XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Static Analysis and Optimization of stiffened plates under pressure loading

Beatriz F. Souza[1], Carla T. M. Anflor[1], Daniel O. Fernandes[1], Ariosto B. Jorge[1], Besim Baranoglu[2],

[1]*Group of Experimental and Computational Mechanics, University of Brasilia*
Gama Campus, Gama, 72.405-610, Brasilia/DF, Brazil
bfesouza@gmail.com, anflorgoulart@gmail.com, danieldeoliveirafernandes@gmail.com,
ariosto.b.jorge@gmail.com
[2]*Metal Forming Center of Excellence, Atilim University*
Atilim University, 06836, Ankara, Turkey
besim.baranoglu@atilim.edu.tr

Abstract. In this work, the Exhaustive Search Method (ESM) was used to optimize a plate structure with stiffeners. The main goal is to establish a numerical simulation procedure in order to identify the optimal geometry for stiffened steel plates, where the objective function to be minimized is the maximum displacement, keeping the total reference volume constant. The search process is allowed to vary the number, thickness, and height of the stiffeners. The optimization and analysis of mechanical behavior were performed by ESM combined with ANSYS Mechanical APDL software which is based on the Finite Element Method (FEM). The stiffened plates were calculated to transform 40% of the total volume into stiffeners. The optimal geometry resulted in a reduction of 98.34% in the maximum displacement if compared to the reference plate showing the good results obtained by the proposed methodology.

Keywords: Stiffened Plates, Optimization, Exhaustive Search, Finite Elements.

1 Introduction

Stiffened plates are widely used in structures with high load capacity such as aircraft, ships, missiles, submersibles, etc. Generally, weight reduction and stiffness conservation are an essential factor in such structures (Chakraborty *et al.* [1]). Stiffeners are long profiles fixed to plate structures in the longitudinal and transverse directions to stiffen them to prevent off-plane deformations. In the aerospace industry, they are used in the construction of aircraft fuselage and wings, while in naval architecture they are used in the construction of ship hulls. Stiffened plates/shells are also used in the construction of bridges, buildings, storage tanks, off-shore structures, and petrochemical processing facilities (Bedair [2]). Structures with stiffeners have greater load capacity with increased stiffness. Without the use of stiffeners, these factors could only be improved by increasing the thickness of the material which would increase the weight of the structure.

The optimization of stiffened plates has been studied by many researchers using different optimization methods (Belblidia *et al.* [3], Wang *et al.* [4], Mensinger [5], Marcelin [6]). Kalassi and Marcelin [7] used the genetic algorithm to optimize the position of the stiffeners using as objective function the deflection in a plate with known load and boundary conditions. Putra et al. [8] used the hybrid genetic algorithm which combines the Genetic Algorithm and subsequent optimization methods to develop a minimum mass design of the stiffened plates used in ship-structures. Tanaka and Bercin [9] evaluate the static bending analysis of stiffened plates with a variety of boundary conditions and a number of stiffeners of the open arbitrary cross-section by the Boundary Element Method. Liu *et al.* [10] proposed the adaptive Morphogenesis Algorithm inspired by the self-adaption of leaf venation by which an effective stiffener layout design that matches the load path can be generated progressively, this endows stiffeners with a capacity of inserting new nodes into an existing Finite Element Model. Helbig *et al.*[11] studied the buckling phenomenon in perforated steel thin plates by Exhaustive Search Method (ESM) to determine which geometries lead to superior mechanical behavior.

In this work, the optimization method used was the ESM, which is one of the oldest methods for solving computational problems; this method generates and inspects all solution configurations in a large search space containing desired solutions (Nievergelt [12]). It is worth remembering that if the search space is too large, the computational time may become unviable and other optimization methods could be used instead. The present article analyzed different geometric configurations that seek to increase the stiffness in the plate. The mechanical behavior was analyzed by ESM combined with ANSYS Mechanical APDL software which is based on the Finite Element Method (FEM).

This study seeks to identify the optimal geometry by ESM that generates the lowest maximum displacement by varying the number, thickness and height of the stiffeners, while keeping the total reference volume constant. This paper is organized as follows: a brief review about Theory of Thin Plates is introduced in Section 2, the ESM is introduced in Section 3, Section 4 presents the numerical results and Section 5 presents the conclusions of this work.

2 A brief review on the Classical Theory of Thin Plates

Exact stress analysis of a thin plate subjected to normal surface loads requires three-dimensional differential equations; however, this approach generates major mathematical difficulties. For this reason, Kirchhoff's Classical Theory is often used, with sufficiently accurate results obtained. The Classical Theory is formulated in terms of transverse deflections $w\,(x,y)$ for which the governing differential equation is 4th order, requiring only two boundary conditions satisfied at each edge (Szilard [13]). The theory proposes some assumptions that simplify the problem for a two-dimensional analysis.

The governing differential equation of Classical Theory is a 4th order partial equation of the elliptical type known as the nonhomogeneous biharmonic equation:

$$\frac{\partial^4 w}{\partial x^4}+\frac{2\partial^4 w}{\partial x^2 \partial y^2}+\frac{\partial^4 w}{\partial y^4}=\frac{P_z(x,y)}{D}, \tag{1}$$

where P_z is the uniformly distributed transverse load and D is the flexural stiffness of the plate.

The plate problem is solved if a deflected surface expression is found $w(x,y)$ that satisfies the boundary conditions imposed and the governing equation (1), simultaneously. The solution to this bending problem considering the simply supported rectangular plates (Fig.1) was proposed by Navier in 1820 using double trigonometric series. The deflections can be expressed by a double sine series:

$$w(x,y)=\frac{16\mathrm{p}_0 a^4}{D\pi^6}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\left(\frac{\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)}{mn[m^2+(n^2/4\,)]^2}\right), \tag{2}$$

$$m=1,3,5,\ldots \qquad n=1,3,5,\ldots$$

being m and n positive odd integers, p_0 it's constant, a and b are the width and length of the plate, respectively.

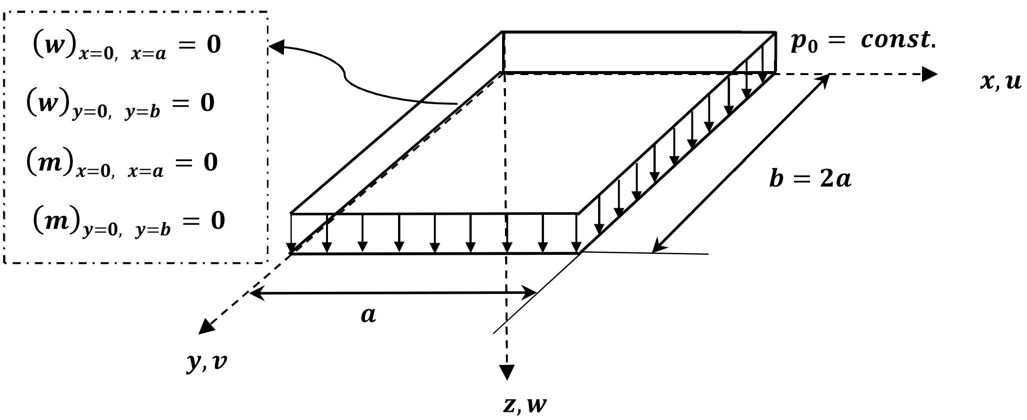


Figure 1: Simply supported rectangular plate

The approach presented previously is applicable only to isotropic plates. In the case of orthotropic plates, such as in this case of the stiffened plates, should use the equation proposed by Huber:

$$D_X \frac{\partial^4 w}{\partial x^4} + 2B \frac{\partial^4 w}{\partial x^2 \partial y^2} + D_Y \frac{\partial^4 w}{\partial y^4} = P_z(x, y), \tag{3}$$

where D_X and D_Y are the rigidities in the x and y directions, respectively and B is effective torsional rigidity of the orthotropic plate. Using the Navier's Method the eq. (3) also can be solved by a double trigonometric series:

$$w(x, y) = \frac{16\mathrm{p}_0 a^4}{D\pi^6} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left(\frac{\sin\left(\frac{m\pi x}{a}\right) \sin\left(\frac{n\pi y}{b}\right)}{mn\left[D_X\left(\frac{m^4}{a^4}\right) + 2B\left(\frac{m^2 n^2}{a^2 b^2}\right) + D_Y\left(\frac{n^4}{b^4}\right)\right]} \right). \tag{4}$$
$$m = 1{,}3{,}5, \dots \qquad n = 1{,}3{,}5, \dots$$

3 Exhaustive Search Method (ESM) for the stiffened plate problem

A flowchart for the ESM is shown in Fig.2. The model starts by generating all possible configurations within the search space for the degrees of freedom initially imposed. Then, some geometric restrictions were imposed and each geometry generated is run by ANSYS and in the post-processing, the design restrictions are imposed to discard unfeasible combinations. When the entire search space is executed, the best geometry is chosen using the proposed criterion.

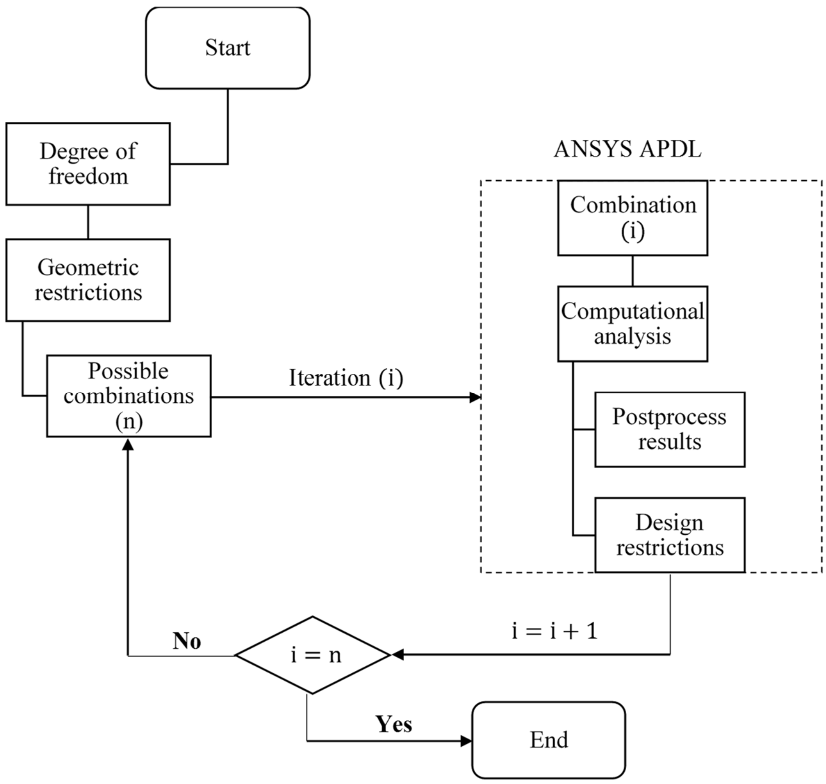


Figure 2: Exhaustive Search Method

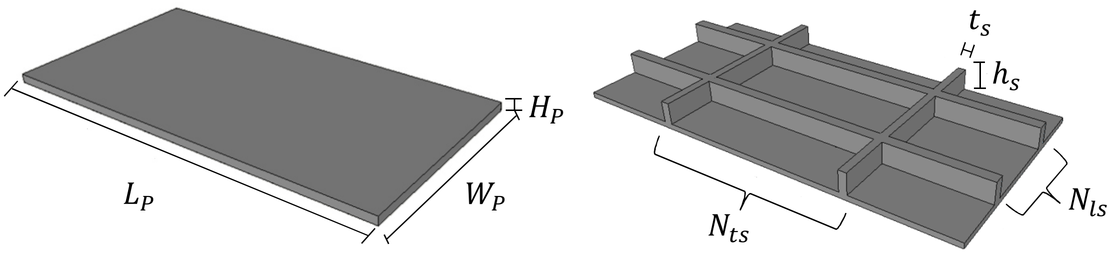
4 Numerical procedures

The ESM (Fig.2) was implemented in MATLAB and ran in parallel with the ANSYS APDL software. The stiffened plates were calculated transforming 40% of the total volume into stiffeners. The objective function used is defined by

$$\begin{gathered} minimize\ w(x,y) \\ subject\ to\colon \phi(N_{ls},N_{ts},h_s,t_s)=\frac{N_{ls}\left(L_p h_s t_s\right)+N_{ts}\left[\left(W_p-N_{ls}t_s\right)h_s t_s\right]}{L_p W_p H_p}=0.4, \\ 1\le N_{ls}\le 10, \\ 1\le N_{ts}\le 10, \\ h_s\le 300, \\ 3\le t_s\le 76, \end{gathered} \tag{5}$$

being ϕ the percentage of the total volume of the reference plate (Fig.3a) that was turned into stiffeners. N_{ls} is the number of longitudinal stiffeners, N_{ts} is the number of the transversal stiffeners. L_P, W_P and H_P are dimensions of the reference plate 2000x1000x20 mm. h_s and t_s are the height and thickness of the stiffeners, respectively.

The degrees of freedom in this study are N_{ls}, N_{ts}, h_s and t_s (Fig.3b) as previously studied by Queiroz *et al.* [14]. The total volume is kept constant in all possible configurations and plate and stiffeners are made of the same material, in this case, ASTM A-36 steel with Young's modulus equal to 200 GPa, Yield strength equal to 250 MPa and Poisson coefficient equal to 0.3. The boundary conditions used were from all edges simply-supported. The applied transverse loading was of the 10 kPa uniform pressure load type.



(a) (b)

Figure 3: a) Reference Plate b) Stiffened Plate ($N_{ls} = N_{ts} = 2$),

The structural analysis is performed using ANSYS APDL by FEM. The element used in modeling the plate and the stiffeners was the SHELL63, which has six degrees of freedom at each node: translations in the nodal x, y, and z directions and rotations about the nodal x, y, and z axes and stress stiffening and large deflection capabilities are included [15].

As design restrictions, the maximum displacement value (w_{max}) was set to a 4 mm limit and the maximum equivalent stress (S_{max}) was limited to 208 MPa, avoiding configurations with excessive displacement and ensuring safety against the yielding of the material.

The mesh convergence was performed by reference plate and the true error (E) was calculated by

$$E=\Phi-\phi, \tag{6}$$

being Φ the analytical solution (eq. 2) and ϕ is the numerical solution by ANSYS. In eq. (6), the error of the variable of interest (w_{max}), is calculated in relation to the analytical solution for each mesh as the model is discretized. The model discretization considered all elements (plate and stiffeners) quadrilateral and for each mesh performed the element length was defined by

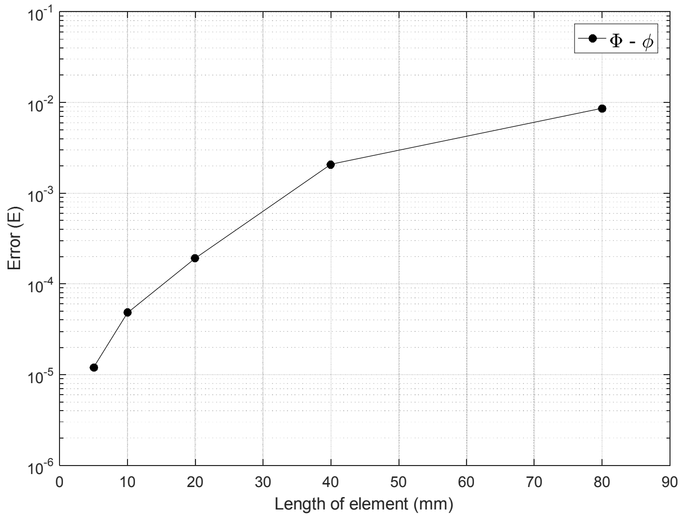
$$\Delta_{h(i)}=\frac{\Delta_{h(i-1)}}{2}, \tag{7}$$

where Δ_h is the element length and i is the iteration number of mesh.

5 Results

5.1 The mesh convergence

The convergence results are shown in Fig.4. For the study of convergence, 5 meshes with different element lengths (Δ_h) were used.



(a)

Figure 4: Mesh convergence for plates

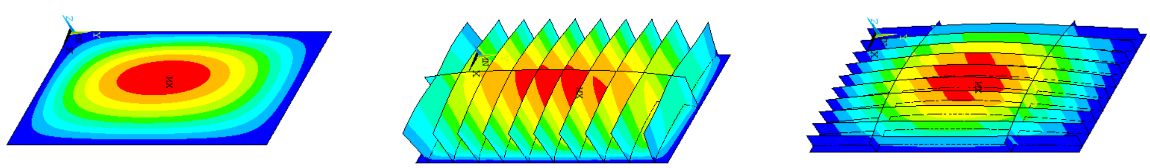
Evaluating the data, it can be verified that for Δ_h equal to 5 mm the numerical result had an error of approximately 10^{-5}. In this study was chosen Δ_h=10 mm, which has less accuracy, but the computational time is more suitable for the type of optimization proposed.

5.2 The ESM results

The exhaustive search was performed considering all possible combinations and resulted in a total of 670 iterations. After obtaining all possible configurations, the optimal geometry can then be defined. Table 1 presents a comparison between the reference plate (Fig.5a), the optimized geometry (Fig.5b) and the non-optimized geometry (Fig.5c). The data shows the relevant influence of the degrees of freedom on the mechanical behavior of the structure.

Table 1: Comparison of results

	h_s/t_s	N_{ls}	N_{ts}	w_{max} (mm)	S_{max} (MPa)
Reference Plate	-	-	-	0.6912	15.2527
Optimal Geometry	53.5565	1	10	0.0115	16.9777
Non-optimized Geometry	11.4469	10	2	0.2608	31.7665



(a) (b) (c)

Figure 5: Results of ESM: a) Reference Plate, b) Optimized geometry and c) Non-optimized geometry

The results show that the maximum displacement decreases as the h_s/t_s ratio increases, as previously verified by Queiroz *et al*. [14]. This mechanical behavior occurs because of the increase in the moment of inertia when $h_s\ /t_s$ increases. The optimal geometry of the stiffened plate allowed a reduction of 98.34% in the maximum displacement when compared to the reference plate. Comparing the optimal geometry with the non-optimized geometry, the influence of a good configuration on the maximum stresses is even more evident; one can note that the displacement decreased by 95.59% and the maximum stress was reduced by 46.56%. Besides that, the analysis of results shows that, for the case of a bad stiffened plate configuration, although the maximum displacement may decrease, stresses much higher than in the case of the reference plate were generated. This greatly justifies the optimization studies evaluating the stress and displacement in stiffened plates.

6 Conclusions

The present work used the exhaustive search to develop the optimization of stiffened plates by analyzing the maximum displacement and the equivalent stress in each configuration while keeping the total reference volume constant and varying the number, thickness and height of the stiffeners. The results showed that the presence of stiffeners decreases the maximum deflection in the plates. Another finding from the results of this work was that bad configurations may generate high equivalent stresses that can cause local material yielding. In addition, one can see from the results presented that the displacement decreases as the $h_s\ /t_s$ ratio increases. Finally, the optimal configuration was determined and compared with the reference plate and with the non-optimized geometry found. The results obtained show a reduction of 98.34% in the maximum displacement when compared to the reference plate. Also, the displacement was decreased by 95.59% and the maximum stress was reduced by 46.56% when compared with the non-optimized configuration.

Acknowledgements. This study was financed in part by the Coordenação de Aperfeiçoamento de Pessoal de Nível Superior-Brasil (CAPES, Brazilian Council for Scientific and Technological Development (CNPq) and Research Support Foundation of Federal District (FAPDF). The authors are grateful to the Group of Experimental and Computational Mechanics at UnB (UnB-FGA/GMEC) for providing experimental and computational resources and for making possible the work development.

Authorship statement. The authors hereby confirm that they are the solely liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] S. Chakraborty, M. Mukhopadhyay, and O. P. Sha, "Determination of physical parameters of stiffened plates using genetic algorithm," *J. Comput. Civ. Eng.*, vol. 16, no. 3, pp. 206–221, 2002.

[2] O. Bedair, "Analysis and Limit State Design of Stiffened Plates and Shells : A," vol. 62, no. March, pp. 1–16, 2009.

[3] F. Belblidia, S. M. B. Afonso, E. Hinton, and G. C. R. Antonino, "Integrated design optimization of stiffened plate structures," *Eng. Comput. (Swansea, Wales)*, vol. 16, no. 8, pp. 934–951, 1999.

[4] B. Wang, K. Tian, P. Hao, Y. Cai, Y. Li, and Y. Sun, "Hybrid analysis and optimization of hierarchical stiffened plates based on asymptotic homogenization method," *Compos. Struct.*, vol. 132, pp. 136–147, 2015.

[5] M. Mensinger, J. Ndogmo, and R. Parra, "Optimization of stiffened plates for steel bridges based on Eurocode 3 Part 1-5 using genetic algorithms," *Steel Constr.*, vol. 4, no. 1, pp. 29–33, 2011.

[6] J. L. Marcelin, "Genetic optimization of stiffened plates and shells," *Int. J. Numer. Methods Eng.*, vol. 51, no. 9, pp. 1079–1088, 2001.

[7] A. Kallassy and J. L. Marcelin, "Optimization of stiffened plates by genetic search," *Struct. Optim.*, vol. 13, no. 2–3, pp. 134–141, 1997.

[8] G. L. Putra, M. Kitamura, and A. Takezawa, "Structural optimization of stiffener layout for stiffened plate using hybrid GA," *Int. J. Nav. Archit. Ocean Eng.*, no. March, pp. 1–10, 2019.

[9] M. Tanaka and A. N. Bercin, "Static bending analysis of stiffened plates using the boundary element method," *Eng. Anal. Bound. Elem.*, vol. 21, no. 2, pp. 147–154, 1998.

[10] H. Liu, B. Li, Z. Yang, and J. Hong, "Topology optimization of stiffened plate/shell structures based on adaptive morphogenesis algorithm," *J. Manuf. Syst.*, vol. 43, pp. 375–384, 2017.

[11] D. Helbig, C. C. C. Silva, M. D. V Real, E. D. Santos, L. A. Isoldi, and L. A. O. Rocha, "Study About Buckling Phenomenon in Perforated Thin Steel Plates Employing Computational Modeling and Constructal Design Method,"

Lat. Am. J. Solids Struct., vol. 13, pp. 1912–1936, 2016.

[12] J. Nievergelt, "Exhaustive Search , Combinatorial Optimization and Enumeration : Exploring the Potential of Raw Computing Power Exhaustive Search , Combinatorial Optimization and Enumeration :," no. January 2000, 2015.

[13] R. Szilard, *Theories and Applications of Plate Analysis: Classical, Numerical and Engineering Methods*, vol. 57, no. 6. 2004.

[14] J. P. T. P. de Queiroz *et al.*, "Geometric evaluation of stiffened steel plates subjected to transverse loading for naval and offshore applications," *J. Mar. Sci. Eng.*, vol. 7, no. 1, pp. 1–12, 2019.

[15] ANSYS Inc, "Elements Reference ANSYS Release 11.0," 2007.