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Optimization of transmission line towers considering the family concept

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Abstract. Transmission line towers (TLTs) are responsible to support the cables of overhead transmission lines (TLs). Because these structures are repeated several times in a line, the application of optimization procedures is highly recommended. In the design of a line, towers with different heights and leg combinations are necessary. However, it would be difficult to individually design or produce these supports. Therefore, for the ease of design, industrial production and erection processes, a TLT can be separated in main parts, such as basic body, body extensions and legs. The so-called tower family is the combination of these minor components in order to obtain the different supports. The present work aims to apply this concept into a size, shape and topology optimization procedure of TLTs. The developed optimization procedure is an extension of the methodology proposed by de Souza et al. [1]. A self-supported 138 kV double circuit tower family composed by tower body, one body extension and two different tower legs was evaluated. The sizing of redundant members and each bolted connection are considered. The final solution is around 9% lighter than the original family design, showing the efficiency of the proposed optimization procedure.

Keywords: Structural optimization, Transmission line tower, tower family

1 Introduction

Optimization procedures are a powerful tool to reduce the mass (cost) of structures. In the last decade, these techniques have been a focus of study on transmission line towers (TLTs), mainly because of its industrial scale production. Initial researches were conducted in this field only for academic purposes, with few dedication to an industrial application ([2–12]).

More recently, some authors have considered important industrial practices to optimization procedures of TLTs, e.g. multiple load cases, code constraints and discrete variables ([1, 13–19]). Among these studies, de Souza et al. [1] proposed a simultaneous size, shape and topology optimization procedure for TLTs. The method consists in splitting the tower by modules that can assume predefined structural elements' arrangements, designated as templates. By means of this approach, the most important industrial aspects can be accounted for. Therefore, the final solution presents constructive feasibility and a proper performance in prototype testing.

Despite these advances, due to the vast number of different tower heights and legs combinations necessary in a TL, to design or produce the supports individually would be quite cumbersome (or even not viable). In order to search for a cost-effective global procedure, for the ease of design, industrial production and erection processes, a TLT structure is split in main parts, such as a basic body, body extensions and leg extensions. Then, a final support is assembled based on these minor components. The basic body is employed for the entire set of towers, while the extensions (body and leg) are added to reach the pre-established height for a specific structure. Figure 1 illustrates these main components. This tower composition is the so-called structure family. The family concept allows avoiding the design to be individually performed. Then, a consequent economy in the global process can be achieved (considering design and fabrication).

In the present work, the size, shape and topology optimization of a TLT considering the family concept is performed. The case study is a self-supported 138 kV double circuit tower family composed of a tower body, one body extension and two types of tower legs. The group of supports is subjected to fourteen different load cases and code constraints. Due to the complexity of the problem, with a great number of design variables, a

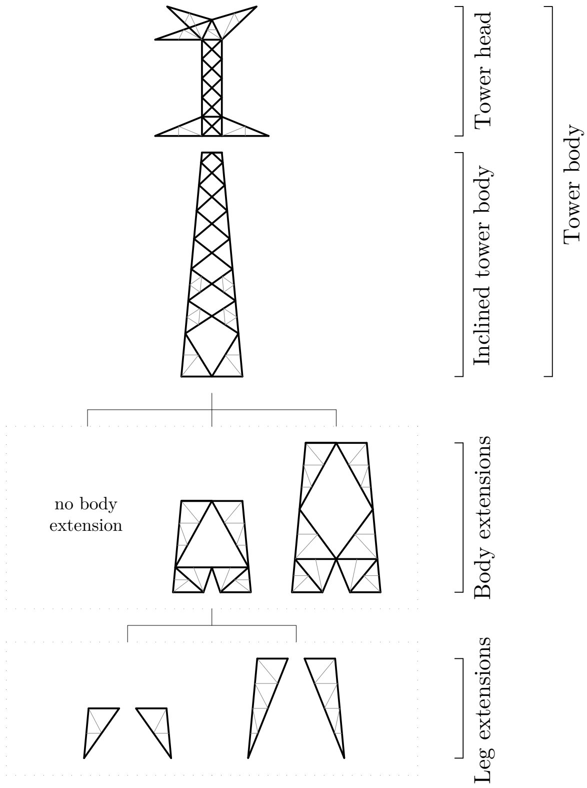

Figure 1. Tower's main components

two-stage optimization approach is employed based on the schemes proposed by Pedro et al. [20] and de Souza et al. [18]. The procedure developed by de Souza et al. [1] is applied to define topological templates for all the main components (inclined tower body, body extensions, tower legs and tower head). In addition, the sizing of the redundant members and bolted connections are also accounted for in the procedure.

The paper is organized in six sections. In Section 2 the problem formulation is defined. In Section 3 a numerical example is described and the results are exhibited. Finally, in Sections 4, 5 and 6 the conclusion, acknowledgments, and references are respectively presented.

2 Optimization problem formulation

The main goal is to apply an optimization procedure to reduce the global mass of a tower family, while respecting code constraints. The objective function is set as the sum of each tower mass multiplied by a coefficient that represents the recurrence of each support in the TL. Therefore, the optimization problem can be described as:

Find x
that minimizes W(x) = $\sum_{i=1}^{nt} n_i \, W_i(\mathbf{x}_i)$,
where $W_i(\mathbf{x}_i) = \sum_{j=1}^{m_i} \rho_j \, l_j(\mathbf{x}_i) \, a_j(\mathbf{x}_i)$,
for each tower subjected to stress constraints
$g_j(\mathbf{x}_i) = |S_{dj}(\mathbf{x}_i)| - R_{dj} \leq 0, \;\; (j = 1, 2, \ldots, m_i)$,
slenderness ratio constraints
$g_{j+m_i}(\mathbf{x}_i) = \lambda_j(\mathbf{x}_i) - \bar{\lambda}_j \leq 0, \;\; (j = 1, 2, \ldots, m_i)$
and cross-sectional constraints
$g_{j+2\,m_i}(\mathbf{x}_i) = \frac{w_{fj}(\mathbf{x}_i)}{t_j(\mathbf{x}_i)} - (w_f/t)_{max} \leq 0, \;\; (j = 1, 2, \ldots, m_i)$

where $\mathbf{x}$ is the vector of design variables, W is the global mass of the tower family, nt is the family size, and for each i tower n_i is the recurrence coefficient, W_i is the mass and $\mathbf{x}_i$ is the vector of design variables. The mass of a tower is a function of each j member's specific material weight (ρ_j), length (l_j) and cross-sectional area (a_j). The constraints are applied to each member by considering S_{dj} as the axial force, R_{dj} as the design capacity, λ_j as the slenderness ratio, $\bar{\lambda}_j$ as the maximum slenderness ratio, w_{fj} as the width, t_j as the thickness and $(w_f/t)_{max}$ as the maximum flat width to thickness ratio.

The vector $\mathbf{x}$ stores all design variables. The cross-sectional areas of the primary members are taken variables for the size optimization and are stored in vector $\mathbf{a} = \{a_1, \ldots, a_m\}$. The vector $\xi = \{\xi_1, \ldots, \xi_q\}$ stores the shape design variables, which are defined as the coordinates of some predefined nodes. Finally, the vector $\tau = \{\tau_1, \ldots, \tau_s\}$ stores design variables for the topology optimization. For the definition of the topological options, the optimization procedure proposed by de Souza et al. [1] is employed in the present paper. Then, the final design vector is represented by $\mathbf{x} = \{\mathbf{a}, \xi, \tau\}$.

Due to the complex of the stated problem, a two-stage optimization routine is proposed. Initially, the tallest TLT of the group is optimized considering size, shape and topology design variables. Then, the resulting topology is fixed and the optimum region is taken as the starting point to proceed with the size and shape family optimization.

The primary members are evaluated according to the ASCE:10-15 [21] recommendations. The evaluation of the constraints for each tower is carried by the penalization strategy described by de Souza et al. [1].

Due to the presence of discrete variables and the existence of many local minima, the metaheuristic Backtracking Search Algorithm (BSA) proposed by Civicioglu [22] is applied in the present study. Recent studies have shown its efficiency in solving real-world structure optimization problems ([1, 18]).

For the numerical example described in the following section, the parameter $mixrate$ was taken as 1, population size as 36 and number of cycles as 8000 for the size optimization, resulting in 288000 objective function evaluations (OFEs). For the first and second stages of the sequential size, shape and topology optimization approach, it was considered 10000 and 7500 cycles, respectively, which is equivalent to 360000 OFEs.

3 Numerical example

A real self-supported 138 kV double circuit tower family is taken as the case study. The original structure family topology is shown in Figure 5. The support family is composed by tower body, one body extension and two tower legs. The steel used is the ASTM A572 g50 and the bolts are 12 mm ASTM A394 Type 0.

The tower family is subjected to fourteen different load cases (LC), in order to follow the original design. The LC include two construction scenarios, rupture of each ground-wire and conductor cable, transverse and longitudinal wind hypothesis. The wind pressure is considered as constant value of 1657 N/m with the tower height and the drag coefficient is equal to 2. Therefore, the total wind force in each tower panel is given by the product of the wind dynamic pressure, drag coefficient and projected area of members in each face of the tower.

3.1 Size optimization

First, a family size optimization is carried out considering the shape and topology original design for all the support components. The size design variables are the cross-sectional areas of the primary members and can assume only discrete values from the set of commercially available profiles displayed in Table 1. For industrial application, the structural members are arranged into 80 different groups (see Fig. 5), which are stored in the vector $\mathbf{x} = \{a_1, \ldots, 80\}$.

Table 1. Data set of commercially available profiles

Profile	Area	Profile	Area	Profile	Area	Profile	Area
L40x40x3	2.35	L60x60x4	4.71	L75x75x6	8.75	L100x100x6	11.64
L45x45x3	2.66	L60x60x5	5.82	L80x80x5	7.75	L100x100x7	13.7
L45x45x4	3.49	L65x65x4	5.13	L80x80x6	9.24	L127x127x6.35	15.73
L50x50x3	2.96	L65x65x5	6.31	L90x90x6	10.6		
L50x50x4	3.89	L75x75x5	7.36	L90x90x7	12.1		

Five independent runs were performed. The global mass of the best solution found is 12910 kg, which corresponds to the sum of the four different family supports. The mean value and standard deviation are 12924 kg and 15.41 kg, respectively. This best solution is taken as a reference to estimate mass reductions in the simultaneous optimization scheme that follows.

3.2 Size, shape and topology optimization

The changes in global geometry are classified in five different variations, similarly to de Souza et al. [1]. Variation 1 (ξ_1 and ξ_2) allows the nodes of the tower base to move horizontally at the longitudinal and transverse faces, respectively. Variation 2 takes the nodes of each internal layer of the inclined tower body as independent design variables and allows them to move vertically. This variation can modify up to 20 nodes (ξ_3 to ξ_{22}) according to the bracing pattern and the number of internal layers. Variation 3 (ξ_{23} to ξ_{26}) allows the bottom and top nodes of the straight tower body to independently move horizontally at the longitudinal and transverse faces, respectively. The top node is constrained to be always equal or inferior to the bottom one. Variation 4 (ξ_{27}) allows the nodes of each internal layer of the straight tower body to move vertically. Then, the cross-arms' height is connect to this definition to keep the vertical distance between two cross-arms (according to the electrical design) unchanged. Finally, Variation 5 allows the leg members' nodes of the body extension to move vertically. This variation can modify up to 2 nodes (ξ_{28} and ξ_{29}) according to the primary members arrangement.

For practical purposes, the shape variables are considered as discrete variables, rounded to centimeters. Electrical clearances requirements are accounted for defining the bounds for Var. 1, Var. 3 and Var. 4 , while for Var. 2 the determination of the bounds aims to avoid the overlap of the internal layers. Then, the respective lower and upper bounds for the geometrical design variables ξ_1, ξ_2, ξ_3 to ξ_{20}, ξ_{21} and ξ_{22}, ξ_{23} and ξ_{24}, ξ_{25}, ξ_{26}, ξ_{27}, ξ_{28} and ξ_{29} are [-1.5, 0] m, [-1, 0] m, [-0.24, 0.24] m, [-0.19, 0.19] m, [0, 0.05] m, [-0.25, 0] m, [-0.2, 0] m, [0, 0.15] m, [0, 0.4] m and [0, 0.5] m.

In respect with the topological variables, the templates presented in Fig. 2 are adopted for the inclined tower body. Variable τ_1 defines the primary bracing patterns, which can be continuous or staggered respectively represented by the values $\{0, 1\}$. The number of internal layers (τ_2) and its lower and upper bounds are determined according to de Souza et al. [1]. For the continuous bracing, options $\{6, 7, 8\}$ are provided, while options with $\{7, 8, 9\}$ layers are given for the staggered bracing. The elevation of each internal layer is calculated through the regression curves shown by the mentioned study. Finally, variable τ_3 defines the redundant members' bracing pattern, depending on the definition of τ_1. When τ_1 is equal to 0, i.e., a continuous pattern of the primary bracing is adopted, the two τ_3 alternatives indicate that redundant members may be disposed in a continuous or staggered bracing configurations. On the other hand, for τ_1 equals to 1, i.e., a staggered pattern of the primary bracing, the two options represent the presence or absence of redundant members.

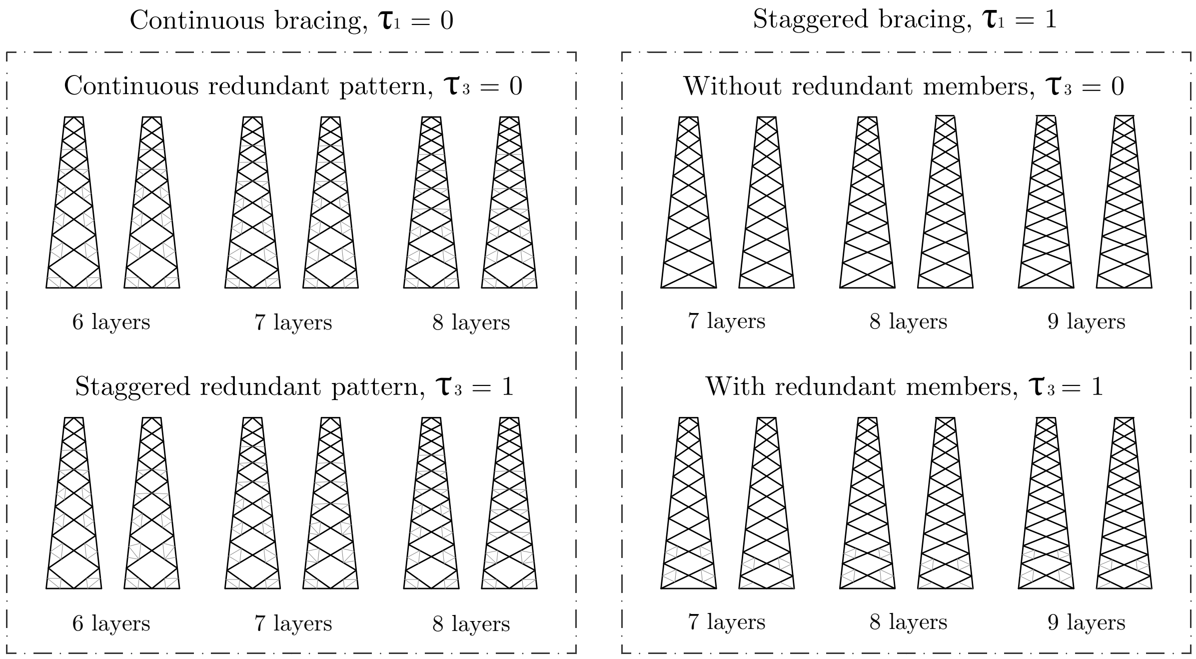

Figure 2. Inclined tower body templates

To create the templates of the straight tower body, two design variables are provided. Options with two and three layers ($\{2, 3\}$) are offered to the optimization problem by τ_4. In addition, the three types of overhead ground-wire (τ_5) shown in Fig. 3 are given as topological possibilities ($\{0, 1, 2\}$).

For the body and leg extensions, the arrangement of the primary members is defined by τ_6, which can assume the topologies shown in Fig, 4, represented by the respective values $\{1, 2\}$. Moreover, two redundant members configurations are provided for variable τ_7, indicated by the values $\{0, 1\}$. Figure 4 also shows the initial heights of each template option and the type of redundant members for both bracing patterns.

Through the combination of all the templates available to each module (provided by these 7 topological design variables), the optimization algorithm can access a total of 384 different structural configurations.

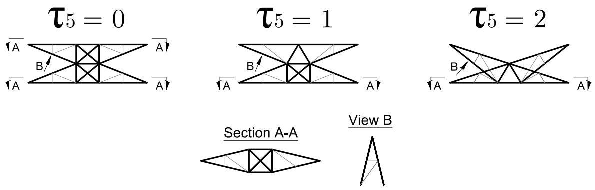

Figure 3. Options of overhead ground-wire

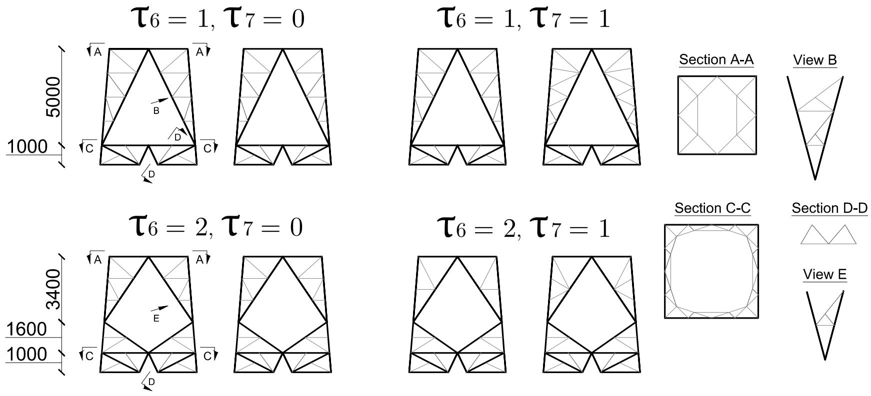

Figure 4. Options of body extension provided

The sequential optimization procedure described in Section 2 is applied. In the first phase, 10 independent runs were executed and the best solution found resulted in 3598 kg of the highest tower. The mean and standard deviation values are 3653 kg and 38.31 kg, respectively. Based on this solution, the simultaneous size, shape and topology family optimization was carried out and the final result was 11741 kg. This represents a reduction of 9.04% (1168 kg) of the global mass when compared to the size optimization. Figure 5 shows the topology of the best solution found. The resulting cross-sectional areas of the primary members and the variable values are presented in Table 2.

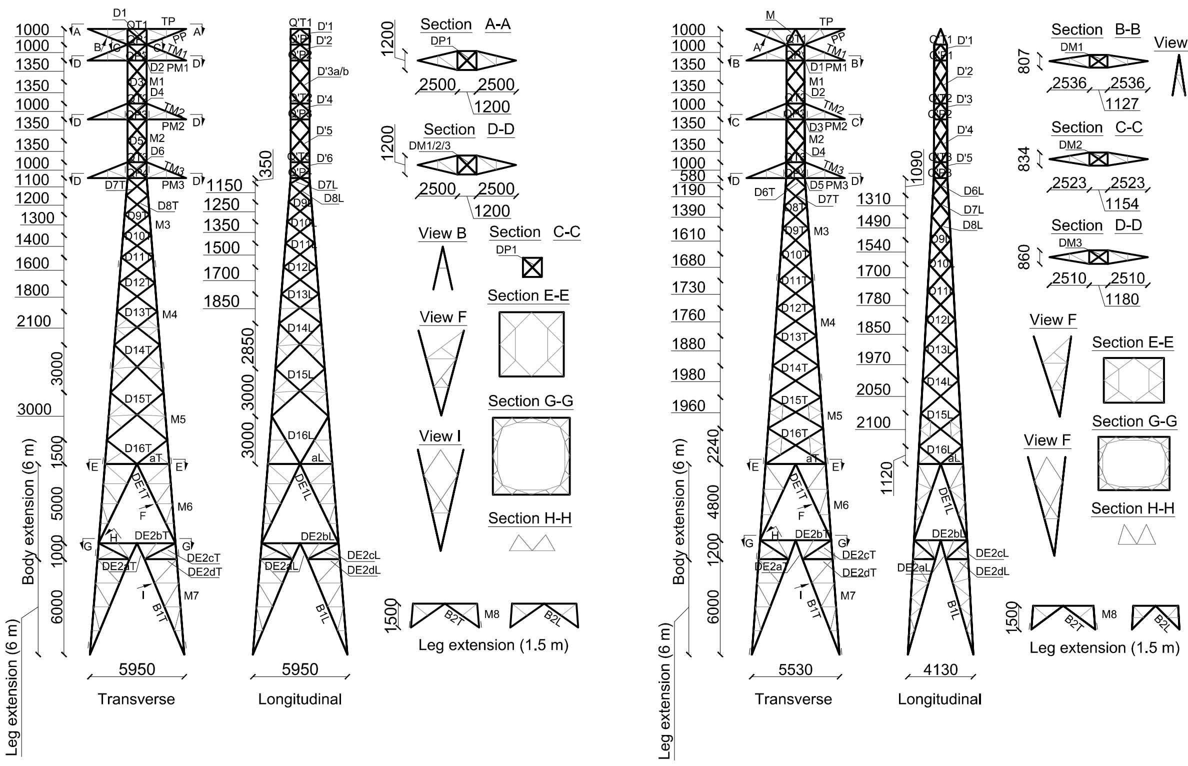

Figure 5. Original design (left) and best solution found for the size, shape and topology optimization

Table 2. Results for the best solution found for the size, shape and topology optimization

Group	Area (cm^2)	Group	Area (cm^2)	Group	Area (cm^2)	Group	Area (cm^2)	Shape var.	Value (m)
M5	10.6	DM2	2.35	D6L	2.35	DE2aL*	7.78	Var. 2.14	0.08
M4	10.6	DM1	2.35	Q'T3	2.35	DE2aT*	7.78	Var. 2.15	0.19
M3	7.36	M	2.35	Q'P2	2.35	DE2bL	2.35	Var. 2.16	0.14
M2	5.13	D16T	2.35	Q'T2	2.35	DE2bT	2.35	Var. 2.17	0.19
M1	2.96	D15T	2.35	Q'P1	2.35	DE2cL	2.35	Var. 2.18	0.11
TM3	2.96	D14T	2.96	Q'T1	2.35	DE2cT	2.35	Var. 2.19	0.15
TM2	2.96	D13T	2.66	Q'P3	2.35	DE2dL	2.35	Var. 2.20	0.02
TM1	2.96	D12T	2.66	aL	2.35	DE2dT	2.96	Var. 3.1	0.03
TP*	2.35	D11T	2.35	QT3	2.35	Shape var.	Value (m)	Var. 3.2	0.03
PM3	2.96	D10T	2.35	QP2	2.35	Var. 1.1	-0.91	Var. 3.3	-0.20
PM2	2.96	D9T	2.35	QT2	2.35	Var. 1.2	-0.21	Var. 3.4	-0.04
PM1	2.96	D8T	2.35	QP1	2.35	Var. 2.1	0.03	Var. 4	0.11
PP	5.13	D7T	2.35	QT1	2.35	Var. 2.2	0.04	Var. 5.1	0.20
D'5	2.35	D6T	2.35	QP3	2.35	Var. 2.3	0	Topology var.	Value
D'4	2.35	D16L	2.35	aT	2.96	Var. 2.4	0.05	τ_1	1
D'3	2.35	D15L	2.35	M8	10.6	Var. 2.5	0.17	τ_2	9
D'2	2.35	D14L	2.66	B2T	2.66	Var. 2.6	0.12	τ_3	1
D'1	2.35	D13L	2.35	B2L	2.35	Var. 2.7	0.24	τ_4	3
D5	2.35	D12L	2.35	M7	11.64	Var. 2.8	0.12	τ_5	1
D4	2.35	D11L	2.35	B1T	3.89	Var. 2.9	0.24	τ_6	1
D3	2.35	D10L	2.35	B1L	4.71	Var. 2.10	0.06	τ_7	1
D2	2.35	D9L	2.35	M6	10.6	Var. 2.11	0.24		
D1	2.35	D8L	2.35	DE1L	4.71	Var. 2.12	0.05		
DM3	2.35	D7L	2.35	DE1T	3.89	Var. 2.13	0.24		
Total global mass (kg):			11742						

4 Conclusions

This paper introduced the family concept to the optimization of TLTs. The case study was a self-supported 138 kV double circuit tower family, composed of tower body, one body extension and two tower legs. Fourteen load cases and code constraints were considered. The final solution of the size, shape and topology optimization provided a reduction of 9.04% (1168 kg) of the global mass (sum of the four tower combination possible), compared to the purely size optimization.

Due to the high computational cost demanded, a sequential optimization procedure based in two phases was proposed. In comparison to the best solution achieved in the first phase, some differences in shape were noticed after the forces envelope of each tower combination was considered. For instance, the base width slightly increased about the longitudinal face, while the top width decreased. In the transverse face the opposite was observed. Therefore, the leg slope in both directions was affected by these changes.

Following the de Souza et al. [1], the template-based optimization strategy was adopted into the definition of the topologies available to each structures component, i.e., extensions, inclined tower body and tower head. Because of its adaptability property, this scheme was easy to be implemented and important industrial aspects could be accounted for the search of the best topological solution.

Finally, the final results showed the efficiency of the metaheuristic algorithm BSA in dealing with complex engineering problems. It is important to emphasize that, in addition to the existence of many local minima, the family concept consideration adds more variables to the size, shape and topology optimization of TLTs.

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