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Global stability and natural frequencies of vibration in multi-objective optimization of 3D steel frames.

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Abstract. In a structural optimization problem, it may be desired not only to minimize the cost of the final design but also to enhance its performance. In this sense, aspects concerning the first natural frequency of vibration and the critical load factor concerning global stability may be desired to be maximized. Also, for instance, the displacements are to be minimized, leading to conflicting objective functions. This paper analyses multi-objective structural optimization of 3D steel frames considering the weight minimization of the structure with the first natural frequency, the critical load factor, and the maximum horizontal displacement in three different multi-objective problems. The analysis presents their Pareto-fronts showing the non-dominated solutions, leaving the task of defining which solution or solutions to be extracted from these curves to a Multi-Tournament method based on the preferences of the decision-maker. The search algorithm adopted is the Third Step Differential Evolution (GDE3) coupled with an Adaptive Penalty (APM) Method to handle the constraints.

Keywords: Multi-objective optimization, Steel frames, Natural frequencies of vibration, Global stability

1 Introduction

In structural engineering, when solving an optimization problem, it may be desired not only to minimize the structure's cost but also to improve its mechanical performance. For that, a multi-objective optimization problem formulation is required to take into account more than one objective function when searching for the most suitable solution. The process of finding this solution is not trivial. In contrast, in the mono-objective problem, the evolutionary process provides one best solution, in the multi-objective problem, the result is a Pareto front of non-dominated solutions. From this curve, the decision-maker has the complex task of extract one or more solutions according to his/her preferences. In steel structures, especially in tall buildings, aspects related to the horizontal displacements due to the wind, the first natural frequency of vibration and the critical load factor concerning the global stability, are parameters of interest to optimize in order to improve its mechanical performance.

Some references in the literature concerning multi-objective optimization of spatial steel frames are listed following. In the late 1900s, Li *et al.* [1] proposed a new application of multi-objective and multilevel optimization concerning the minimization of the total strain energy and the weight of steel frames. A multi-objective optimization strategy and a decision-making method in the process of steel frame optimization, are described by Cui *et al.* [2]. The study deals with structural volume and horizontal displacement minimization through a fast Non-dominated Sorting Genetic Algorithm (NSGA II), proposed by Deb *et al.* [3], and high-level multiple attribute decision-making. Another multi-objective optimization study, concerning the implementation of a performance-based design of steel frames, is conducted by Gholizadeh and Baghchevan [4]. In this study, a comparison of different meta-heuristics is made on displacement and weight minimization as conflicting objectives. Recently, Tu *et al.* [5] investigated the multi-objective optimization of steel frames with buckling-restrained braces, proposing a hybrid code scheme to modify the NSGA II, where the objectives are to minimize the maximum frame energy dissipation and the structure's cost.

The remainder of this paper is organized as follows: Section 2 describes the formulation of the optimization problem discussed in this paper. Section 3 presents the basic steps of the Differential Evolution and the constraint-handling technique. The Multi-criteria decision-making used to extract the solutions from the Pareto sets are briefly described in Section 4. Numerical experiments are presented in Section 5 and their analysis in Section 6. Finally, the concluding remarks and future works are reported in Section 7.

2 Multi-objective structural problem

The multi-objective structural optimization presented in this paper is written as: find a set of commercial steel profiles, designated by an integer index vector (design variables), $\mathbf{x} = \{I_1, I_2, ..., I_i\}$ that minimizes the first objective function ($of_1(\mathbf{x})$) and also minimize (or maximize) the second conflicting objective function ($of_2(\mathbf{x})$), subjected to structural design constraints, as shown in eq. (1).

$$\begin{aligned} \min \quad of_1(\mathbf{x}) \quad \text{and} \quad \min \quad of_2(\mathbf{x}) \\ \text{s.t.} \quad \text{structural constraints} \end{aligned} \tag{1}$$

The first objective function ($of_1(\mathbf{x})$) is the structure's total weight, defined by eq. (2). Where L_i, A_i, and ρ_i are the length, the cross-sectional area, and the specific mass of the i-th member, respectively, and N is the number of elements.

$$W(\mathbf{x}) = \sum_{i=1}^{N} \rho_i A_i Li \tag{2}$$

Three cases are analyzed in this paper, in which the second objective function varies: (i) case 1: the second objective is to minimize the maximum horizontal displacement ($\delta_{max}(\mathbf{x})$); (ii) case 2: the second objective is to maximize the first natural frequency of vibration ($f_1(\mathbf{x})$); (iii) case 3: the second objective is to maximize the critical load factor ($\lambda_{crt}(\mathbf{x})$). $\mathbf{x}^L$ and $\mathbf{x}^U$ are the lower and the upper bounds of the search space, respectively (eq. (3)).

$$\begin{aligned} \text{case 1}: \quad & \min \quad W(\mathbf{x}) \quad \text{and} \quad \min \quad \delta_{max}(\mathbf{x}) \\ \text{case 2}: \quad & \min \quad W(\mathbf{x}) \quad \text{and} \quad \max \quad f_1(\mathbf{x}) \\ \text{case 3}: \quad & \min \quad W(\mathbf{x}) \quad \text{and} \quad \max \quad \lambda_{crt}(\mathbf{x}) \\ & \text{s.t.} \quad \text{structural constraints} \\ & \quad \mathbf{x}^L \leq \mathbf{x} \leq \mathbf{x}^U \end{aligned} \tag{3}$$

The constrains of the problem are the maximum horizontal displacement, the inter-story drift, the LRFD interaction equations for combined axial force and bending moments, the LRDF shearing equation, the first natural frequency of vibration, the critical load factor concerning the global stability and geometric constraints referring to column-column connection. It is important to point that when a constraint turns to be an objective functions, it stops being a constraint.

The displacement constraints are the maximum horizontal displacement and the maximum inter-story drift. Equations (4) and (5) define the displacement related constraints, in which $\delta_{max}(\mathbf{x})$ is the maximum horizontal displacement computed, $\bar{\delta}$ is the maximum allowable horizontal displacement, $d_{max}(\mathbf{x})$ is the maximum inter-story drift computed and $\bar{d}$ is the maximum allowable inter-story drift. The values of the maximum allowable horizontal displacement and the maximum inter-story drift are taken as $\bar{\delta} = H/400$ and $\bar{d} = h/500$, according with both Brazilian ABNT [6] and American ANSI [7] codes. Where H is the building height and h is the height between two consecutive stories.

$$\frac{\delta_{max}(\mathbf{x})}{\bar{\delta}} - 1 \leq 0 \tag{4}$$

$$\frac{d_{max}(\mathbf{x})}{\bar{d}} - 1 \leq 0 \tag{5}$$

The first natural frequency of vibration is determined by the evaluation of a generalized eigenvalue problem considering the stiffness and mass matrix (Bathe [8]). The structure must present a first natural frequency ($f_1(\mathbf{x})$) that is higher than a minimum allowable ($\bar{f_1}$). Equation (6) describes the natural frequency constraint.

$$1 - \frac{f_1(\mathbf{x})}{\bar{f_1}} \leq 0 \tag{6}$$

To ensure the structure's global stability, the critical load factor ($\lambda_{crt}(\mathbf{x})$) must be higher than one, as defined in eq. (7). The critical load factor is obtained by solving an eigenvalue problem concerning the elastic and geometric stiffness matrices (McGuire *et al.* [9]).

$$1 - \frac{\lambda_{crt}(\mathbf{x})}{1} \leq 0 \tag{7}$$

The members of the structure are sized to satisfy the LRDF interection equation for combined axial and bending (eq.(8)) and the LRDF shearing equation (eq. (8)). P_r, M_{rx}, and M_{ry} are the required axial strength, required flexural strength about the major axis and the minor axis, respectively. The available axial and flexural members strength are named as P_c, M_{cx}, and M_{cy}. For the allowable shearing strength equation, V_r is the required shearing strength, and V_c is the available shearing strength. The methodology of determining the allowable strengths are similar in both ABNT [6] and ANSI [7], and adopted in this paper.

$$\begin{cases} \dfrac{P_r}{P_c} + \dfrac{8}{9}\left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \dfrac{P_r}{P_c} \geq 0.2 \\ \\ \dfrac{P_r}{2P_c} + \left(\dfrac{M_{rx}}{M_{cx}} + \dfrac{M_{ry}}{M_{cy}}\right) - 1 \leq 0 \quad if \quad \dfrac{P_r}{P_c} < 0.2 \end{cases} \tag{8}$$

$$\frac{V_r}{V_c} - 1 \leq 0 \tag{9}$$

The geometric constraints refer to the column-column connection, in order to establish that the upper column must not have, neither the profile depth nor the mass, higher than the lower column. Equations (10) and (11) show the geometric constraints, where $dp_i(\mathbf{x})$ and $dp_{i-1}(\mathbf{x})$ are the depth of the W section selected for the group of columns i and $i-1$, respectively. $ms_i(\mathbf{x})$ and $ms_{i-1}(\mathbf{x})$ are the unit weight of W section selected for the group of columns i and $i-1$, respectively. NG_c is the number of groups of columns.

$$\frac{dp_i(\mathbf{x})}{dp_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \tag{10}$$

$$\frac{ms_i(\mathbf{x})}{ms_{i-1}(\mathbf{x})} - 1 \leq 0 \quad i = 1, NG_c \tag{11}$$

3 The GDE3 and constraint-handling technique

The Third Evolution Step of Generalized Differential Evolution (GDE3), proposed by Kukkonen and Lampinen [10], extended the DE proposed by Storn and Price [11]. The GDE3 starts randomly generating an initial population and improves it using DE's selection, mutation, and crossover operations. The crossover rate ($CR \in [0, 1]$), the mutation factor ($F \in \mathbb{R}$) and the population size (N_p) as parameters.

Let P_G be a population of N_p decision vectors $\mathbf{x}_{i,G}$ in generation G, where $i \in \{1, 2, 3, \ldots, N_p\}$ is a vector index. Each $\mathbf{x}_{i,G}$ of the population in generation G is a n-dimensional vector and $\mathbf{x}_{j,i,G}$ is its j-th component ($j \in \{1, 2, 3, \ldots, n\}$). A decision vector $\mathbf{x}_{i,G}$ creates the corresponding trial vector $\mathbf{u}_{i,G}$ applying mutation and crossover operations (Storn [12]). After that, the trial vector $\mathbf{u}_{i,G}$ is compared to the decision vector $\mathbf{x}_{i,G}$ using the constraint domination concept. A vector $\mathbf{x}$ dominates a vector $\mathbf{y}$ (denoted by $\mathbf{x} \succeq_c \mathbf{y}$) if one, and only one, of the following conditions is true: (i) both are unfeasible and $\mathbf{x} \succ \mathbf{y}$ in the constraint function violation space; (ii) $\mathbf{x}$ is feasible and $\mathbf{y}$ is unfeasible, and (iii) $\mathbf{x}$ and $\mathbf{y}$ are feasible and $\mathbf{x} \succ \mathbf{y}$ in the objective function space. The trial vector $\mathbf{u}_{i,G}$ is selected to replace the decision vector $\mathbf{x}_{i,G}$ in the next generation P_{G+1} (population in generation $G+1$) if $\mathbf{u}_{i,G} \succeq_c \mathbf{x}_{i,G}$. If $\mathbf{x}_{i,G} \succeq_c \mathbf{u}_{i,G}$, $\mathbf{u}_{i,G}$ is discarded and $\mathbf{x}_{i,G}$ remains in the population. Otherwise, both are included in P_{G+1}. A complete and detailed description of the entire GDE3 algorithm can be found in Vargas *et al.* [13].

The Adaptive Penalty Method (APM) proposed by Barbosa and Lemonge [14] is adopted in this paper to handle the constraints. From the feedback of the evolutionary process, the method automatically sets a higher penalty coefficient on those constraints that seem to be more difficult to satisfy.

4 Multi-criteria decision making

After obtained the Pareto front of non-dominated solutions, its not trivial to simply extract a solution without any kind of predetermined methodology. For that, the Decision Maker (DM) can extract a solution, for instance, by assigning weight of preferences on the objective functions (Zhang *et al.* [15]), generating different scenarios and extracting one solution for each scenario. The decision making in this paper is aided by a multi-criteria tournament proposed by Parreiras and Vasconcelos [16]. According to the objective functions and their respective importance weights (w_i), established by the Decision Maker, a Multi Tournament Decision Method (MTD) ranks the best and the worst possible solutions in the Pareto front. The complete and detailed description of the MTD method can be found in Parreiras and Vasconcelos [16] and examples in multi-objective structural optimization in Carvalho *et al.* [17].

5 Numerical Experiments

The numerical experiments conducted in this paper concern the multi-objective optimization of a 78 member six-story spatial steel frame illustrated in Figure 1. The structure is subjected to gravity loads of 10 kN/m on the outer beams and 20 kN/m on the inner beams. The wind pressure acts on the larger facade, resulting in a mean load of 3.17 kN/m for the corner columns and 6.34 kN/m for the outer columns, calculated for a reference velocity of 35 m/s in accordance to ABNT [18]. The maximum displacement constraint is $\bar{\delta} = 45$ mm, the minimum allowed frequency of vibration is $\bar{f}_1 = 2$ Hz and the maximum allowed inter-story drift is $\bar{d} = 6$ mm. The members of the frame are linked as follows: CC (corner columns), OC (outer columns), OB (outer beams), and IB (inner beams). The group changes for every three stories resulting in eight groups.

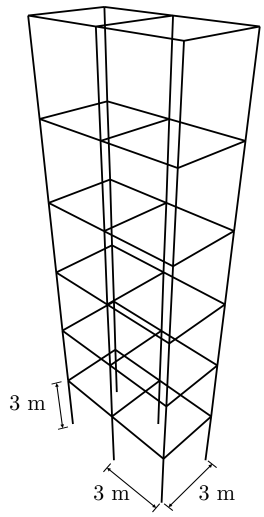

Figure 1. Six-story spatial steel frame.

To analyze different solutions for distinct multi-objective problems, three cases, already detailed in Section 2, are investigated. For each case, the evolutionary process provides a Pareto front, composed of non-dominated solutions. Three solutions are extracted by the Multi-criteria Tournament Decision, resulting in nine designs. The MTD is made based on importance weights for each of the objective functions. For each experiment, three scenarios are considered: (i) scenario 1: the extracted solution has the structure's weight w_1 = 0.3 of importance and w_2 = 0.7 for the second objective function; (ii) scenario 2: the extracted solution has both objective functions with the same importance i.e. w_1 = w_2 = 0.5; (iii) scenario 3: the extracted solution has the structure's weight w_1 = 0.7 of importance and w_2 = 0.3 for the second objective.

Ten independent runs with 200 generations and a population of 50 candidate vectors are set for the three cases. The DE parameters adopted here are: $C_r = 0.9$ is the crossover ratio, $M = 0.1$ is the mutation probability, and $F = 0.4$ is the scale factor. The Pareto front of non-dominated solutions obtained for cases 1, 2 and 3, as well as its extracted solutions are illustrated in Figures 2, 3 and 4, respectively. Table 1 presents the results found for the three extracted solutions from each one of the three cases. The highest values found for the LRDF interaction equation, LRFD shearing equation and inter-story drift are denoted by $LRFD_{max}$, V_{max} and d_{max}, respectively.

Table 1. The best results found for the three cases of the multi-objective problems presenting details of the profiles assigned to each member group, constraints, and objective function values.

	Case 1			Case 2			Case 3		
	Extracted Solutions			Extracted Solutions			Extracted Solutions		
Scenario	1	2	3	1	2	3	1	2	3
Group (Stories)	W Profiles								
CC (1-3)	360x122	360x122	360x91	310x107	310x107	310x79	310x117	310x117	310x97
CC (4-6)	360x91	310x79	250x62	250x62	250x62	250x62	250x73	250x62	250x62
OC (1-3)	360x122	360x122	360x122	310x117	310x117	310x107	310x117	310x117	310x97
OC (4-6)	360x101	360x101	360x91	310x79	310x79	250x62	310x79	310x79	250x73
OB (1-3)	610x101	530x82	530x72	530x66	460x52	460x52	530x72	460x52	460x52
OB (4-6)	530x72	410x38.8	410x38.8	310x21	310x21	310x21	360x32.9	360x32.9	360x32.9
IB (1-3)	610x113	610x125	610x113	410x38.8	410x38.8	460x52	360x72	310x38.7	360x64
IB (4-6)	610x125	610x113	530x85	310x28.3	310x28.3	360x32.9	310x38.7	200x26.6	360x32.9
Constraints and objective functions values									
$LRFD_{max}(\mathbf{x})$	0.31	0.34	0.37	0.97	0.97	0.87	0.59	0.90	0.68
$V_{max}(\mathbf{x})$	0.10	0.12	0.12	0.23	0.24	0.22	0.22	0.29	0.25
$d_{max}(\mathbf{x})$ (mm)	2	3	4	6	6	6	4	6	5
$f_1(\mathbf{x})$ (Hz)	3.42	3.67	3.59	4.44	4.29	4.02	4.41	4.31	4.06
$\lambda_{crt}(\mathbf{x})$	34.78	34.25	28.44	31.73	30.97	27.56	46.97	40.78	35.73
$\delta_{max}(\mathbf{x})$ (mm)	11	13	16	35	33	33	24	36	29
$W(\mathbf{x})(kg)$	23252	20323	17159	14945	14225	12872	17051	15217	14281

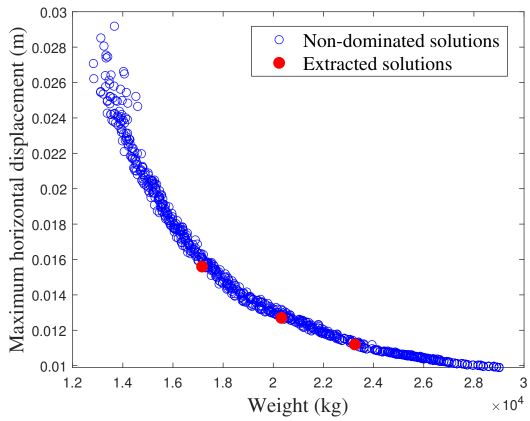

Figure 2. Pareto front for the multi-objective optimization problem of weight and maximum horizontal displacement minimization – case 1.

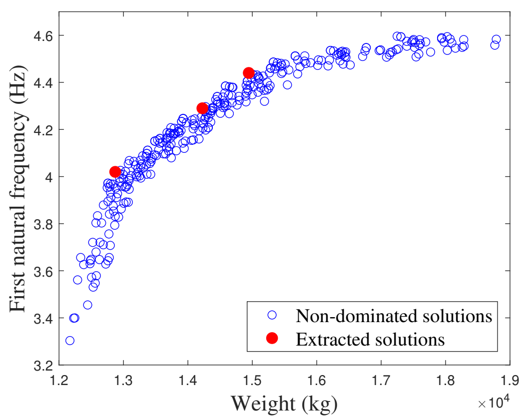

Figure 3. Pareto front for the multi-objective optimization problem of weight minimization and first natural frequency maximization – case 2.

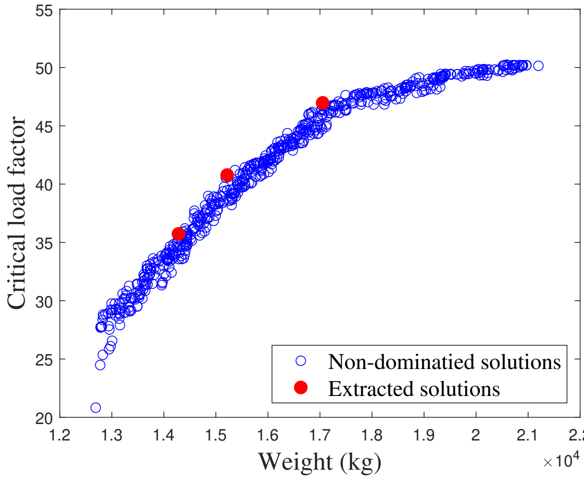

Figure 4. Pareto front for the multi-objective optimization problem of weight minimization and critical load factor maximization – case 3.

6 Analysis of results

Table 1 provides the results extracted from the Pareto fronts obtained from the three cases studied in this paper. Firstly, analyzing case 1 results, one can observe that from the extracted solution of the scenario 1 (importance weight w_1 = 0.3 for $W(\mathbf{x})$ and w_2=0.7 for $\delta_{max}(\mathbf{x})$) to the scenario 3 (importance weight w_1 = 0.7 for $W(\mathbf{x})$ and w_2=0.3 for $\delta_{max}(\mathbf{x})$), the weight decreased 26% (from $W(\mathbf{x})$ = 23252 kg to $W(\mathbf{x})$ = 17159 kg) and the maximum horizontal displacement increased 45% (from $\delta_{max}(\mathbf{x})$ = 11 mm to $\delta_{max}(\mathbf{x})$ = 16 mm).

In the second case, the weight decreased 14% (from $W(\mathbf{x})$ = 14945 kg to $W(\mathbf{x})$ = 12872 kg) and the first natural frequency of vibration decreased 9.5% (from $f_1(\mathbf{x})$ = 4.44 Hz to $f_1(\mathbf{x})$= 4.02 Hz) from scenario 1 to 3. The third case provided a weight variation of 16% (from $W(\mathbf{x})$ = 17051 kg to $W(\mathbf{x})$ = 14281 kg) and a critical load factor variation of 24% (from $\lambda_{crt}(\mathbf{x})$ = 46.97 to $\lambda_{crt}(\mathbf{x})$ = 35.73), when comparing scenario 1 with scenario 3.

One can observe that: (i) the lightest structure was obtained in scenario 3 (importance weight w_1 = 0.7 for $W(\mathbf{x})$ and w_2=0.3 for $f_1(\mathbf{x})$) of case 2 (weight minimization and the first natural frequency maximization), and presented $W(\mathbf{x})$ = 12872 kg; (ii) the structure with highest first natural frequency of vibration, presented $f_1(\mathbf{x})$ = 4.44 Hz and it was found in case 2 in scenario 1, which is intuitive since in case 2, the second objective, is to maximize $f_1(\mathbf{x})$ and in scenario 1, $f_1(\mathbf{x})$ has importance weight of w_2 = 0.7; (iii) the most stable structure, with highest critical load factor, presented $\lambda_{crt}(\mathbf{x})$ = 46.97, and it was found in case 3, scenario 1. This is also expected for the same reason of observation (ii), i.e., this case concerns the maximization of the critical load factor and, it in scenario 1, presents an importance weight w_2=0.7 for $\lambda_{crt}(\mathbf{x})$; (iv) The structure with highest $LRFD_{max}(\mathbf{x})$, i.e., with highest flexural and bending solicitation-resistance ratio, are both scenarios 1 and 2 from case 2, and presented $LRFD_{max}$ = 0.97; (v) the structure with highest shearing solicitation-resistance ratio ($V_{max}(\mathbf{x})$ = 0.29) is extracted from case 3 in scenario 2; (vi) all of the three scenarios of case 2 presented the maximum allowable inter-story drift, $d(\mathbf{x})$ = 6 mm.

7 Conclusions

This paper discussed a multi-objective structural optimization of a six-story spatial steel frame, where three different cases with conflicting objective functions were investigated. The first one concerned the minimization of both weight and the maximum horizontal displacement. The second one involved the weight minimization and the maximization of the first natural frequency of vibration. The last one referred to the weight minimization and the maximization of the critical load factor concerning global structural stability. For each case, three different scenarios were considered with different importance weight for the objective functions, in the first scenario the structure weight was given an importance of 30%, in the second 50% and in the third 70%.

The Pareto fronts obtained in the numerical experiments analyzed in this paper show coherent aspects, as expected. The analyzes obtained here are preliminary and will be extended to large-scale problems. It is expected to consider other types of loads acting on the structure, and their combinations, other design variables, additional constraints, etc. Further comparisons should be made with other evolutionary algorithms, and a more in-depth analysis of the algorithm will be discussed using performance metrics such as the Empirical Attainment Function (EAF) Fonseca *et al.* [19] and the Hypervolume (Zitzler and Thiele [20]) that provides a qualitative measure of convergence and diversity.

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