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A tri-objective truss design optimization using a Multi-objective Craziness based Particle Swarm Optimization

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Abstract. Recently there has been a growing interest in evolutionary multi-objective optimization algorithms due to its applicability in problems from several fields, especially those of applied engineering and mathematics. In this context, there are many of algorithms applied to these types of problems, such as differential evolution, genetic algorithms, particle swarm, among others. This paper deals with a multi-objetive sizing, discrete or continuous, strucutural optimization problem with respect to: i) the minimization of the mass of truss structures; ii) the maximization of the first natural frequency of vibration and iii) the minimization of the maximum displacement, considering stress constraints. A multi-objective particle swarm algorithm called Multi-objective Craziness based Particle Swarm Optimization (MOCRPSO) is the search algorithm adopted here and an Adaptive Penalization Method (APM), which has been successfully applied to solving mono-objective optimization problems, is used to handle the constraints. Some computational experiments are analyzed, presenting very interesting results providing pareto fronts between the objectives.

Keywords: Multi-objective Truss Optimization, Particle Swarm Optimization, Multiple natural frequencies of vibration

1 Introduction

Commonly in structural design of trusses, for example, designers are interested in finding the minimal weight of the structure subject to displacements, axial stresses, critical load factors, and so on. Besides these constraints, optimizing a truss structure can be formulated considering multiple and conflicting objective functions, for instance, to minimize the weight of the structure and its maximum nodal displacement. Also, it is common to search for light and economic structures that meet safety criteria, which leads to constraints such as minimum values for natural frequencies of vibration. For most applications staying away from excitations frequencies may be much more important.

Evolutionary Algorithms (EAs), especially the population-based metaheuristics, have become attractive approaches used to solve multi-objective optimization problems in several areas. Many metaheuristics have been developed by researchers and the most popular ones are GA, PSO, HS, ACO, and so on [1]. In the field of structural optimization, many successful applications of these algorithms have been reported in literature [2–4].

Particle Swarm Optimization (PSO) is a popular metaheuristic introduced by Eberhart & Kennedy [5] which provides computational models based on the concept of collective intelligence. A modified version of the algorithm called Craziness based Particle Swarm Optimization (CRPSO), proposed by Kar *et al.* [6], is used in this paper as the search engine. The multi-objective version of the CRPSO, called Multi-objective Craziness Particle Swarm Optimization (MOCRPSO), incorporates a crowding distance mechanism, non-dominated solutions, and an external archive, together with a mutation operator based on the MOPSO-CD developed by Raquel & Naval [7].

In this paper, a tri-objective structural optimization problem is formulated considering conflicting objectives: i) the minimization of the mass of truss structures; ii) the maximization of the first natural frequency of vibration, and iii) the minimization of the maximum displacement. The axial stresses are the constraints. A Pareto front composed by the non-dominated solutions is searched as the expected solution. The constrained optimization problem is replaced by an unconstrained problem by introducing an Adaptive Penalty Method (APM) proposed by Lemonge & Barbosa [8]. A 10-bar truss using continuous and discrete design variables is analyzed in the computational experiments.

The remainder of the paper is organized as follows. Section 2 describes the formulation of the multi-objective structural optimization problem. Section 3 presents the constraint-handling technique and the MOCRPSO algorithm. The computational experiments are presented in Section 4. Finally, the paper ends with conclusions in Section 5.

2 Multi-objective optimization

In engineering, most decision problems are Multi-objective Optimization Problems (MOP) [9]. A MOP has some objective functions that are to be minimized (or maximized) simultaneously. Although single-objective structural optimization problems are commonly found in the literature; the formulation of optimization problems involving multiple objectives appears naturally due to the presence of two or more conflicting objectives.

The structural multi-objective problem discussed in this paper is formulated as:

$$\begin{array}{lc} \min\ W(\mathbf{x}) \quad \text{and} \quad \max\ \omega_1(\mathbf{x}) \quad \text{and} \quad \min\ \text{maximum}(u_j(\mathbf{x})),\ j=1,...,n_{dof}, & \\ \text{subject to} \qquad\qquad\qquad\qquad \sigma_i(\mathbf{x}) \leq \overline{\sigma} & \\ \qquad\qquad\qquad\qquad\qquad\qquad \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U, & \end{array} \tag{1}$$

where $W(\mathbf{x})$ is the weight of the structure, $\omega_1(\mathbf{x})$ is the first natural frequency of vibration, $u_j(\mathbf{x})$ is the displacement at the j-th node, n_{dof} is the number of degree of freedom, and $\sigma_i(\mathbf{x})$ is the axial stress at the i-th bar. The design variables are $\mathbf{x} = \{A_1, A_2, ..., A_N\}$, where A_i are the sizing design variables indicating the cross-sectional areas of the N bars (continuous or discrete) that must be in the lower $\mathbf{x}^L$ and upper $\mathbf{x}^U$ bounds. $W(\mathbf{x})$ is written as:

$$W(\mathbf{x}) = \sum_{i=1}^{N} \rho A_i L_i, \tag{2}$$

where ρ is the specific mass of the material and L_i is the length of the i-th bar of the structure. $\omega_1(\mathbf{x})$ is obtained by the evaluation of the eigenvalues of the matrix

$$\Big[(\omega_{m_f}^2 \times [M]) + [K]\Big] = 0, \tag{3}$$

where $[M]$ is the mass matrix and ω_{m_f} are the equivalent eigenvalues with respect to the m_f natural frequencies of vibration of the structure [10]. The nodal displacements $\{u\}$ are obtained by the equilibrium equation for a discrete system of bars, which is written as:

$$[K]\{u_j(\mathbf{x})\} = \{p\}, \tag{4}$$

where $\{p\}$ are the load components.

3 Methods

3.1 Constraint-Handling technique

The constrained structural optimization problem is replaced by an unconstrained optimization problem adding a penalty function. An analysis of relevant types of constraint-handling techniques that have been adopted with nature-inspired algorithms is presented in Mezura-Montes & Coello [11] and Barbosa *et al.*. [12].

An Adaptive Penalty Method (APM) has been proposed by Barbosa & Lemonge [8] to handle constraints. This method is adopted in this study to handle all the constraints of the test-function in the numerical experiments. Considering each objective separately, the fitness function $F(x)$ can be written as

$$F(x) = \begin{cases} f(x), & \text{if} \quad x \quad \text{is feasible} \\ \bar{f}(x) + \sum_{j=1}^{n_c} k_j v_j(x), & \text{otherwise} \end{cases} \tag{5}$$

and

$$\bar{f}(x) = \begin{cases} f(x), & \text{if} \quad f(x) > \langle f(x) \rangle \\ \langle f(x) \rangle, & \text{if} \quad f(x) \leq \langle f(x) \rangle, \end{cases} \tag{6}$$

where $\langle f(x) \rangle$ is the average value of the objective function of the current population and nc is the number of constraints. The penalty parameter k_j is defined as

$$k_j = |\langle f(x) \rangle| \frac{\langle v_j(x) \rangle}{\sum_{l=1}^{n_c} [\langle v_l(x) \rangle]^2}, \tag{7}$$

where $\langle v_j(x) \rangle$ means the violation of the j-th constraint averaged over the current population considering only unfeasible individuals.

3.2 Multi-objective algorithm

Swarming behavior is a collective behavior found in birds, fish, bees, and other types of insects. Life in society offers more chances of survival as it facilitates hunting and gathering food, reduces the possibility of attack by predators, among others [13]. Particle Swarm Optimization (PSO) is a population algorithm, introduced by Eberhart & Kennedy [5], inspired by the social behavior of birds flocking in search of food and widely used in the literature. PSO has been successfully applied to different types of structural optimization problems [14, 15].

A modified version of the standard PSO called Craziness-based Particle Swarm Optimization (CRPSO) and proposed by Kar *et al.* [6] is used in this paper. The velocity expression v_i and the position x_i is written as follows [6]:

$$\begin{aligned} v_j^{(i)}(t+1) = \quad & r_2 \cdot sign(r_3) \cdot v_j^{(i)}(t) + (1 - r_2)c_1 \cdot r_1(x_{pbest}^{(i)} - x_j^{(i)}) + (1 - r_2) \cdot c_2 \cdot \\ & (1 - r_1)(x_{gbest} - x_j^{(i)}) + P(r_4) \cdot sign2(r_4) \cdot v_j^{craziness} \end{aligned} \tag{8}$$

$$x_j^{(i)}(t+1) = x_j^{(i)}(t) + v_j^{(i)}(t+1) \tag{9}$$

where r_1, r_2, r_3 and r_4 are the random parameters uniformly taken from the interval [0,1), $sign(r_3)$ is a function defined as

$$sign(r_3) = \begin{cases} -1, & r_3 \leq 0.5 \\ 1, & r_3 > 0.5, \end{cases} \tag{10}$$

$v_j^{craziness}$, the craziness velocity, is a user defined parameter from the interval [v^{min}, v^{max}] and $P(r_4)$, $sign2(r_4)$ are defined, respectively, as

$$P(r_4) = \begin{cases} 1, & r_4 \leq Pcr \\ 0, & r_4 > Pcr, \end{cases} \tag{11}$$

$$sign2(r_4) = \begin{cases} -1, & r_4 \geq 0.5 \\ 1, & r_4 < 0.5, \end{cases} \tag{12}$$

and Pcr is a predefined probability of craziness. Although the parameter Pcr is fixed, $P(r_4)$ is defined every time the velocity is calculated.

The multi-objective version of CRPSO algorithm entitled Multi-objective Craziness based Particle Swarm Optimization (MOCRPSO) is based on the MOPSO-CD[1] algorithm proposed by Raquel & Naval [7]. The new velocity of the MOCRPSO is written as

$$\begin{aligned} v_j^{(i)}(t+1) = \quad & r_2 \cdot sign(r_3) \cdot v_j^{(i)}(t) + (1 - r_2)c_1 \cdot r_1(x_{pbest}^{(i)} - x_j^{(i)}) + (1 - r_2) \cdot c_2 \cdot \\ & (1 - r_1)(A[gbest] - x_j^{(i)}) + P(r_4) \cdot sign2(r_4) \cdot v_j^{craziness}. \end{aligned} \tag{13}$$

The algorithm uses the concept of an external archive ARQ to store the non-dominated solutions. Also, some mechanisms are incorporated in MOCRPSO to maintain the diversity of non-dominated solutions [7]: mutation, crowding distance, and global best selection. An operator of a mutation is used to increase diversity in the swarm. It helps prevent premature convergence due to existing local Pareto fronts in some optimization problems. The

[1] https://sites.google.com/site/prosnaval/codes

crowding distance is incorporated into the algorithm, specifically on the global best selection and in the external archive. The global best of the particles is selected from those non-dominated solutions with the highest crowding distance values. Whenever the archive is full, crowding distance is again used to select the solution that should be replaced in the external archive. More details can be found in [7, 16].

4 Computational experiments

The computational experiment refers to a well-known structural optimization problem, named as a 10-bar truss considering continuous and discrete variables. The first objective is to minimize the weight of the structure, the second is to maximize the first natural frequency of vibration, and the third is to minimize the maximum displacement, considering axial stresses as constraints.

The following section presents a description of the problem presented above. For this test-problem, the initial population was randomly generated considering the maximum number of objective function evaluations is 50000 (50 particles and 1000 generations), the number of independent runs is 100, and all of the presented solutions are rigorously feasible. The parameters of the MOCRPSO are: $c_1 = c_2 = 2.05$, $v^{craziness} = 0.001$, $Pcr = 0.5$, global neighborhood topology. The external file limit $ARQ = 500$. The code was developed using C language and the structure was analyzed by the Finite Element Method (FEM) [17] during the evolutionary process.

4.1 The 10-bar truss

The 10-bar truss [18], illustrated in Fig. 1, has ρ = 0.1 lb/in^3 and E = 10^4 ksi. Vertical downward loads of 100 kips are applied at nodes 2 and 4, and the stress in each bar is limited to $\pm$ 25 ksi. A non-structural mass of 1000 lb is attached to the free nodes. For the continuous case, the lower and upper bounds for the cross-sectional areas are defined by $[0.1; 40]$ (in^2). For the discrete case, the values of the cross-sectional areas are chosen from the set (in^2): $\{$1.62, 1.80, 1.99, 2.13, 2.38, 2.62, 2.63, 2.88, 2.93, 3.09, 3.13, 3.38, 3.47, 3.55, 3.63, 3.84, 3.87, 3.88, 4.18, 4.22, 4.49, 4.59, 4.80, 4.97, 5.12, 5.74, 7.22, 7.97, 11.50, 13.50, 13.90, 14.20, 15.50, 16.00, 16.90, 18.80, 19.90, 22.00, 22.90, 26.50, 30.00, 33.50$\}$, resulting in 42 options.

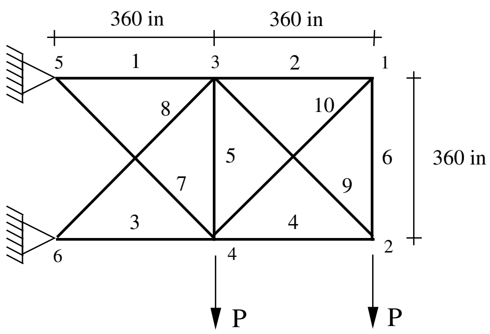


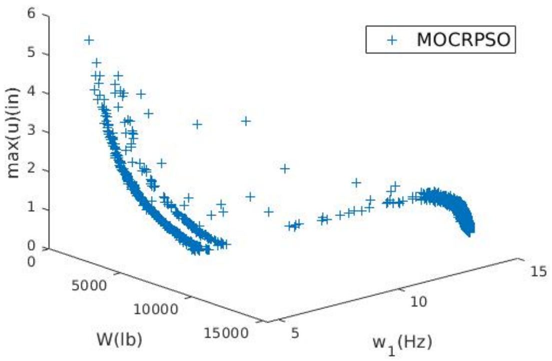
Figure 1. 10-bar truss, taken from [19].

4.2 Results and discussions

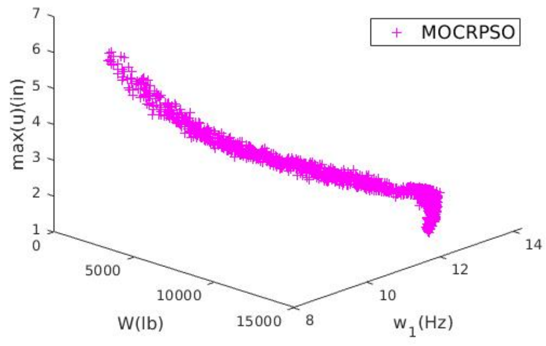
Figure 2(a) and (b) presents the Pareto front of the 10-bar truss continuous and discrete case. No comparison is made, since to the best of our knowledge, no results are found in the literature comparable with those obtained in this study.

However, an analysis takes into account all the information obtained by the Pareto fronts was performed. The Decision Maker (DM) has a nontrivial task of extracting a solution from the Pareto set. Based on that, a tournament-based method that ranks the best and the worst possible solutions in the Pareto set according to objectives and preferences (weights) established by the DM was proposed by Parreiras & Vasconcelos [20] and named as Multicreiteria Tournament Decision (MTD) Method. More details and pseudocode for the MTD can be found in [20].

This method is used in this study to find the best solutions, according to some importances. Four decision scenarios are used considering three criteria: (i) the weight, (ii) the first natural frequency, and (iii) the maximum



(a) Continuous case



(b) Discrete case

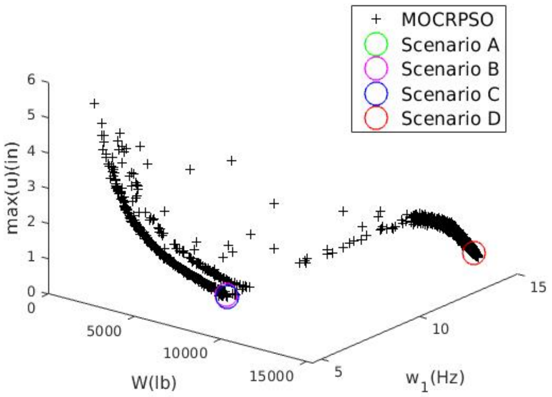
Figure 2. Pareto front of the MOCRPSO for the 10-bar truss.

displacement. The scenarios are described as follows:

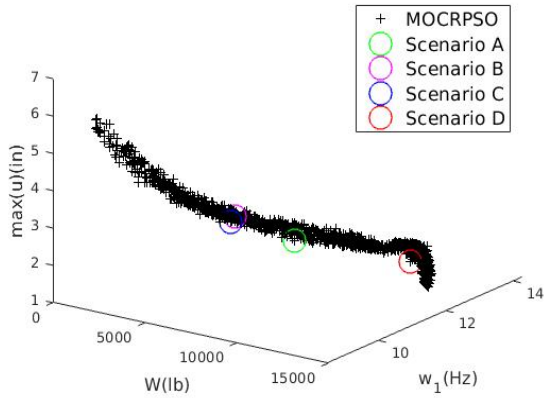
- Scenario A: all criteria have the same importance, i.e. $(w_1, w_2, w_3) = (0.3333, 0.3333, 0.3333)$.
- Scenario B: criterion (i) is the most important and criteria (ii) and (iii) have the same importance, i.e. $(w_1, w_2, w_3) = (0.6, 0.2, 0.2)$.
- Scenario C: criterion (ii) is the most important and criteria (i) and (iii) have the same importance, i.e. $(w_1, w_2, w_3) = (0.2, 0.6, 0.2)$.
- Scenario D: criterion (iii) is the most important and criteria (i) and (ii) have the same importance, i.e. $(w_1, w_2, w_3) = (0.2, 0.2, 0.6)$.

Figure 3(a) and (b) show the non-dominated solutions for the 10-bar truss continuous and discrete cases, respectively. The large circle in this figure represents the solutions extracted by the MTD method corresponding to each scenario. It is possible to observe the effect of the importance of w in the MTD results. In Fig. 3(a), with $w_3 = 0.6$, in scenario D, it leads to the lowest value for the displacement and, on the other hand, the highest values for the weight and the first frequency. In scenarios A, B, and C, the three values found for the objective functions are almost identical.

For the discrete case, in Fig. 3(b), with the importance $w_1 = 0.6$ for scenario B, and $w_2 = 0.6$ for scenario C, all the objective functions values are the same. When $w_3 = 0.6$, in scenario D, it leads to the lowest value for the displacement and the highest values for the weight and the first frequency, as also occurred in the continuous case. Moreover, when all criteria have the same importance, in scenario A, the three objective functions have intermediate values compared to the other scenarios. Finally, the values of the three objective functions and their respective design variables are presented in Table 1.



(a) Continuous case



(b) Discrete case

Figure 3. MTD solutions according to four different scenarios for the 10-bar truss.

Table 1. Design variables (dv) and objective function values of the MTD solutions (Scenarios (Sc.) A, B, C, and D) of the 10-bar truss. $W(\mathbf{x})$ in lb, $\omega_1(\mathbf{x})$ in Hz, and $u(\mathbf{x})$ in inches.

	Continuous case				Discrete case			
dv	Sc. A	Sc. B	Sc. C	Sc. D	Sc. A	Sc. B	Sc. C	Sc. D
A_1	40.00	40.00	40.00	40.00	14.2	11.50	11.50	14.20
A_2	0.10	0.10	0.10	30.54	1.62	1.62	1.62	4.18
A_3	40.00	40.00	40.00	40.00	11.5	7.97	5.74	14.20
A_4	40.00	40.00	40.00	40.00	7.97	4.59	4.97	14.20
A_5	0.10	0.10	0.10	0.10	1.62	1.62	1.80	1.62
A_6	0.10	0.10	0.10	36.43	1.62	2.38	1.62	7.22
A_7	23.97	23.97	24.52	40.00	4.59	4.49	4.59	14.20
A_8	40.00	40.00	39.93	40.00	7.22	5.12	4.80	14.20
A_9	40.00	40.00	40.00	40.00	14.2	7.22	11.50	14.20
A_{10}	5.18	5.18	8.14	40.00	1.62	1.8	1.62	4.80
$W(\mathbf{x})$	9888.0253	9888.0253	10063.8185	14880.8092	6220.0026	4368.3981	4568.3662	10341.3724
$\omega_1(\mathbf{x})$	4.9340	4.9340	4.8239	13.1817	12.2542	11.5010	11.2640	13.4602
$u(\mathbf{x})$	1.1390	1.1390	1.1393	0.981464	1.8373	2.5864	2.5453	1.2501

5 Conclusions

In the present study, a multi-objective algorithm entitled MOCRPSO has been used to solve structural engineering design problems considering three conflicting objectives. An adaptive penalty method was used to handle the constraints of the constrained optimization problem. The algorithm's performance was evaluated using a 10-bar truss take into account continuous and discrete search spaces.

No studies were found in the literature, since to the best of our knowledge, comparable with those conducted in this study. Hence, an analysis was conducted to evaluate the Pareto set using an MTD method to allow the DM to indicate his/her preferences simulating different criteria. Additionally, the values of the objective functions and their respective design variables obtained by the MTD were presented, indicating the best solutions according to some criteria.

For future works, the proposed algorithm will be applied for solving other test-problems considering large-scale optimization problems.

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