckling results presented in this paper have been obtained through the application of Galerkin's method which is an extremely advantageous technique for simple support purlins– indeed since the eigenfunctions are known to display pure sinusoidal shapes of the type:

$$\varphi_k(x) = d_k \sin\left(\frac{\eta\pi x}{L}\right) \quad .(10)$$

This technique can be readily used to obtain exact solutions for the problem, and longitudinal discretization can be avoided without sacrificing accuracy. In Eq. (10), d_k is the amplitude of mode k (problem unknown) and η is the number of longitudinal half-waves.

3 Illustrative Example

The illustrative example concerns the buckling behaviour of simply supported steel purlins (E=205GPa and ν=0.3) with Z 200x75x25x2.5 (mm) cross-section and connected on sheeting under wind uplift loading, as shown in Fig. 2(a). The longitudinal pre-buckling (reference) stress applied to the cross-section in all analysis (based on FSM, and GBT) is calculated assuming only (restrained) bending about the centroidal axis perpendicular to undeformed web configuration – Fig. 2(b) shows the pre-buckling longitudinal stress distribution, where the upper (connected) and bottom (free) flanges are acted by uniform tensile and compressive stresses, respectively. It is important to highlight that, in practical, the wind uplift loading is applied at the purlin top flange mid-width region, as illustrated in Figure 5(a). Moreover, the authors are aware that the load point of application effects play a relevant role in the response of purlins under wind uplift and, therefore, must necessarily be included in more realistic models, which take into account the pre-buckling states involving non-uniform bending and/or torsion combining with lateral-distortional. However, in the context of design routine, which require accurate and simple procedures, the latest edition of the North American specification [7] proposed a DSM-based design criterion for purlin-sheeting systems, which consists of performing elastic buckling analysis of restrained purlins subjected to longitudinal pre-buckling stress equal to those adopted here (Fig. 2(b)) and the non-uniform bending and initial torsion effects are predicted by well-known moment gradient factor C_b and so-called Reduction factor R, respectively. In order to avoid further complexities, this pre-buckling longitudinal stress distribution is used here just for example purposes – since this work basically aims at the development and application of a formulation for the buckling analysis of braced members by considering GBT restrained deformation modes.

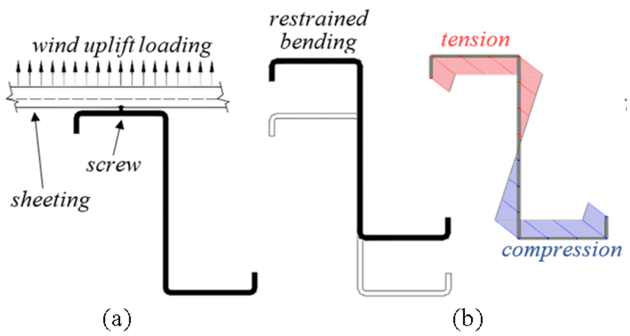

Figure 2. (a)Purlin under wind uplift loading (b) Restrained bending pre-buckling stress

For comparison purposes, while Fig. 3 depicts the first 10 (conventional) deformation modes yielded by code GBTUL2.0 for the unrestrained Z-section (so-called **P0**), Fig. 4(a) displays the 9 most relevant deformation modes obtained by means of the proposed restrained cross-section analysis procedure (mode 1 stands for axial extension) for two possible combinations of continuous translational and rotations springs with infinity and real stiffness values, i.e., K_{TG} equal to ∞ kN/cm – constraints **P1** ($K_{TG} = \infty$, $K_R = \infty$), Fig. 4(b) shows the structurally meaningful restrained deformation modes corresponding to cross-section with a rigid translational restraint ($K_{TG} = \infty$) and an elastic rotational restraint of stiffness K_R = 1.285 kNcm/rad/cm (constraint **P2**) – whose values were calculated by Gao and Moen [10] using a typical metal sheeting and standard fastener locations.

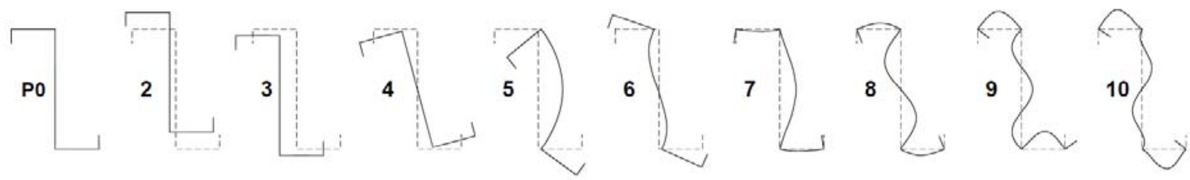

Figure 3. P_0 (unrestrained) deformation modes

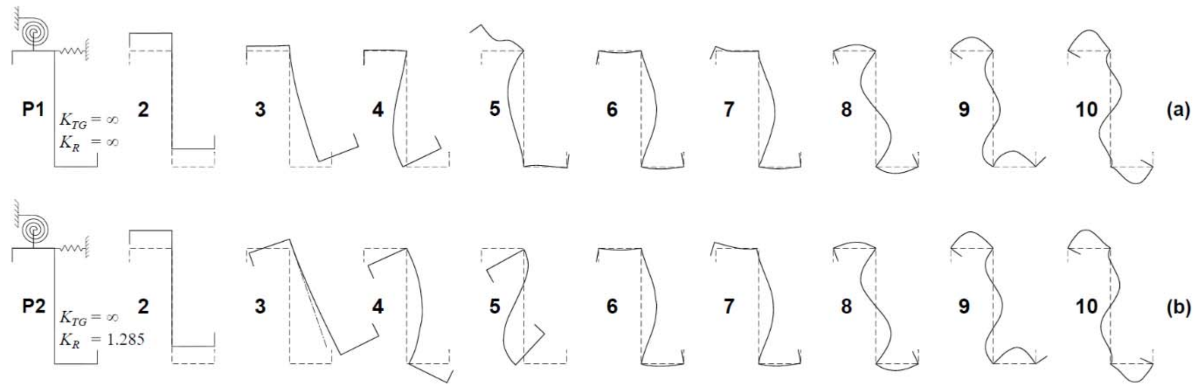

Figure 4. Most relevant deformation modes: (a) P1 ($K_{TG} = \infty$, $K_R = \infty$) (b) P2 ($K_{TG} = \infty$, $K_R = 1.285$ kNcm/rad/cm)

Using one sine function with a single half-wave (η=1 in Eq. (10)) to approximate the mode amplitude functions and performing buckling analyses for a wide range of lengths (L), one obtained the results (signature curves) shown in Fig. 5, where the variation of the critical buckling moment M_{cr} with the beam length L (logarithmic scale) are depicted for the unrestrained member and purlins exhibiting the two sheeting restraint conditions – the "minimum points" at the signature curve corresponds to critical moments that lead to either local, distortional or lateral-distortional buckling, while the minimum associated with a global dominant buckling mode nature does not exist. Besides the results obtained through restrained-mode GBT analyses (solid and dashed curves), the figure also displays the critical buckling mode shapes yielded by CUFSM and, for validation and comparison purposes, buckling moment values determined by means of GBTUL2.0 (circles) and CUFSM (symbol x) analyses. Figure 6 provides the buckling modal participation diagram concerning the restraint conditions indicated and mid-span cross-section buckled shape related to minimum point lengths of the signature curve while the left-hand side figures are diagrams and buckling shapes obtained by GBTUL (see Fig. 3), their right-hand side counterparts display the diagrams and buckling shapes yielded by proposed GBT-based formulation considering restrained deformation modes (see Fig. 4).

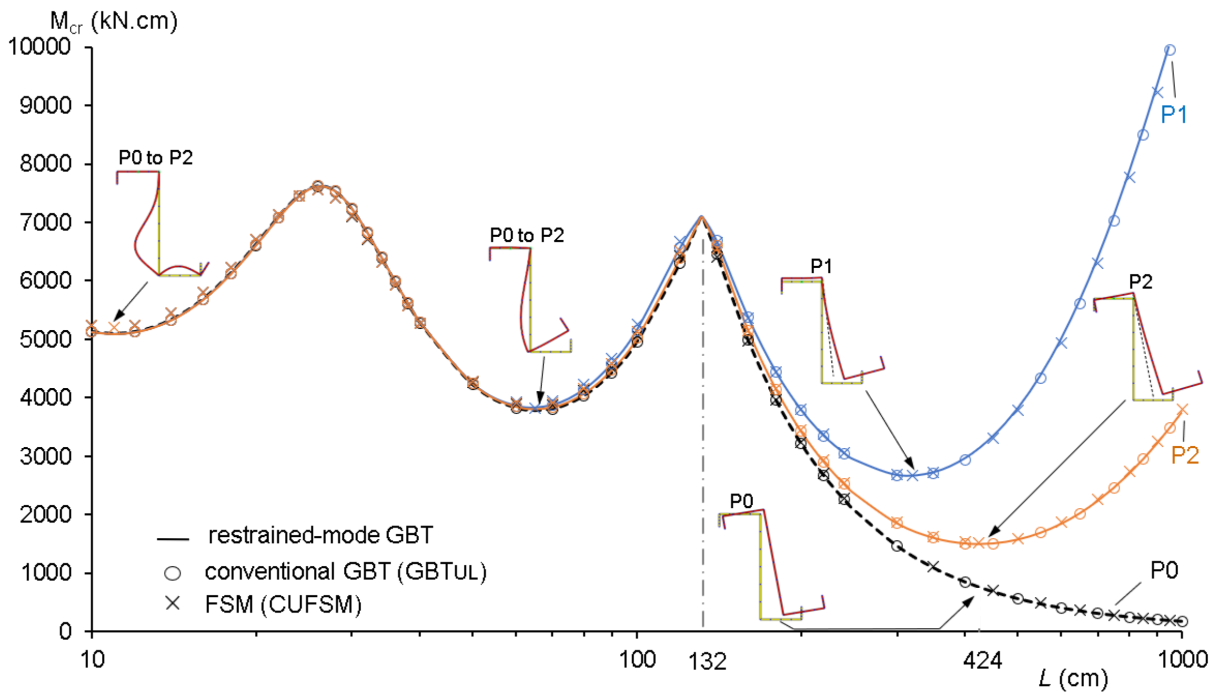

Figure 5. Critical moment variation and their respective deformed shape

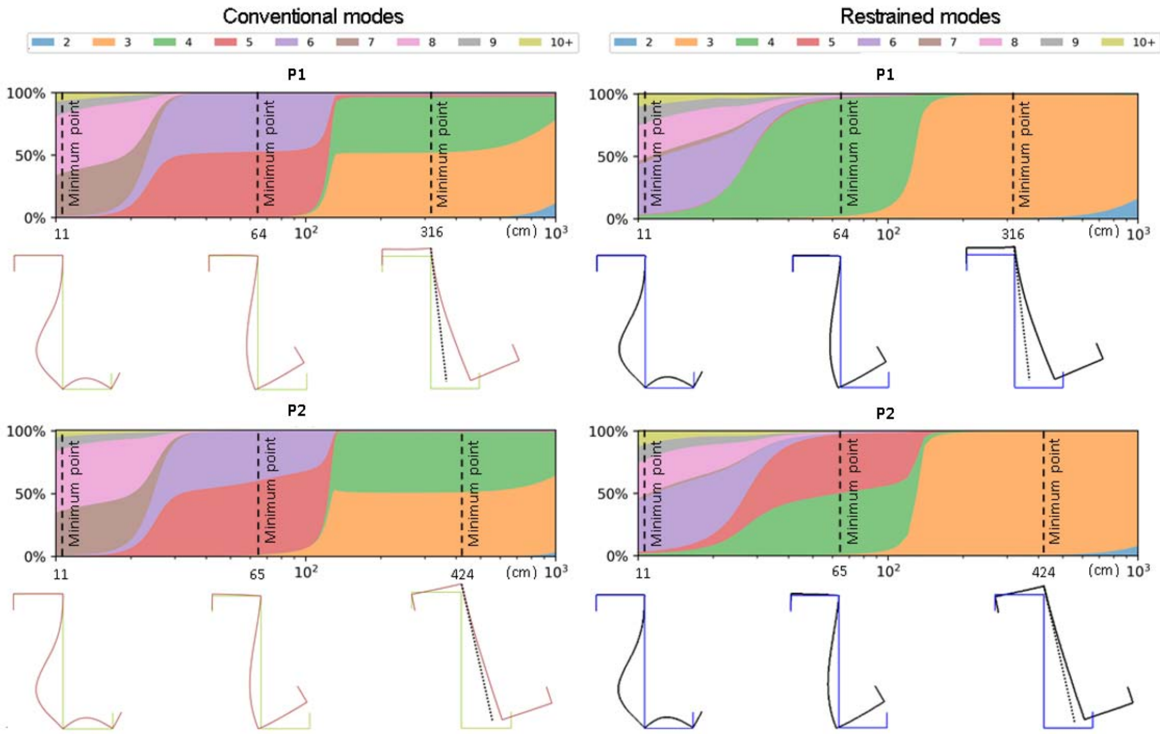

Figure 6 - Modal Participation comparison of restrained purlins.

The close observation of the buckling results displayed in all these figures prompts the following remarks:

(i) First of all, the buckling moment values yielded by the FSM and the two GBT analyses (including all deformation modes) are virtually coincident – all differences below 2.5%. Moreover, there is also very close agreement between the buckling mode shapes provided by the two analyses.

(ii) For $L \leq 132$ cm, the buckling behaviour does not depend on the restraint conditions (the solid and dashed curves coincide). This is because the critical buckling modes combine local ($L = 11$ cm) and/or distortional ($L = 64 \pm 1$ cm) deformations that do not involve upper flange horizontal displacements or rotations – see the buckled shapes.

(iii) For restrained purlins ($K_{TG} \neq 0$ and $K_R \neq 0$) and $L > 132$ cm, the buckling mode shape is clearly lateral-distortional – this instability phenomenon was investigated among others by [5,10,13,14].

(iv) Besides the virtual coincidence of the critical buckling loadings and mode shapes, it turns out that the proposed GBT formulation is much more efficient than the conventional one. Indeed, by evaluating the modal participation diagrams, one may conclude that it is possible to obtain accurate buckling results with a single (mostly) or two restrained deformation modes for purlins exhibiting the distortionally and lateral-distortionally buckled configurations.

4 Conclusions

The paper reported the results of an ongoing investigation on the development of a GBT formulation for the buckling analysis of restrained thin-walled members, differing from the conventional one in the cross-section analysis (it already incorporates the elastic restraints). Special attention was paid to the procedures involved in the determination of the restrained deformation modes. The application and capabilities of the above GBT formulation were illustrated through the presentation and discussion of numerical results concerning purlins restrained by sheeting. For validation and assessment purposes, some results were compared with values provided by the codes GBTUL2.0 (conventional GBT) and/or CUFSM5.0 (FSM). Besides the expected virtual coincidence of the results, it was found the proposed GBT formulation is much more efficient than the conventional one. In particular, it becomes possible to obtain accurate buckling results with a single deformation mode, which enables the development of analytical formulae to calculate critical buckling loadings of restrained members - this feature, currently being exploited by the authors, will be addressed in future works.

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