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Optimal Search Space for Topological Optimization of Transmission Line Towers

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Abstract. Due to the vast territorial extension of Brazil and its hydroelectric potential, many power plants are built far from consumption centers and in order to deliver electricity to these centers, extensive transmission lines (TL) are needed. One of the main components of these lines are the steel lattice towers, which end up being repeated several times in a TL. For this reason, an opportunity to reduce costs presents itself through the optimization of these structures, especially when the dimension, geometry and topology of the tower are optimized simultaneously. However, with the inclusion of topological variables an uncertainty arises due to the definition of the search space for the process of optimization. On one hand, if many variables are considered, the convergence may be impaired. On the other hand, if just a few variables are chosen, great possibilities of mass reduction might be left aside. Hence, the present work aims to relate the structure mass and its search space, finding a general rule for templates selection by analyzing the optimization results of eight towers with three design spaces for each, where the difference between these is the number of templates considered.

Keywords: Structural Optimization, Transmission Line Towers, Meta-heuristics

1 Introduction

In countries with large territorial extension and hydroelectric potential as Brazil, where the power plants have to be built far from consuming centers, it is necessary the creation of far-reaching transmission lines (TL) so that the produced energy reaches an electrical substation, which is then distributed to the consumers. In order to achieve this, two options are available: underground and overhead TLs. The former is by far the most expensive and also has some other disadvantages (e.g., difficulty to pinpoint the damage and repair), which makes it unsuitable for lengthy TLs. The latter is the most usual option, but requires structures to support the cable conductors and ground wires, and to suspend them, guaranteeing the minimum electrical clearances required. These functions in an overhead TL are exercised by transmission line towers (TLT), latticed structures typically formed by steel angle sections. In long TLs, it is common the repetition of one or few designs of such towers. As a consequence, an opportunity to reduce costs arises by optimizing the structure, i.e., reducing the mass while maintaining its safety.

In a structural optimization, three types of variables are usually employed: size, shape and topology, where, respectively, the elements cross-sectional areas, nodal position and structure layout are altered. In the context of truss towers optimization, some studies have been performed Taniwaki and Ohkubo [1], Sivakumar et al. [2], Shea and Smith [3], Mathakari et al. [4], Kaveh et al. [5], Guo and Li [6], Noilublao and Bureerat [7], París and Colomínas [8], Gomes and Beck [9], with primary focus in academic research, providing infeasible results for industrial application or lacking common topologies employed by the industry as possibilities. In contrast to these limitations, Souza et al. [10] developed a methodology which divides the structure in main modules that can assume pre-established topologies (templates), creating topology variables that are based in terms of the design practice and feasibility of prototype testing. Besides, size and shape variables were considered as discrete to better suit industry practice. It is also important to highlight that topological optimization showed advantage over only size or size and shape optimization, emphasizing the potential of increasing the search space with topological variables.

On the other hand, the consideration of discrete variables, especially topological ones, creates a design space that is disjoint and non-convex, more difficult to solve than continuous spaces, except in some trivial cases Arora et al. [11]. But the optimization problems presented in this work - which uses TLTs with more than 100 variables and over 10^{100} possibilities for each tower - are far from trivial. Such a huge design space starts to become a problem by impairing convergence and sometimes making it impossible to achieve great results, i.e., high standard deviation. On the other hand, if just a few variables are chosen, great possibilities of mass reduction might be left aside.

In this work, a total of eight vertical self-supported, with single and double circuit, 130 and 230 kV TLTs were optimized. For each tower, it was considered three search spaces, with their main difference being the addition of topological variables. The primary objective here is to find a general rule for templates selection in order to optimize the search space, i.e. reduced computational cost accompanied by great reduction in mass.

2 Optimization problem description

Initially, the information contained in the project - such as the elevations of the legs, extensions, common body and head, groups of bars, openings of the base and head and distance of the conductors to the tower axis - were described in the MATLAB software and based on this, the variables and their lower and upper limits were established, giving sequence to the optimization process.

The selected optimizer employed was an evolutionary algorithm called BSA (Backtracking Search Algorithm). This meta-heuristic was chosen because it showed little sensitivity to the dimension or type of the problem Civicioglu [12] and its success in TLT optimization as seen in Souza et al. [10].

First, the BSA initializes, generating a random population and evaluating it. Then starts the iterations and stops when the maximum number of cycles (*maxcycle*) defined by the user is achieved. Inside the iterations, a historical population *histPop* has a 50% chance of assuming values of the last evaluated population (*pop*) or it is not changed. Next, a mutation process in the *pop* occurs, which is, at the same time, combined (crossover) with the *histPop*, finishing with the evaluation of this recombined population.

BSA pseudo-code

for $iteration = 1, 2, \ldots, maxcycle$ **do**
    SELECTION:
        50% chance of keeping histPop ***or*** *histPop = pop*

    RECOMBINATION (MUTATION + CROSSOVER):
        Mutation of pop and combination with histPop

    OBJECTIVE FUNCTION EVALUATION:
        Evaluates pop
end for

The population was composed of 36 individuals and *maxcycle* is equal to 8,000, totaling 288,000 objective function evaluations (OFEs) per run. After running each search space five times (total of fifteen runs), the "winner" received five more runs. Of all these twenty runs, the one with minimum weight received 2000 more cycles (72000 OFEs). This was done for all the eight TLTs and the results achieved are described in Section 3.

For every function evaluation, the following was done:

1. The TLT was modeled according to the project plus its variations generated by the BSA;
2. Angles between the diagonal bars in the common body were analyzed and restricted so that the optimization process resulted in TLTs with adequate aesthetics. Thus, a penalty was applied if the angles were too distant from each other, which narrowed the range of optimal shape possibilities. The penalty here sums thousands of kilograms to the tower's mass, ensuring proper aesthetics;
3. Another penalty was applied for the main members which the profile width or thickness was greater than the main member immediately above;
4. Wind forces were calculated according to IEC 60826 (2017) and ten or more load cases for each tower were considered, representing the main cases in real projects of transmission line towers, as extreme wind conditions with transversal, longitudinal and inclined incidence, rupture of conductor cables and lightning rods, construction and assembly, and containment of the cascade effect were included;
5. The structure was analyzed using the finite element method with the main bars modeled as spatial frame elements. Redundant members were not considered in the structural model, being used only to determine the buckling lengths of the main bars; and
6. Extra penalties were applied if the TLT had not followed restrictions of ASCE 10-15 (2015) standard project criteria regarding axial forces, slenderness ratio and cross-sectional areas of main bars. The penalties applied

to infeasible towers added at least billions of kilograms to the TLT mass, more than enough to guarantee a safe structure.

Doing all of this 288,000 times (i.e, one run) costs about 5 hours and the additional 72000 OFEs described earlier takes roughly 1 hour and 15 minutes to completion.

2.1 Search space

As mentioned before, there are three types of optimization variables: size, shape and topological. A brief description about the implementation of these variables is carried out in this section.

Size variables

The profiles available during the optimization were the ones that were very often selected during size optimizations, in order to accelerate convergence. The bars were divided in groups, according to the original projects. An average of 18 profiles per tower were considered in the optimization, which any group could assume one of these profiles and there were around 84 groups in each tower ($\approx 2.8 \times 10^{105}$ possible size combinations per tower).

Shape variables

The head and base opening in both transverse and longitudinal face was considered. The vertical position of the bars in the common body and extensions was also considered as a variable, not changing the vertical position of legs and beginning or ending of extension and common body. The range of shape variables were calculated based on the electrical clearances and the vertical disposition of nodes in the common body of the TLTs took in consideration node overlapping. Towers that had their base or the head too open, only a reduction of these parts were allowed, since the optimization always leads to slimmer towers. This way, a bit of the search space was able to be reduced.

All the shape variables were discrete with an interval of 0.01 m.

Topological variables

Topology variables were treated as templates, using the same methodology as Souza et al. [10], which brings the advantage of providing only templates employed by the industry, as topological possibilities, to fulfill constructional requirements, and limiting the search space to reduce computational cost as well.

For the inferior part of the towers, only one leg template and usually two to four extensions were considered, some changing the design of main elements and others the bracing of redundant members (staggered or continuous). For the common body, templates could assume a continuous or staggered bracing, the number of layers could be altered and redundant members could be present or not in staggered bracing and they could be continuous or staggered too when using continuous bracing. The edges of the common body and the head were fixed, aiming to follow the recommendations of de Souza et al. [13], in order to ensure that, besides the linear analysis employed here, the optimization process generated TLTs less susceptible to the nonlinear bolt slippage effect. This consideration helped to simultaneously improve security and convergence, by decreasing topological possibilities. Most towers could assume different configurations of lightning rods (except the TIS5) and straight body, altering the design of the former and the number of layers and vertical nodal position of the latter.

The number of templates in relation to the search space is expressed in Table 1, where the numbers inside the brackets of the SB column is the quantity of straight body templates possible when a certain lightning rod is chosen. CB stands for Common Body, Ext for Extension, LR equals to Lightning Rod and SB is Straight Body. The addition of different lightning rods and extensions created a few more size variables. This and examples of templates used in the optimization process and the parts of a TLT are illustrated in Fig. 1.

3 Results

Five runs of size optimization were done for every TLT and the gains brought by adding shape and topology variables are demonstrated in Table 3. A summary of the main findings is described down below and in Table 4.

- Only two TLTs (EA1 and BRS5) had their best result in the smallest search space, being one of them due to luck, as it can be seen in the average and standard deviation of BRS5 tower in Table 5. The best of DFS was in the intermediate search space and the others had their best in the biggest;
- For those TLTs with the best result inside the biggest search space, if this result was eliminated, the best search space would remain the same, while this is not true when the smallest or intermediate search space

Table 1. Number of templates per tower part

	Smallest (SS1)				Intermediate (SS2)				Biggest (SS3)			
Towers	CB	Ext	LR	SB	CB	Ext	LR	SB	CB	Ext	LR	SB
A2C	12	2	1	1	12	4	3	1	12	4	3	[4, 1, 1]
TIS5	12	2	1	1	12	4	1	2	12	4	1	5
SY	12	2	1	1	12	4	3	[1, 1, 1]	12	4	3	[2, 2, 2]
TS1	12	2	1	1	12	4	3	[1, 1, 1]	12	4	3	[3, 3, 3]
EA1	12	2	1	1	12	4	3	[1, 1, 1]	12	4	3	[2, 2, 3]
DFS	12	2	1	1	12	3	3	[1, 1, 1]	12	3	3	[1, 1, 3]
APS5	12	2	1	1	12	2	3	[2, 2, 2]	12	2	3	[4, 4, 5]
BRS5	12	2	1	1	12	4	3	[1, 1, 1]	12	4	3	[4, 2, 2]

Table 2. Total number of possibilities

	Templates			Size		Shape	Total	
Towers	SS1	SS2	SS3	SS1	SS3	Any SS	SS1	SS3
A2C	24	144	288	1.5×10^{99}	1.2×10^{111}	1.7×10^{38}	6.1×10^{138}	5.8×10^{151}
TIS5	24	96	240	7.9×10^{100}	7.9×10^{100}	5.3×10^{45}	1.0×10^{148}	1.0×10^{149}
SY	24	144	288	5.2×10^{111}	9.4×10^{112}	4.6×10^{47}	5.8×10^{160}	1.3×10^{163}
TS1	24	144	432	1.4×10^{107}	2.4×10^{108}	10×10^{44}	3.5×10^{152}	1.0×10^{155}
EA1	24	144	336	2.8×10^{82}	6.7×10^{89}	2.8×10^{40}	1.8×10^{124}	6.2×10^{132}
DFS	24	108	180	1.4×10^{121}	2.1×10^{122}	1.8×10^{36}	5.8×10^{158}	6.5×10^{160}
APS5	24	144	312	9.9×10^{120}	3.2×10^{127}	3.9×10^{46}	9.2×10^{168}	3.9×10^{176}
BRS5	24	144	384	1.9×10^{96}	9.7×10^{107}	7.1×10^{44}	3.2×10^{142}	2.6×10^{155}

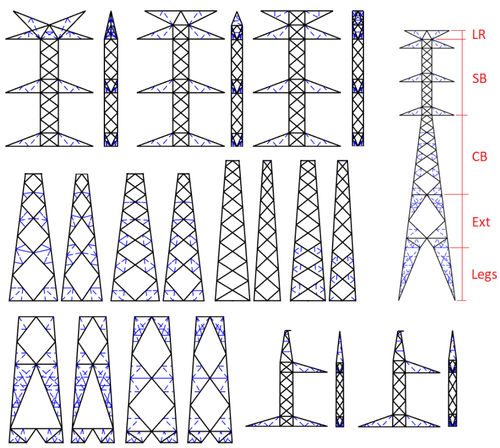

Figure 1. Examples of templates employed in the optimization process

Table 3. Best size vs best topology run

	A2C	TIS5	SY	TS1	EA1	DFS	APS5	BRS5
Size	12375	5169	4033	5746	5947	5478	13700	7230
Topology	11530	4413	3595	5233	5437	5247	11559	6459
Reduction	6.8%	14.6%	10.9%	8.9%	8.6%	4.2%	15.6%	10.7%

was the best;

- For five towers (TIS5, TS1, EA1, APS5 and BRS5), the additional 2000 cycles helped to reduce around 0.5% the weight. As for the other three (A2C, SY and DFS), less than 0.1% was reduced;
- The lightning rod templates helped to reduce the weight in five towers (A2C, SY, TS1, DFS and APS5), choosing the original template only three times, including the TIS5 which had no template other than the original. One type of lightning rod present in three projects was clearly a counterproductive option, since the best results in these towers avoided it and it was also barely chosen during other runs that generated adequate results;
- The possibility of staggered bracing was chosen in six of the towers. In TIS5 the continuous bracing (original) was chosen only three out of the twenty times and they composed the worst results, while in APS5 it was never chosen continuous bracing, even though it was the original template;
- Continuous bracing was chosen for the SY and A2C, but for the SY the original template was staggered;
- Different templates for the straight body were chosen in four of the eight TLTs and in two of these the templates greatly improved results;
- Different extensions were chosen only twice, but for BRS5 it allowed an optimal result that was significantly better when compared to the other runs (more than 2% difference); and
- Observing the best results within certain search spaces, it was possible to achieve similar results (less than 1% difference) with very different designs.

Table 4. Percentage of reduction caused by the consideration of additional templates

	LR	SB	Ext
A2C	0.04	-	-
TIS5	-	1.85	-
SY	1.37	0.08	-
TS1	2.09	1.67	-
EA1	-	-	-
DFS	1.83	-	0.10
APS5	3.17*	1.76	-
BRS5	-	-	3.76

**with the influence of the SB*

Increasing templates number causes the average to be reduced and makes it easier to find an optimal result, but due to the creation of multiple local minimums the standard deviation tends to be raised. Figure 2 express these results, where for each TLT the search spaces were compared to the one that did worse - i.e., lower minimum and mean and higher standard deviation - for that respective tower.

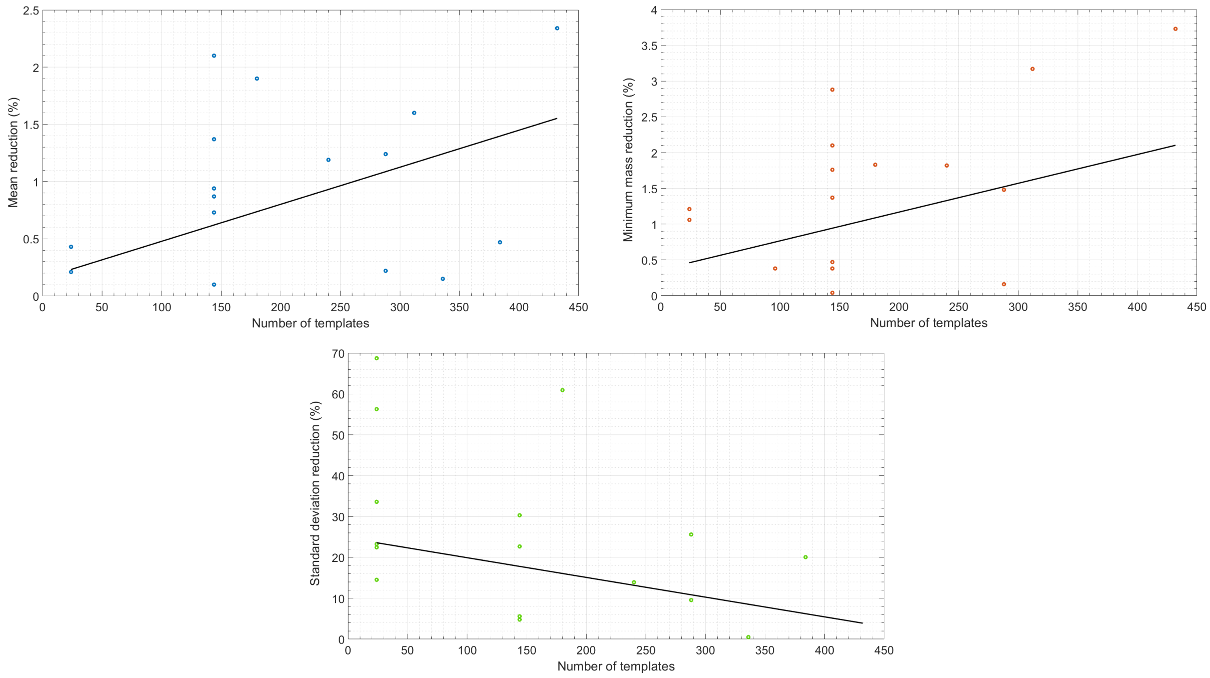

Figure 2. Reduction of minimum weight, average and standard deviation and their tendency

Table 5. Search space results

Tower	A2C			TIS5			SY			TS1		
SS	24	144	**288**	24	96	**240**	24	144	**288**	24	144	**432**
Min	11549	11544	**11530**	4513	4496	**4413**	3649	3599	**3595**	5436	5322	**5233**
% Red	-	0.05	**0.17**	-	0.36	**2.20**	-	1.35	**1.46**	-	2.09	**3.72**
Mean	11630	11618	**11605**	4561	4571	**4517**	3699	3667	**3653**	5473	5358	**5345**
Std	**62.6**	81.6	73.8	**61.6**	72.1	62.1	39.9	51.5	**38.3**	**21.4**	46.6	48.9
Tower	**EA1**			**DFS**			**APS5**			**BRS5**		
SS	**24**	144	336	24	**108**	180	24	144	**312**	**24**	144	384
Min	**5437**	5469	5495	5311	**5158**	5214	11937	11727	**11559**	**6459**	6513	6538
% Red	**1.06**	0.47	-	-	**2.87**	1.82	-	1.76	**3.17**	**1.20**	0.38	-
Mean	**5518**	5543	5534	5348	5275	**5247**	12005	11917	**11812**	6688	**6625**	6656
Std	47.3	**44.7**	47.1	**34.8**	111.1	43.4	**113.8**	119.4	171.3	105.3	**81.4**	84.2

4 Conclusions

In five out of the eight TLTs the biggest search space achieved best average and minimum results. For the BRS5 tower the intermediate search space had the best average and the smallest reached the best minimum as a lucky shot. For the DFS the minimum weight was in the intermediate search space, but the best average was in the biggest and there was a great reduction in standard deviation between the intermediate and the biggest. The smallest space stood out in all statistics for the EA1, except the standard deviation, which was generally smaller in reduced search spaces for all the TLTs, but it could improve in bigger ones if the templates added allowed more optimal possibilities.

Shape and topology variables should not be left out of the optimization as Table 3 shows, because even increasing largely the number of possibilities, as seen in Table 2, the reduction in mass obtained is noteworthy. The computational cost could be even a bit higher since in five towers the 2000 extra cycles done for the best result of each tower decreased the weight about 0.5% for roughly 1 extra hour. An engineer designing a tower in an office might take up to one week to come up with a TLT blueprint. So a few extra hours to reduce the weight - which will be transmitted to several other towers in an extensive line - might be worthy.

So, in general, a wider search space allows better results, because even with the increase of standard deviation, the reduction of average usually compensates for that, though the standard deviation can be improved even when increasing search space, as long as enough optimal variable combinations are brought as new possibilities in the optimization. Thus, a broader search space is also the best choice if the designer does not have enough experience to determine which templates are better alternatives. Regarding the templates employed, the only one that could be left out of the optimization is the lightning rod of top right in Fig. 1, the one that is a square in the center in both transverse and longitudinal faces, because it achieved the worst result in all the TLTs that considered it.

Further studies considering even bigger search spaces, composed by thousands adequate templates or more may be worthy to draw a clearer line outlining the limits of topological optimization.

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References

[1] Taniwaki, K. & Ohkubo, S., 2004. Optimal synthesis method for transmission tower truss. *Struct Multidisc Optim*, vol. 26, pp. 441–454.

[2] Sivakumar, P., Rajaraman, A., Samuel Knight, G. M., & Ramachandramurthy, D. S., 2004. Object-oriented optimization approach using genetic algorithms for lattice towers. *Journal of Computing in Civil Engineering*, vol. 18, pp. 162–171.

[3] Shea, K. & Smith, I. F. C., 2006. Improving full-scale transmission tower design through topology and shape optimization. *Journal of Structural Engineering*, vol. 132, pp. 781–790.

[4] Mathakari, S., Gardoni, P., Agarwal, P., Raich, A., & Haukaas, T., 2007. Reliability-based optimal design of electrical transmission towers using multi-objective genetic algorithms. *Computer-Aided Civil and Infrastructure Engineering*, vol. 22, pp. 282–292.

[5] Kaveh, A., Gholipour, Y., & Rahami, H., 2008. Optimal design of transmission towers using genetic algorithm and neural networks. *International Journal of Space Structures*, vol. 23, pp. 1–19.

[6] Guo, H. Y. & Li, Z. L., 2011. Structural topology optimization of high-voltage transmission tower with discrete variables. *Struct Multidisc Optim*, vol. 43, pp. 851–861.

[7] Noilublao, N. & Bureerat, S., 2011. Simultaneous topology, shape and sizing optimisation of a three-dimensional slender truss tower using multiobjective evolutionary algorithms. *Computer and Structures*, vol. 89, pp. 2531–2538.

[8] París, J. & Colomínas, I., 2012. Structural optimization of high voltage transmission line towers considering continuum and discrete design variables. *WIT Transactions on The Built Environment*, vol. 125, pp. 59–69.

[9] Gomes, W. J. d. G. & Beck, A. T., 2013. Global structural optimization considering expected consequences of failure eand using ann surrogates. *Computer and Structures*, vol. 126, pp. 56–68.

[10] Souza, R. R. d., Miguel, L. F. F., Lopez, R. H., Miguel, L. F. F., & Torii, A. J., 2016. A procedure for the size, shape and topology optimization of transmission line tower structures. *Engineering Structures*, vol. 111, pp. 162–184.

[11] Arora, J. S., Huang, M. W., & Hsieh, C. C., 1994. Methods for optimization of nonlinear problems with discrete variables: A review. *Structural optimization*, vol. 8, pp. 69–89.

[12] Civicioglu, P., 2013. Backtracking search optimization algorithm for numerical optimization problems. *Applied Mathematics and Computation*, vol. 219, n. 15, pp. 8121 – 8144.

[13] de Souza, R. R., Fadel Miguel, L. F., Kaminski, J., & Lopez, R. H., 2019. Topology design recommendations of transmission line towers to minimize the bolt slippage effect. *Engineering Structures*, vol. 178, pp. 286 – 297.