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A Comparative analysis of structural optimization of spatial steel frames considering different bracing systems

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Abstract. In tall steel buildings, aspects related to natural frequencies of vibration, global stability, and horizontal displacements due to wind pressure become more relevant when compared to sizing strength constraints. In this sense, the bracing systems may be crucial to improve the dynamic behavior increasing the structure stiffness. This paper shows a comparative analysis between four bracing systems used in six-story spatial steel frames subjected to weight minimization. This study shows which bracing system is most suitable for the final design of the structure. The constraints imposed in the structural optimization problem are the first natural vibration frequency, the critical load factor concerning to global stability, and horizontal displacements. The search algorithm is the Differential Evolution (DE) coupled with an Adaptive Penalty Method (APM) to handle the constraints.

Keywords: Structural optimization, Steel frames, Bracing systems, Differential evolution

1 Introduction

In tall steel buildings, aspects related to the dynamic behavior, global structural stability, and horizontal displacements due to the wind are crucial to consider in the design. Bracing systems are adopted to stiffen the structure, making it work as a vertical truss to transmit the lateral loads to the ground, avoiding undesired vibration and horizontal displacements. There are several different consolidated topologies of bracing system, and the most common ones widespread in practice, are diagonal, "Z", "V" and "X". In an optimization problem, the search is for a cost-efficient structure in which it is not possible to define what kind of bracing system will be applied previously. This study focuses on finding the best structural design of a six-story spatial steel frame with four different bracing systems topologies. The additional constraints consider the first natural frequency of vibration and the critical load factor concerning global stability as well as those typically employed in most published works. The search algorithm used is the differential evolution (DE) coupled with an adaptive penalty method (APM) to handle the constraints. The results are evaluated with an implemented code in MATLAB ® and confronted with SAP2000 ®, also used to generate the images.

Several works can be found in the literature about steel structure optimization concerning the bracing system's configuration. Although it is not the objective of this paper to provide a vast bibliography, one can cite some relevant works. Memari and Madhkhan [1] studied optimum design of two-dimensional steel frames presenting different bracing systems configuration, as well as a rigid connected structure with no bracing system under gravity and lateral seismic forces. The constraints were related to allowable stresses and inter-story drift ratio. Liang *et al.* [2] researched a performance-based optimization method for optimal topology design of bracing systems applied in multistory steel frames. Kameshki and Saka [3] presented a Genetic Algorithm for multistory non-swaying optimization with different bracing systems considering constraints concerning strength and serviceability. This study investigated the efficiencies of “Z”, “V” and “X” bracing systems in pin-jointed planar frames and rigidly connected planar frames without any bracing system. Huang and Wang [4] studied the application of continuum structural topology optimization methods in the layout design of bracing systems under earthquake loads, and He and Wang [5] presented a topology optimization design of steel frames bracing systems based on a discrete model, also known as ground structures. Through a cost-efficiency analysis, Hasançebi [6] presented an optimization study taking into consideration 13 different topologies of spatial steel frames, including different kinds of bracing systems.

The remainder of this paper is organized as follow: Section 2 describes the formulation of the optimization problem discussed in this paper. Section 3 presents the basic steps of the Differential Evolution and the constraint-handling technique. Numerical experiments are presented in Section 4 and their analyzes in Section 5. Finally, the concluding remarks and future works are reported in Section 6.

2 Formulation of the optimization problem

The structural optimization problem consists in the weight minimization of spacial steel frames. The design variables are integer indexes of a vector $\mathbf{x}$, defined by eq.(1), that points to commercial steel profiles.

$$\mathbf{x} = \{I_1, I_2, ..., I_i\} \tag{1}$$

The objective function ($W(\mathbf{x})$), to be minimized, is defined by the whole weight of a structure composed of N elements (eq.(2)). Where L_i, A_i, and ρ_i are the length, the cross-sectional area, and the specific mass of the i-th member, respectively, and $\mathbf{x}^L$ and $\mathbf{x}^U$ are the lower and the upper bounds of the search space, respectively.

$$W(\mathbf{x}) = \sum_{i=1}^{N} \rho_i A_i Li \tag{2}$$

The maximum horizontal displacement and the maximum inter-story drift are defined by eqs. (3) and (4), respectively. Where $\delta_{max}(\mathbf{x})$ is the maximum horizontal displacement computed, $\bar{\delta}$ is the maximum allowable horizontal displacement, $d_{max}(\mathbf{x})$ is the maximum inter-story drift computed and $\bar{d}$ is the maximum allowable inter-story drift. The maximum allowable horizontal displacement and the maximum inter-story drift are taken as $\bar{\delta} = H/400$ and $\bar{d} = h/500$, according to Brazilian ABNT [7] and American ANSI [8] codes. Where H is the building height, and h is the height between two consecutive stories.

$$\frac{\delta_{max}(\mathbf{x})}{\bar{\delta}} - 1 \leq 0 \tag{3}$$

$$\frac{d_{max}(\mathbf{x})}{\bar{d_{max}}} - 1 \leq 0 \tag{4}$$

The dynamic behavior constraint is taken in consideration ensuring the solution to have a first natural frequency of vibration ($f_1(\mathbf{x})$) higher than a minimum allowable value ($\bar{f_1}$), as it is defined in eq.(5). The natural frequencies of vibration are obtained by solving an eigenvalue problem (Bathe [9]).

$$1 - \frac{f_1(\mathbf{x})}{\bar{f_1}} \leq 0 \tag{5}$$

The structure’s global stability is guaranteed if the critical load factor ($\lambda_{crt}(\mathbf{x})$) is higher than one, as defined in eq. (6). The critical load factor, as the first natural frequency of vibration, is obtained by solving an eigenvalue problem (McGuire *et al.* [10]).

$$1 - \frac{\lambda_{crt}(\mathbf{x})}{1} \leq 0 \tag{6}$$

The strength constraints define that every member of the structure must satisfy the LRDF interection equation for combined axial and bending (eq.(7)) and the LRDF shearing equation (eq. (7)). P_r, M_{rx}, and M_{ry} are the required axial strength, required flexural strength about the major axis and the minor axis, respectively. The available axial and flexural members strength are named as P_c, M_{cx}, and M_{cy}. For the allowable shearing strength equation, V_r is the required shearing strength, and V_c is the available shearing strength. Both ABNT [7] and ANSI [8] have the same approach in defining the allowable strengths, which are followed in this work.

$$\begin{cases} \frac{P_r}{P_c} + \frac{8}{9}\left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \frac{P_r}{P_c} \geq 0.2 \\ \frac{P_r}{2P_c} + \left(\frac{M_{rx}}{M_{cx}} + \frac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \frac{P_r}{P_c} < 0.2 \end{cases} \tag{7}$$

$$\frac{V_r}{V_c} - 1 \leq 0 \tag{8}$$

The geometric constraint applied concerns the column to column fitting up, in which is undesirable to have a profile with higher depth or mass placed above one with lower. Equations (9) and (10) show the geometric constraints, where $dp_i(\mathbf{x})$ and $dp_{i-1}(\mathbf{x})$ are the depth of the W section selected for the group of columns i and $i-1$, respectively. $ms_i(\mathbf{x})$ and $ms_{i-1}(\mathbf{x})$ are the unit weight of W section selected for the group of columns i and $i-1$, respectively. NG_c is the number of groups of columns.

$$\frac{dp_i(\mathbf{x})}{dp_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \tag{9}$$

$$\frac{ms_i(\mathbf{x})}{ms_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \tag{10}$$

3 Search algorithm and the constraint-handling technique

The search algorithm adopted in this paper is the Differential Evolution (DE), introduced by Storn and Price [11]. The DE is based on an evolutionary process of a candidate vector population defined by its upper and lower bounds. Details of DE can be found in Price *et al.* [12].

The Adaptive Penalty Method (APM), proposed by Barbosa and Lemonge [13], is the penalty scheme adopted to handle the constraints. The APM adapts the value of each constraint's penalty coefficients by using information collected from the population, such as the average of the objective function and the level of violation of each constraint. Details on how the APM works is provided by Lemonge and Barbosa [14].

4 Numerical Experiments

The numerical experiments addressed here concerns a six-story spatial steel frame. Five models are considered to study the optimization with different kinds of bracing systems: (i) Model 1 is a six-story spatial steel frame without bracers; (ii) Model 2 has a diagonal (D) bracing system; (iii) Model 3 has a "Z" bracing system; (iv) Model 4 has an "X" bracing system and (v) Model 5 has a "V" bracing system. Figure 1 illustrates from left to right the six-story spatial steel frames and the five models, from Model 1 to Model 5, respectively.

The structure is subjected to gravity loads of 10 kN/m on the outer beams and 20 kN/m on the inner beams. The wind pressure acts on the larger facade, resulting in a mean load of 3.17 kN/m for the corner columns and 6.34 kN/m for the outer columns, calculated for a reference velocity of 35 m/s in accordance to ABNT [15].

The maximum displacement is $\bar{\delta} = 45$ mm, the minimum allowed frequency of vibration is $\bar{f}_1 = 2$ Hz and the maximum allowed inter-story drift is $\bar{d} = 6$ mm. Ten independent runs with 250 generations and a population of 50 candidate vectors are set. The DE parameters adopted here are: $C_r = 0.9$ is the crossover ratio, $M = 0.1$ is the mutation probability, and $F = 0.4$ is the scale factor.

The best result found for the five models are detailed in Tables 1 to 5, also providing the highest values found for the LRDF interaction equation, LRFD shearing equation and inter-story drift are named $LRFD_{max}$, V_{max} and d_{max}, respectively. Figure 2 shows the best results found matching the color with the corresponding profiles in the Tables 1 to 5.

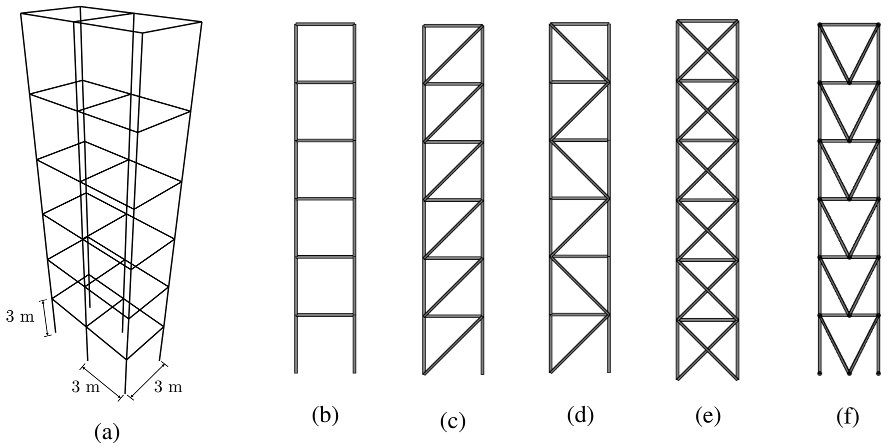


Figure 1. (a) The six-story spatial steel frame; (b) Model 1; (c) Model 2; (d) Model 3; (e) Model 5; (f) Model 5.

Table 1. The best result found for Model 1 detailing the profiles assigned to each group of members and the constraint values.

Group	Characteristics	Floors	Profile	Color
1	Corner Columns	1,2,3	W 250x62	Blue
2	Corner Columns	4,5,6	W 250x62	Blue
3	Outer Columns	1,2,3	W 360x91	Green
4	Outer Columns	4,5,6	W 200x35.9	Cyan
5	Outer Beams	1,2,3	W 460x52	Red
6	Outer Beams	4,5,6	W 310x23.8	Orange
7	Inner Beams	1,2,3	W 530x66	Yellow
8	Inner Beams	4,5,6	W 360x32.9	Magenta
$\delta_{max}(\mathbf{x})$= 42 mm		$LRFD_{max}$= 0.84		
$f_1(\mathbf{x})$= 3.36 Hz		V_{max}= 0.22		
$\lambda_{crt}(\mathbf{x})$= 18.79		d_{max}= 6.0 mm		$W(\mathbf{x})$= 11795 kg

Table 2. The best result found for Model 2 detailing the profiles assigned to each group of members and the constraint values.

Group	Characteristics	Floors	Profile	Color
1	Corner Columns	1,2,3	W 150x29.8	Cyan
2	Corner Columns	4,5,6	W 150x22.5	Blue
3	Outer Columns	1,2,3	W 250x62	Green
4	Outer Columns	4,5,6	W 150x22.5	Blue
5	Outer Beams	1,2,3	W 250x17.9	Red
6	Outer Beams	4,5,6	W 200x26.6	Orange
7	Inner Beams	1,2,3	W 410x38.8	Yellow
8	Inner Beams	4,5,6	W 250x17.9	Red
9	Bracers	1 to 6	W 150x13	Black
$\delta_{max}(\mathbf{x})$= 39 mm		$LRFD_{max}$= 0.99		
$f_1(\mathbf{x})$= 2.11 Hz		V_{max}= 0.24		
$\lambda_{crt}(\mathbf{x})$= 4.97		d_{max}= 5.6 mm		$W(\mathbf{x})$= 7053 kg

Table 3. The best result found for Model 3 detailing the profiles assigned to each group of members and the constraint values.

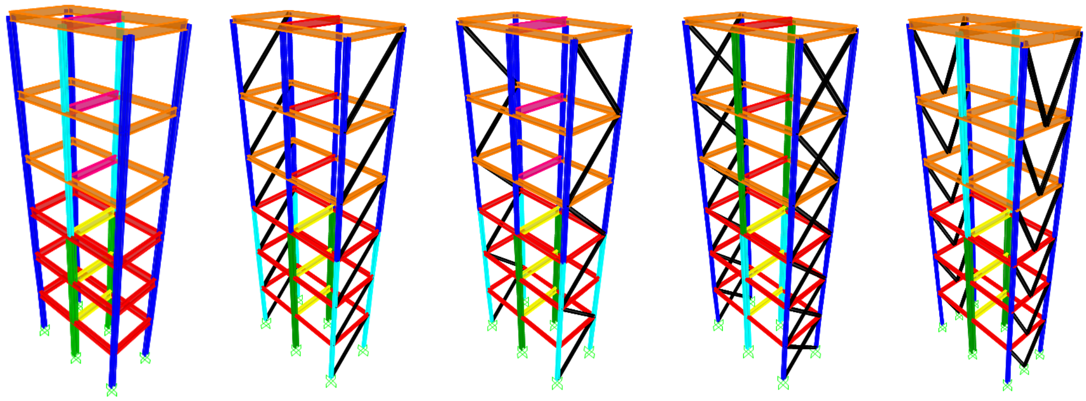
Group	Characteristics	Floors	Profile	Color
1	Corner Columns	1,2,3	W 150x29.8	Cyan
2	Corner Columns	4,5,6	W 150x22.5	Blue
3	Outer Columns	1,2,3	W 250x62	Green
4	Outer Columns	4,5,6	W 150x22.5	Blue
5	Outer Beams	1,2,3	W 250x17.9	Red
6	Outer Beams	4,5,6	W 200x26.6	Orange
7	Inner Beams	1,2,3	W 410x38.8	Yellow
8	Inner Beams	4,5,6	W 310x23.8	Magenta
9	Bracers	1 to 6	W 150x13	Black
$\delta_{max}(\mathbf{x})$= 38 mm		$LRFD_{max}$= 0.99		
$f_1(\mathbf{x})$= 2.10 Hz		V_{max}= 0.23		
$\lambda_{crt}(\mathbf{x})$= 5.31		d_{max}= 5.3 mm		$W(\mathbf{x})$= 7107 kg

Table 4. The best result found for Model 4 detailing the profiles assigned to each group of members and the constraint values.

Group	Characteristics	Floors	Profile	Color
1	Corner Columns	1,2,3	W 150x22.5	Blue
2	Corner Columns	4,5,6	W 150x22.5	Blue
3	Outer Columns	1,2,3	W 250x62	Cyan
4	Outer Columns	4,5,6	W 200x35.9	Green
5	Outer Beams	1,2,3	W 250x17.9	Red
6	Outer Beams	4,5,6	W 200x26.6	Orange
7	Inner Beams	1,2,3	W 360x39	Yellow
8	Inner Beams	4,5,6	W 250x17.9	Red
9	Bracers	1 to 6	W 150x13	Black
$\delta_{max}(\mathbf{x})$= 38 mm		$LRFD_{max}$= 0.98		
$f_1(\mathbf{x})$= 2.01 Hz		V_{max}= 0.25		
$\lambda_{crt}(\mathbf{x})$= 5.79		d_{max}= 5.6 mm		$W(\mathbf{x})$= 7683 kg

Table 5. The best result found for Model 5 detailing the profiles assigned to each group of members and the constraint values.

Group	Characteristics	Floors	Profile	Color
1	Corner Columns	1,2,3	W 150x22.5	Blue
2	Corner Columns	4,5,6	W 150x22.5	Blue
3	Outer Columns	1,2,3	W 310x79	Green
4	Outer Columns	4,5,6	W 200x35.9	Cyan
5	Outer Beams	1,2,3	W 250x17.9	Red
6	Outer Beams	4,5,6	W 310x28.3	Orange
7	Inner Beams	1,2,3	W 410x38.8	Yellow
8	Inner Beams	4,5,6	W 310x28.3	Orange
9	Bracers	1 to 6	W 150x13	Black
$\delta_{max}(\mathbf{x})$= 45 mm		$LRFD_{max}$= 0.99		
$f_1(\mathbf{x})$= 2.10 Hz		V_{max}= 0.17		
$\lambda_{crt}(\mathbf{x})$= 7.87		d_{max}= 5.8 mm		$W(\mathbf{x})$= 7886 kg



(a) (b) (c) (d) (e)

Figure 2. Best solution found for all models: (a) Model 1; (b) Model 2; (c) Model 3; (d) Model 4; (e) Model 5.

5 Analysis of results

This section aims to analyze and compare aspects related to numerical experiments conducted in this paper. Table 6 gathers information concerning the weight and constraints results of the five experiments carried out here. The first and imminent point to note refers to the structure's weight, which is desired to minimize. It is easy to note that the model without a bracing system (Model 1) presented the heaviest structure ($W(\mathbf{x})$=11795 kg) comparing to other models. It shows that the braced structures, although with more elements, reached lighter designs. The lightest structure found is the one with a diagonal bracing system ($W(\mathbf{x})$=7053 kg) proposed in Model 2.

Interesting analyses can be made by observing constraint values. (i) The LRFD interaction equation ratio is an active constraint in Models 2, 3, 4, and 5; (ii) the maximum inter-story drift is an active constraint for Model 1, but all models presented a maximum drift near the maximum allowable $\bar{d_{max}} = 6$ mm; (iii) the maximum horizontal displacement is an active constraint in Model 5, where $\bar{\delta_{max}} = 45$ mm; (iv) the first natural frequency of vibration is an active constraint for Model 4, and all braced models presented the first frequency near the minimum allowable, which is $\bar{f_1} = 2$ Hz.

The first model presented the highest global stability ($\lambda_{max} = 11.78$) and first natural frequency of vibration ($f_1 = 3.36$ Hz), but this is because the best result found was too heavy, which led to heavy profiles. Hence, it is an unfair comparison with braced models that presented a better solution with lighter profiles. By analyzing only the braced models (Model 2, 3, 4, and 5), it is possible to observe that the most stable design is Model 5 ($\lambda_{max} = 7.87$) with the first natural frequency of vibration oscillating around ($f_1 = 2,1$ Hz).

Table 6. Comparative analysis concerning the weight and constraints values.

Model	Bracing System	LRFD$_{max}$	V$_{max}$	drift$_{max}$ (mm)	$\delta_{max}(\mathbf{x})$ (mm)	$f_1(\mathbf{x})$ (Hz)	$\lambda_{crt}(\mathbf{x})$	$W(\mathbf{x})$ (kg)
1	No Bracing System	0.84	0.22	6.0	42	3.36	18.78	11795
2	D Bracing System	0.99	0.24	5.6	39	2.11	4.97	7053
3	Z Bracing System	0.99	0.23	5.3	38	2.10	5.31	7107
4	X Bracing System	0.98	0.25	5.6	38	2.01	5.79	7683
5	V Bracing System	0.99	0.17	5.8	45	2.10	7.87	7886

6 Concluding remarks and future works

A comparative study of structural optimization with different bracing systems was applied on a six-story spatial steel frame. Constraints concerning the first natural frequency of vibration and the critical load factor were taken into consideration in addition to the usual constraints employed in the literature. The solution that led to the lightest structure presented a diagonal bracing system configuration for the experiments addressed here. The most stable model, with the highest critical load factor, among the bracing system, was the model with "V" configuration, the ”Z” configuration presented the lowest maximum inter-story drift and the ”X” configuration the lowest maximum horizontal displacement. It is important to highlight that the model without bracing systems presented a much heavier solution than the others, showing that a bracing system configuration is needed. The problem is to find which one would give the lightest solution. Large-scale frames and the evaluation of more types of different bracing system's configuration will be investigated in future works.

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