Tri-objective truss structural optimization considering sizing design variables

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Abstract. Structural multi-objective optimization problems are common in the Engineering field's real-world problems where one or more objective functions, in general conflicting, may be considered to be optimized, leading to complex optimization problems. Two objective functions problems represent the great majority of formulations in this context where Pareto fronts can be easily represented and provided to the Decision Maker (DM). For instance, in the trusses' structural optimization, these conflicting objective functions may be the weight and the maximum nodal displacements to be minimized. This paper presents the formulation of multi-objective truss structural optimization problems considering three objective functions, such as the weight, the maximum nodal displacement, and the first natural frequency of vibration. The constraints refer to the allowable axial stresses in the bars. Several experiments inspired in the benchmark mono-objective optimization are analyzed in this paper, presenting their Pareto surfaces showing the non-dominated solutions. The structural optimization problems consider sizing and shape design variables simultaneously, and they can be continuous, discrete, or mixed. The search algorithm adopted to solve these problems is a modified version of the Differential Evolution called Third Evolution Step Differential Evolution (GDE3). One of the most important steps, after obtaining the Pareto surfaces, is the definition of which solution or solutions will be chosen by the decision-maker (DM), a non-trivial task. A Multi-Tournament Decision method, commonly used in bi-objective optimization, is adapted in this paper to extract the solutions from the Pareto surfaces based on DM preferences.

Keywords: Multi-objective optimization, Truss structural optimization, Differential Evolution, Multi-Tournament Decision method.

1 Introduction

Structural optimization in Engineering is a powerful tool that can lead to the economy in real-world designs. Usually, a single objective is employed on its process where the goal is to minimize or maximize any desired objective function, such as cost, weight, subject to displacement, stress, natural frequencies of vibrations as constraints, and so on. At the end of this process, one single optimized solution is presented, and it's taken as a final product to be made.

As a similar process, multi-objective optimization can handle two or more objectives to be reached. However, in the end, the Decision-Maker (DM) has a set of non-dominated solutions (usually called Pareto) and need to take the final decision choosing one among them. Another feature of multi-objective is obtaining "great" improvements in one objective function when having "small" losses on another one depending on the aspect of Pareto (i.e., "small" weight increase leading to "high" displacements decrease).

Classical approaches treats multi-objective optimization problems as multiple mono-objective optimization problems (Reyes-Sierra and Coello [1],Coello et al. [2]). On the other hand, more recent approaches modify original algorithms to handle the multi-objective problems: Mokarram and Banan [3] proposed a new version of the Particle Swarm Optimization (PSO) for multi-objective optimization called the FC-MOPSO. The algorithm can handle continuous and discrete variables by extending the usual PSO formulation for multi-objective optimization; several benchmark functions and some benchmark trusses are analyzed. Pareto surfaces are provided for the case of 3 objective-functions, which is similar to the work proposed here. Kaveh and Mahdavi [4] presents a new multi-objective algorithm called MOCBO based on energy released by colliding bodies. Again, several benchmark functions are first tested, and then two multi-objective optimizations of trusses are analyzed. Tejani et al. [5] also proposes a new algorithm based on heat transfer called MOHTS to handle the multi-objective optimization problems; again, some benchmark of double objective-functions optimization of trusses are analyzed to validate the new algorithm. Similar works where an adaption of pre-existing algorithms is adopted to handle the multi-objective optimization applied to truss optimization problems can be found in Moradi et al. [6], Kaveh and Laknejadi [7]. Since most works in the literature present just two objective functions, this paper presents an example of tri-objective truss structural optimization showing an amplified set of non-dominated solutions by a Pareto surface.

The remainder of this paper is organized as follows: Section 2 describes the formulation of the multi-objective structural optimization problem discussed in this paper. Section 3 summarizes the basic steps of a strategy to extract desired solutions, based on designer's preferences, from the Pareto surface. The methodology and computational experiment discussed in this paper are provided in Section 4 followed by the analysis of results presented in Section 5. Finally, the conclusions and future works are reported in Section 6.

2 Multi-objective structural optimization

The tri-objective structural optimization problem can be defined as follows:

$$\begin{array}{lllll} \min/\max \quad of_1 & \text{and} & \min/\max \quad of_2 & \text{and} & \min/\max \quad of_3 \\ \text{s.t.} & & constraints & & \end{array} \tag{1}$$

where of_1, of_2, and of_3 are the three conflicting objective functions. In this work, four structure's properties are considered and combined between objective functions and or constraints: the weight, the natural frequency of vibration, the elastic critical load factor, and the nodal displacements. Three multi-objective structural optimization (MOSO), further defined in subsection 4.1, will describe which one is set as an objective function and which one is set as a constraint. Stresses on bars are always treated as constraints since the global stability is verified, checking the elastic critical load factor.

Defining **x** as the sizing design variables concerning the cross-sectional areas, the following items are defined:

- The weight of structure is obtained by the sum:

$$W(\mathbf{x}) = \sum_{i=1}^{N} \rho A_i L_i \tag{2}$$

  where ρ is the specific mass of the material, A_i and L_i are the cross-sectional areas and the length of the i-th bar of the structure, respectively. The number of bars of the structure is denoted by N.

- The nodal displacements $\{\delta\}$ are obtained by the equilibrium equation for a discrete system of bars, which is written as:

$$[K]\{\delta\} = \{F\} \tag{3}$$

  where $[K]$ is the elastic stiffness matrix of structure and $\{F\}$ is the nodal force vector.

- The natural frequencies of vibration are obtained by evaluating the eigenvalues of matrix

$$\left([K] - f_{m_f}^2 [M]\right) \phi_{m_f} = 0 \tag{4}$$

  where $[M]$ is the mass matrix and ϕ_{m_f} is the m_f-th eigenvector corresponding to the m_f-th eigenvalue (Bathe [8]).

- The load factors λ concerning the global stability are obtained by the evaluation of eigenvalues of matrix

$$([K] + \lambda_{m_\lambda} [K_G]) \nu_{m_\lambda} = 0 \tag{5}$$

where $[K_G]$ is the geometric matrix of structure and λ_{m_λ} are the equivalent eigenvalues with respect to the m_λ load factors of the structure. The lowest value λ_{cr} of λ_{m_λ} gives the buckling load factor or critical load factor. The eigenvector ν_{m_λ} represents the corresponding instability modes for the load factors λ_{m_λ} (McGuire et al. [9]).

3 Multicriteria decision making

Once the optimization process is carried out, the DM might want to choose a solution between those one presented in the Pareto front according to its preferences. There are several strategies for this task, and the extraction scheme used in this work is detailed on Parreiras and Vasconcelos [10] and Angelo et al. [11]. Briefly, the DM must set weights (in the meaning of importances) for objective functions according to its desire, and the sum of them should be equal to 1 (representing 100%). For instance, since in this work there are 3 objective functions to be analyzed, one example of weights could be w_1 = 0.7, w_2 = 0.2 and w_3 = 0.1 (representing 70%, 20% and 10% of importance for each objective function). Once the weights are set, the mathematical formulation presented in the works cited above provides a final solution given to the DM (See the scheme details therein these references).

4 Methodology and computational experiment

In this section, a benchmark example is presented to assess the proposed methodology. The search algorithm is the Third Evolution Step of Generalized Differential Evolution (GDE3) proposed by Kukkonen and Lampinen [12], which is a modified version of traditional Differential Evolution (DE) proposed by Storn and Price [13, 14]. The multi-objective optimization is based on ranking by crowding distance (Raquel and Naval Jr [15]) and coupled to the Adaptive Penalty Method (APM), proposed by Barbosa and Lemonge [16], to handle the mechanical constraints. The optimization process was done in 20 independent runs throughout 200 generations with 50 candidate vectors. Only non-dominated solutions (Pareto front) are shown as the final results.

4.1 10 bar truss

This is one of the most popular benchmark examples of structural optimization corresponding to the 10-bar truss shown in Figure 1. The design variables are the cross-sectional areas of bars, which lower and upper bounds are respectively 6.45x10^{-4} m^2 and 258.06x10^{-4} m^2. This truss has Young's modulus equal to E = 68.95GPa, and specific mass equals 2700 kg/m^3. A nonstructural mass of 454 kg is added at each free node (not considered in the final weight of the optimized trusses).

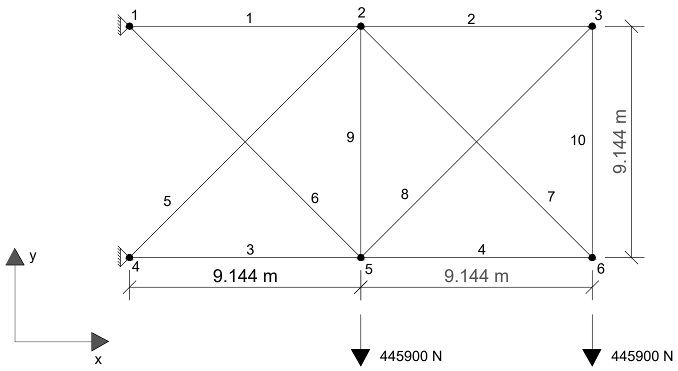

Figure 1. The 10-bar truss.

The MOSO studied in this work is defined as to minimize weight and the maximum displacement and to maximize the first natural frequency of vibration, setting the stresses and the elastic critical load factor as constraints. Table 1 shows the limits for each constraint:

5 Analysis of result

The obtained results showed interesting 3d curves as the Pareto front. For the studied MOSO (Figure 2), one can observe the decrease of maximum displacement with an increase in the structure's weight, which was expected.

Constraint	Value
σ_{max}	$\leq$ 1.72 x 10^8 N/m^2
λ_{cr}	≥ 1

Table 1. Limits for each constraint.

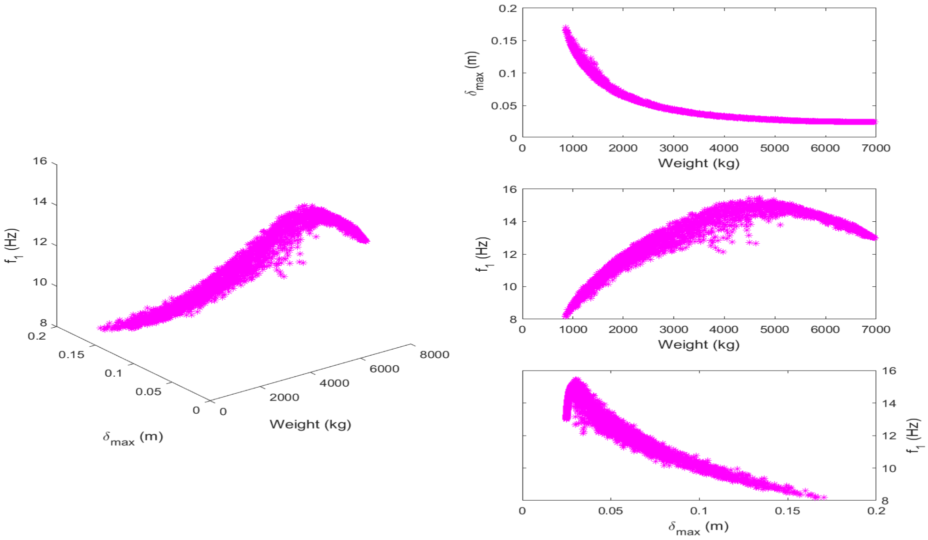

Figure 2. Pareto front of the 10 bar truss.

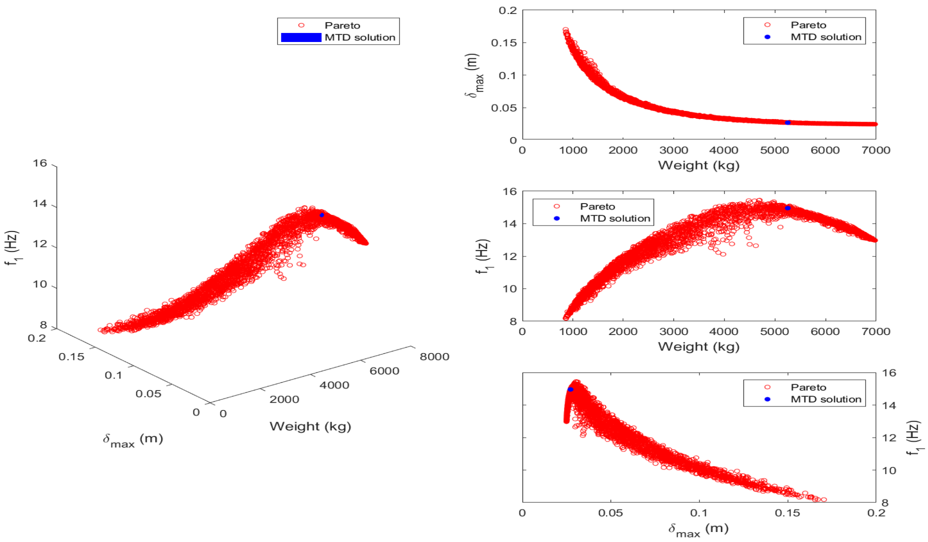

Figure 3. Solution extraction via MTD - w_1 = 0.2, w_2 = 0.7 and w_3 = 0.1.

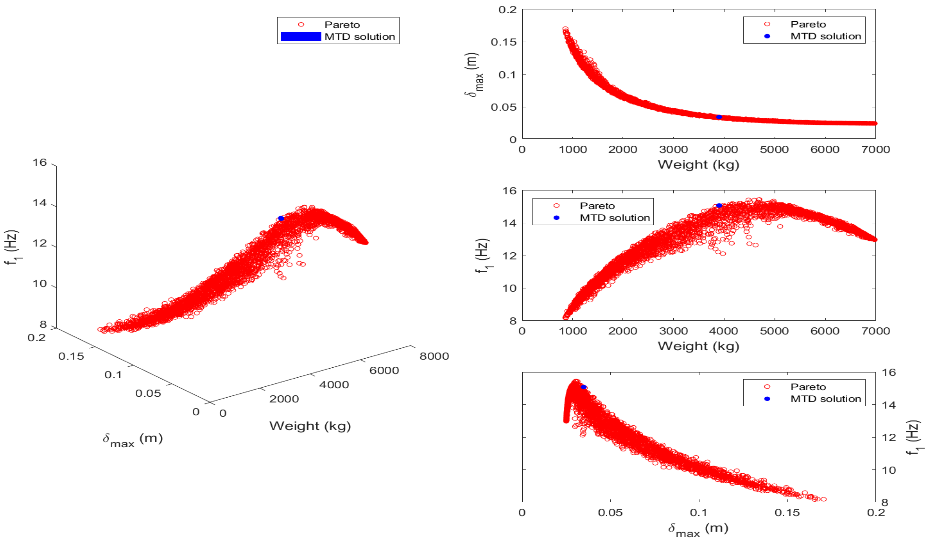

Figure 4. Solution extraction via MTD - $w_1 = w_2 = w_3 = 1/3$.

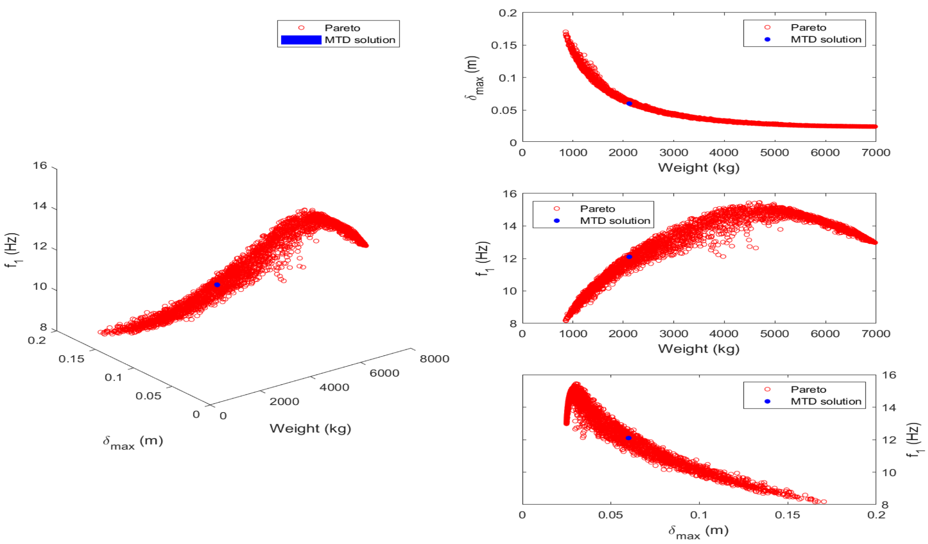

Figure 5. Solution extraction via MTD - $w_1 = 0.7$, $w_2 = 0.2$ and $w_3 = 0.1$.

On the other hand, one can observe a stretch where the first natural frequency of vibration increases, whereas the structure's weight decreases and a stretch where the first natural frequency decreases with the structure's weight increase, which is not intuitive. This same remark can be seen on the bottom right graph of the same figure, but this time for maximum displacement versus the first natural frequency.

The solution extraction via MTD showed also intuitive results: when changing the DM preferences, the position of the selected solution changed on an expected way in the three objective function space. One can observe similar the first natural frequency of vibration but different weight and maximum displacement (around 14 Hz) for both chosen structures when w=$[0.2 \;\; 0.7 \;\; 0.1]$ and w=$[1/3 \;\; 1/3 \;\; 1/3]$ (Figures 3 and 4). The third objective weight defines a plane on another two-objective space where few solutions might exist, possibly leading to an unfair and inaccurate extraction.

6 Conclusions and future works

It can be concluded that the proposed methodology worked for this kind of problem, although there's still no comparison in literature. The Pareto fronts in most of the cases are intuitive and seemed to have a smooth geometry (i.e., no spikes). The extraction method also seemed to work and showed intuitive results when changing the weights (importance).

As future works, new metrics to evaluate the performance of this methodology will be used, and comparison with other experiments found in the literature is going to be carried out. Also, an improvement in MTD extraction for three objective function surfaces will be studied, and large-scale multi-objective structural optimization will be investigated.

Acknowledgements

This study was supported in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior – Brasil (CAPES) – Finance Code 001, and Conselho Nacional de Desenvolvimento Científico e Tecnológigo (CNPq) - Finance Codes 306186/2017, 142266/2019-1 and 306069/2016-4. The authors also thank the Civil Engineering Program of the COPPE/Federal University of Rio de Janeiro, Brazil.

Authorship statement

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References

[1] Reyes-Sierra, M. & Coello, C. A. C., 2006. Multi-objective particle swarm optimizers: A survey of the state-of-the-art. *International Journal of Computational Intelligence Research.*, vol. 2, pp. 287–308.
[2] Coello, C. A. C., Lamont, G. B., & van Veldhuizen, D. A., 2007. *Evolutionary Algorithms for Solving Multi-Objective Problems*. Springer.
[3] Mokarram, V. & Banan, M. R., 2018. A new pso-based algorithm for multi-objective optimization with continuous and discrete design variables. *Structural and Multidisciplinary optimzation*, vol. 57, pp. 509–533.
[4] Kaveh, A. & Mahdavi, V. R., 2019. Multi-objective colliding bodies optimization algorithm for design of trusses. *Journal of Computational Design and Engineering*, vol. 6, pp. 49–59.
[5] Tejani, G. G., Kumar, S., & Gandomi, A. H., 2019. Multi-objective heat transfer search algorithm for truss optimization. *Engineering With Computers*, vol. 1.
[6] Moradi, A., Nafchib, A. M., & Ghanbarzadeha, A., 2015. Multi-objective optimization of truss structures using the bee algorithm. *Scientia Iranica*, vol. 22, pp. 1789–1800.
[7] Kaveh, A. & Laknejadi, K., 2011. A hybrid multi-objective optimization and decision making procedure for optimal design of truss structures. *IRANIAN JOURNAL OF SCIENCE AND TECHNOLOGY*, vol. 35, pp. 137–154.
[8] Bathe, K.-J., 2006. *Finite element procedures*. Prentice Hall, Pearson Education, Inc.

[9] McGuire, W., Gallagher, R. H., & Ziemian, R. D., 2014. *Matrix structural analysis*. John Wiley & Sons, New York. 2nd Edition.

[10] Parreiras, R. & Vasconcelos, J., 2009. Decision making in multiobjective optimization aided by the multicriteria tournament decision method. *Nonlinear Analysis: Theory, Methods & Applications*, vol. 71, n. 12, pp. e191–e198.

[11] Angelo, J. S., Bernardino, H. S., & Barbosa, H. J., 2015. Ant colony approaches for multiobjective structural optimization problems with a cardinality constraint. *Advances in Engineering Software*, vol. 80, pp. 101–115.

[12] Kukkonen, S. & Lampinen, J., 2005. Gde3: The third evolution step of generalized differential evolution. In *IEEE Congress on Evolutionary Computation (CEC 2005)*, pp. 443–450. IEEE.

[13] Storn, R. & Price, K., 1995. Differential evolution a simple and efficient adaptive scheme for global optimization over continuous spaces. Tech. Rep. 95-012, Univ. of California, Berkeley, CA.

[14] Storn, R. & Price, K., 1997. Differential evolution a simple and efficient adaptive scheme for global optimization over continuous spaces. *Journal of Global Optimization*, vol. 11(4), pp. 341–359.

[15] Raquel, C. R. & Naval Jr, P. C., 2005. An effective use of crowding distance in multiobjective particle swarm optimization. In *Proceedings of the 7th annual conference on Genetic and evolutionary computation*, pp. 257–264. ACM.

[16] Barbosa, H. J. C. & Lemonge, A. C. C., 2002. An adaptive penalty scheme in genetic algorithms for constrained optimization problems. In *GECCO'02: Proceedings of the Genetic and Evolutionary Computation Conference.*, pp. 287–294, New York.