# Structural Optimization of Reinforced Concrete Beams using GA method.

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**Abstract.** In the civil construction sector, especially in the structural area, the search for optimized designs that satisfy architectural, constructive, safety criteria, being at the same time economic viable are the challenges experienced by designers, who are constantly looking for tools and methods to find the ideal structure. In reinforced concrete structures, beams are elements designed to support and distribute the loads, generally from slabs, to the columns and correspond to a significant portion of the structural design. Therefore, the search for optimal beams can be advantageous, since the final cost can be positively impacted. The main objective of this work is the optimization of cross sections of reinforced concrete beams, having as an objective function the final cost of reinforced concrete inputs. For computational implementation of routines for dimensioning longitudinal and transverse reinforcement, the programming language Matlab was used. The dimensioning process of longitudinal and transverse reinforcement areas were implemented using the commercial software Matlab. This process, as well as all imposed constraints are in accordance with the Brazilian standard NBR6118/2014. The optimization was achieve using the heuristic method GA (*Genetic Algorithm*). Diverse situations were analyzed as variations in spans and loads, having the height of the section, the resistance of the concrete (*Fck*) and the area of the transverse and longitudinal reinforcements as design variables.

**Keywords:** Civil Engineering; Optimization; Beams; Reinforced Concrete; GA.

## 1 Introduction

The elaboration of a structural design of reinforced concrete starts from a basic project, in which the quantity and structural elements to be used are determined, as well as their specifications and dimensions through a pre-dimensioning. Usually, the determination of these parameters is based on the experience and skill of the design engineer and on previously executed projects. These dimensions are necessary to start the calculation process and are adjusted during the development of the project [1].

Commonly, this process does not lead to the most economical structure, given the various solutions for an equally safe structure and in accordance with the prescriptions established by quality and safety standards. For the same loading, there are numerous possible configurations that meet the regulatory requirements. In addition, the short deadlines for the development of structural projects make it impossible to refine the structure to reach the economic and safety criteria concurrently. Therefore, several solutions are considered acceptable [2].

Due to the large number of solutions and a relatively short term involved, professionals seek optimization techniques in order to minimize the consumption of economic resources while all the construction criteria are met. Thus, optimization methods associated with computational programming allow, through a systematic and iterative process, to seek the best possible solution for a structure, or at least to a close value, when considering a wide range of solutions [3].

Structural optimization techniques are based on an objective function, through which it is intended to determine an optimal solution, which may be related to weight, cost, cross-sectional area or another desired parameter, based on criteria and restrictions of the project. Such elements are related to the normative parameters and requirements required for a safe structural design that leads to rationality and the lowest possible cost.

The optimization methods known as metaheuristics have been widely applied to solve optimization problems in different areas of science. Such methods stand out for not requiring gradient information or the convexity of the objective function as in gradient-based algorithms; in addition, metaheuristics use probabilistic transition rules, rather than deterministic. Metaheuristic algorithms are inspired by strategies taken by nature, social culture, biological or physics laws that conduct the process. They are solution methods that coordinate local search procedures with higher level strategies, in order to create a process capable of escaping local minimums and perform a robust search in space in order to find almost optimal solutions, if not the global optimum for the problem [4], [5]. Proposed and developed by John Holland, in 1975, and disseminated by David Goldberg, in the late 1980s, Genetic Algorithms (GAs) are based on the principle of natural selection and survival of the fittest. Among the known metaheuristics, GAs stand out for their popularity due to their robustness regarding optimization of complex problems and their ease of application in various fields of science. As in the evolutionary process of species, GAs manipulate a population of individuals, each with an associated fitness value, for a new generation of individuals, using the Darwinian principles of reproduction and survival of the fittest, performing genetic selection operations, crossover and mutation [2]. Therefore, the individuals of a population in each generation, represent possible solutions; therefore, the algorithm searches within the set of solutions of the search space, always in the direction of the global optimum point, the individual of greatest aptitude.

The Brazilian standard NBR 6118 [6] defines beams as linear elements in which bending is preponderant; and are present in the vast majority of reinforced concrete buildings. The design of the beam is based on the determination of the height and width of its cross section, analyzing the amount of steel necessary for the assembly of the reinforcements [7]. The cost of the beams involves the materials that compose it such as simple concrete, longitudinal and transverse reinforcement, such as formworks, shoring and also the labor involved in the process. Therefore, as the final dimensions of the element, as well as the calculation and choice of reinforcement directly influence the final cost. Due to the great importance of beams in civil construction, its study is justified to the extent that a small reduction in the cost of each element can have a positive impact, generating great savings in the final structure.

The purpose of this work is to implement a computational program to optimize sections of reinforced concrete beams subjected to simple bending, using Genetic Algorithms as the optimization technique. Thus, we seek to define a solution for the lowest possible consumption of materials (steel, concrete and formwork), meeting the safety requirements and current standard regulations.

## 2 Methodology

For the optimization study of reinforced concrete beams, of rectangular section, bi-supported, submitted to a uniformly distributed load, a design routine was developed using the MATLAB software version R2015a (8.5.0.197613). The developed code performs the dimensioning of longitudinal and transversal reinforcements, the determination of deformations, the calculation of reinforcement rates, the detailing of the cross section, in addition to the cost related to the production of the beam under study.

In the optimization field, a study was carried out on bi-supported rectangular beams of reinforced concrete, subjected to simple bending, applying the Genetic Algorithms through the MATLAB predefined function "*ga ( )*". Equation (1) describes the objective function to be minimized regarding the final beam cost.

$$CostBeam = (V_{Con} \cdot Pre_{Con}) + (P_{Aço} \cdot Pre_{Aço}) + (A_{For} \cdot Pre_{For}). \tag{1}$$

Where $V_{Con}$ refers to the volume of concrete (m$^3$); $Pre_{Con}$ at the unit cost of concrete per cubic meter (R\$/m$^3$); $P_{Aço}$ steel weight (Kg); $Pre_{Aço}$ the unit cost of steel per square meter (R\$/m$^2$); $A_{For}$ the formwork area (m$^2$); $Pre_{Form}$ at the unit price of the formwork per square meter (R\$/m$^2$). The material values were obtained by the National System of Research of Costs and Indices of Civil Construction (SINAPI) for the month of January 2018 in the city of Cuiabá-MT.

The constraints adopted in the optimization process of this work meet the requirements of Brazilian standard NBR 6118 [6]. They are as follow: design resistance higher than the requested resistance so that the structure respects the ultimate limit state (ELU); maximum beam deformation equal to $L/250$, where $L$ is beam span; maximum longitudinal reinforcement rate of 4% of the cross-sectional area.

Standard beams were established for the study, in which there was a variation of parameters for each analysis, finding an optimal height/span ratio, analyzing the effects of the beam span, the variation of the characteristic resistance of the concrete to compression ($Fck$) and the uniformly distributed load ($q$) in the final cost of the optimized beam. Table 1 contains the information and defines the general beam model herewith proposed.

Table 1. Input data for optimizing the beam height.

| Data | Values | Unit | Data | Values | Unit |
|---|---|---|---|---|---|
| Width (*b*) | 0.15 | m | Unit weight of steel | 7850 | kg/m³ |
| *Q* | Variable | kN/m | Tractioned long. reinf. | 10 | mm |
| *L* | Variable | m | Compressed long. reinf. | 6,3 | mm |
| *Fck* | Variable | MPa | Transverse reinf. | 5 | mm |
| *Fyk* | 500 | MPa | Armor cover | 3 | cm |

For the first analysis, the beam height was optimized as a function of the *Fck*. The span of the beam was fixed at 5 meters, with an uniformly distributed load of 12 kN/m. Through the information collected regarding the optimal height as a function of the *Fck*, it was possible to verify the impacts of the variation in the resistance of the concrete on the costs of a specific beam. The second problem studied was based on height optimization due to the variation of loads applied on the beam and different *Fck*, under a constant span of 4 meters. Thus, it was possible to analyze the results in search of which concrete resistances are better suited to the most varied degrees of distributed loads. Finally, an optimized analysis relating the height and cost according to the variation in the resistance of the concrete and the span of the beam, with a distributed load of 12 kN/m being applied. From the results, it was possible to establish a relationship between height/span of the beam and perform a cost analysis in relation to which concrete resistances are better suited to the variations of the spans.

# 3 Results

## 3.1 Beam height optimization as a function of *Fck*

Based on the results of this analysis, it is possible to determine the influence of *Fck* on the final beam cost. The determination of concrete resistance is commonly a topic that raises doubts in less experienced engineers, and has an important impact on the final performance of the structure.

Table 2 shows the results obtained from the height variation and optimal costs as a function of the concrete resistance variation, considering the span of the beam fixed at 5 meters and a distributed load of 12 kN/m.

Table 2. Height optimization as a function of concrete *Fck* by GA.

| ***Fck*** **(MPa)** | **Optimal heigth (cm)** | **Concrete** | | **Formwork** | | **Steel** | | **Optimal cost (R$)** |
|---|---|---|---|---|---|---|---|---|
| | | **R$** | **%** | **R$** | **%** | **R$** | **%** | |
| **20** | 50 | 111.82 | 24% | 243.73 | 53% | 104.03 | 23% | 459.58 |
| **25** | 48 | 111.83 | 25% | 235.22 | 52% | 104.35 | 23% | 451.41 |
| **30** | 46 | 110.75 | 25% | 226.71 | 51% | 104.61 | 24% | 442.07 |
| **35** | 45 | 112.21 | 26% | 222.45 | 50% | 104.31 | 24% | 438.97 |
| **40** | 43 | 111.23 | 26% | 213.94 | 50% | 105.30 | 24% | 430.47 |
| **45** | 43 | 125.05 | 28% | 213.94 | 48% | 106.88 | 24% | 445.88 |
| **50** | 42 | 144.84 | 31% | 209.69 | 46% | 107.31 | 23% | 461.84 |

The preponderant input in the total cost of reinforced concrete beams were formwork, which accounted for between 46% to 53% of the total cost, followed by concrete and steel, which varied from 24% to 31% and from 23% to 24% of the total cost, respectively. It was found that in the course of increasing the resistance of concrete, the share of this material in the composition of the total cost increased, while the formwork reduced and steel remained practically constant. This fact is justified by the reduction in the height of the beams, which reduces the consumption of formwork and the fact that the value of the concrete is approximately proportional to its resistance.

Although steel has a practical constant share in the total cost of the beams, varying only 1%, its value has increased with the increment of *Fck*, a consequence of the increase in the steel area required for the section of the beams. That is, although there is a decrease in the optimum height with an increase in the resistance of the concrete and, consequently, a reduction in the beam's own weight, there was an increase in the steel area necessary to maintain the balance of the cross section of the structural element.

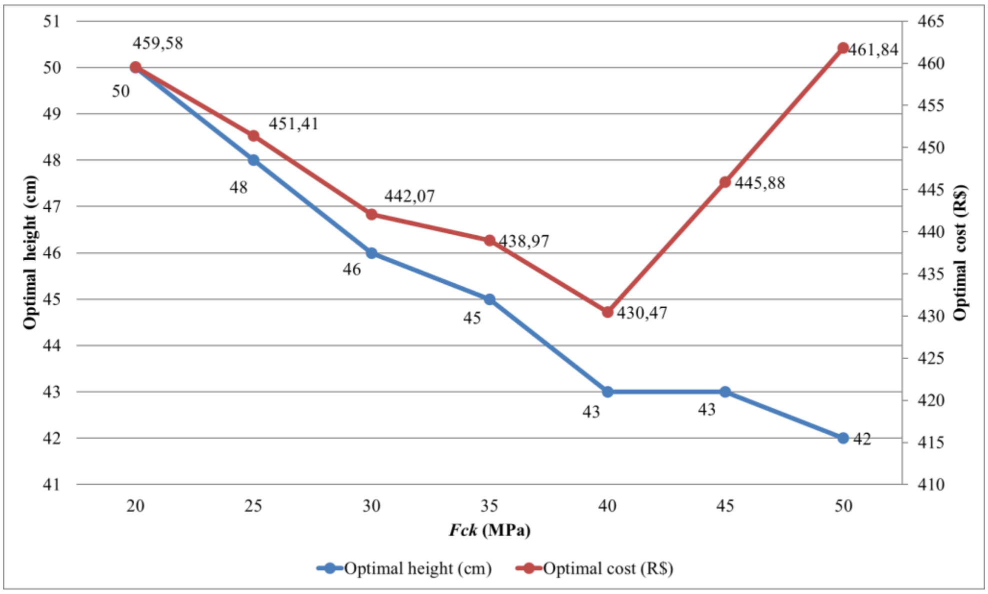


Figure 1. Height optimization as a function of *Fck* by AG.

Figure 1 shows an approximately linear and considerable reduction in the optimum height of the beam as the resistance of the concrete used in the design increases. The cost factor follows the behavior of the optimum height up to the concrete class C40, in which it represents the most economical resistance obtained by the optimization. After this value, the increase in *Fck* is no longer advantageous, although the optimum height continues to decrease, as there is an increase in beam costs, which can be explained by the increased participation of concrete, as seen in Table 2, and by the costs of classes C45 and C50 are discrepant from the others.

Throughout the increase in the resistance of the concrete in a span and constant load, the participation of the concrete in relation to the optimum final cost increased, while the formwork presented a reduction of the participation since the optimum height becomes smaller and, consequently, the surface area of the beam as well, which directly implies the final cost of the form.

### 3.2 Relationship between applied load and *Fck*

Through cost analysis it is possible to have a perspective of the impact of the *Fck* variation along with the load increase in the final cost of each optimized cross section, through a span of 4 m for the beam. From the analysis of the results, it is possible to determine which class of concrete is best suited, from an economic point of view, for each degree of load applied in the calculations.

Table 3. Height optimization as a function of *Fck* and applied load.

| Applied Load (kN/m) | *Fck* 20 | | *Fck* 25 | | *Fck* 30 | | *Fck* 35 | | *Fck* 40 | | *Fck* 45 | | *Fck* 50 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | ***h*** **(cm)** | **R$** | ***h*** **(cm)** | **R$** | ***h*** **(cm)** | **R$** | ***h*** **(cm)** | **R$** | ***h*** **(cm)** | **R$** | ***h*** **(cm)** | **R$** | ***h*** **(cm)** | **R$** |
| **10** | 37 | 290.61 | 35 | 283.55 | 34 | 280.05 | 33 | 277.28 | 32 | 274.55 | 31 | 277.28 | 30 | 284.72 |
| **20** | 47 | 353.41 | 45 | 346.69 | 44 | 344.71 | 42 | 348.86 | 41 | 346.23 | 40 | 351.46 | 39 | 362.98 |
| **30** | 52 | 412.45 | 50 | 403.26 | 49 | 398.95 | 48 | 394.00 | 47 | 391.83 | 47 | 405.59 | 46 | 420.24 |
| **40** | 55 | 465.15 | 54 | 461.55 | 53 | 456.26 | 54 | 449.36 | 51 | 448.32 | 50 | 453.43 | 49 | 466.65 |
| **50** | 60 | 514.94 | 58 | 506.53 | 57 | 501.18 | 57 | 502.15 | 55 | 493.32 | 55 | 504.54 | 55 | 525.39 |

Table 3 presents the respective values of the optimal heights found for the beams and their cost as a function of the increase in the applied load and the concrete class. For all the applied loads, the reduction of the optimum height is observed in the increase of the concrete resistance. Analyzing the height ratio obtained between the concrete classes C20 and C50 for an applied load of 10 kN/m, there was a reduction in the optimal height of 18.9%. For loads of 20 kN/m and 30 kN/m there was a contraction of 17.02% and 11.5%, respectively. While for loads of 40 kN/m and 50 kN/m the reduction was equivalent to 10.9% and 8.3%, in that order. Despite this decrease in the optimum height, the same does not occur with the optimal cost of the structure, with the exception of the applied load of 10 kN/m, the other loads resulted in a higher final cost for the concrete class C50 than the class concrete C20.

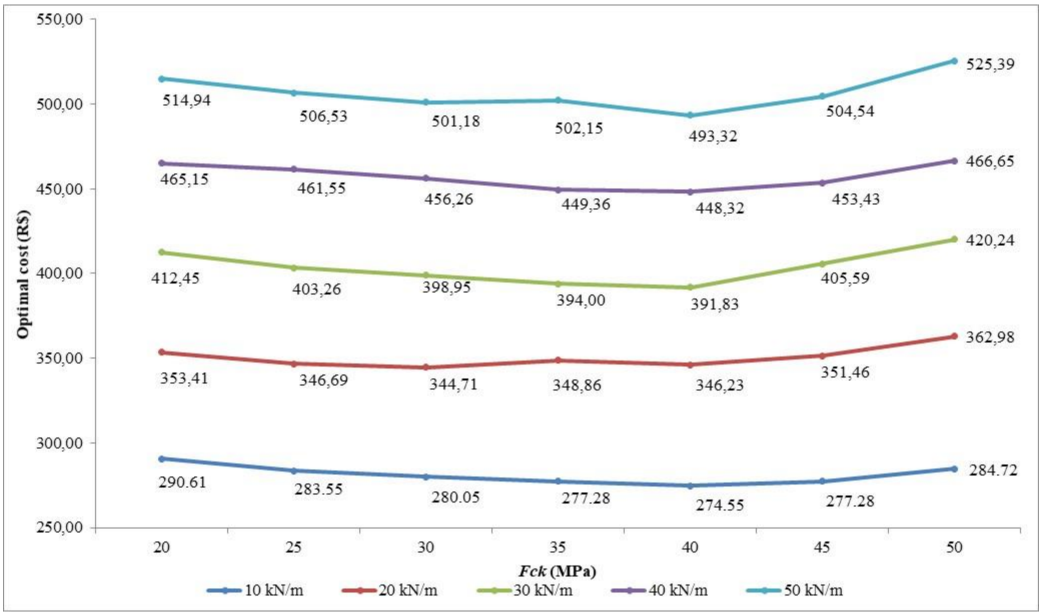


Figure 2. Optimal cost as a function of concrete resistance.

Figure 2 shows the optimal costs due to the resistance of the concrete The results presented demonstrate that the classes of intermediate resistance of concrete presented more economical values in relation to the final cost of the beam, while the classes C20 and C50 expressed the highest values. The concrete class C40 expressed in most results to be the most profitable, with the lowest costs per beam in relation to the applied load.

### 3.3 Beam span ratio as a function of height and optimal cost.

Through this analysis it was possible to assess the optimum height over different spans, by varying the resistance of the concrete and considering a constant distributed load of 12 kN/m. From the results obtained, it is possible to establish height/span relationships found for each concrete resistance class, in addition to obtaining a perspective of the impact of the *Fck* variation along the increment of the span in the final cost of each optimized cross section.

Table 4 and Figure 3 show the results obtained for the optimal height and the height/span ratio found for each concrete resistance class.

Table 4. Height optimization as a function of *Fck* and span.

| Vão (m) | ***Fck* 20** | | ***Fck* 25** | | ***Fck* 30** | | ***Fck* 35** | | ***Fck* 40** | | ***Fck* 45** | | ***Fck* 50** | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | **h (cm)** | **h/L (%)** | **h (cm)** | **h/L (%)** | **h (cm)** | **h/L (%)** | **h (cm)** | **h/L (%)** | **h (cm)** | **h/L (%)** | **h (cm)** | **h/L (%)** | **h (cm)** | **h/L (%)** |
| **3** | 30 | 10 | 28 | 9.33 | 27 | 9.00 | 26 | 8.67 | 26 | 8.67 | 26 | 8.67 | 25 | 8.33 |
| **4** | 40 | 10 | 38 | 9.50 | 37 | 9.25 | 35 | 8.75 | 34 | 8.50 | 34 | 8.50 | 34 | 8.50 |
| **5** | 50 | 10 | 48 | 9.60 | 46 | 9.20 | 44 | 8.80 | 43 | 8.60 | 42 | 8.40 | 40 | 8.00 |
| **6** | 60 | 10 | 58 | 9.67 | 55 | 9.17 | 54 | 9.00 | 52 | 8.67 | 51 | 8.50 | 49 | 8.16 |
| **7** | 70 | 10 | 68 | 9.71 | 65 | 9.29 | 63 | 9.00 | 61 | 8.71 | 61 | 8.71 | 58 | 8.28 |
| **8** | 81 | 10.13 | 78 | 9.75 | 75 | 9.38 | 73 | 9.13 | 71 | 8.88 | 68 | 8.50 | 66 | 8.25 |
| **9** | 92 | 10.22 | 89 | 9.89 | 85 | 9.44 | 83 | 9.22 | 79 | 8.78 | 77 | 8.56 | 75 | 8.33 |

The behavior of the graph represented in Figure 3, a linear evolution of the optimal height for each resistance class analyzed, maintaining an increasing height/span ratio along the span increase, with values varying between 8.00% and 10.22% of the span length according to *Fck* considered, as can be seen in Table 4.

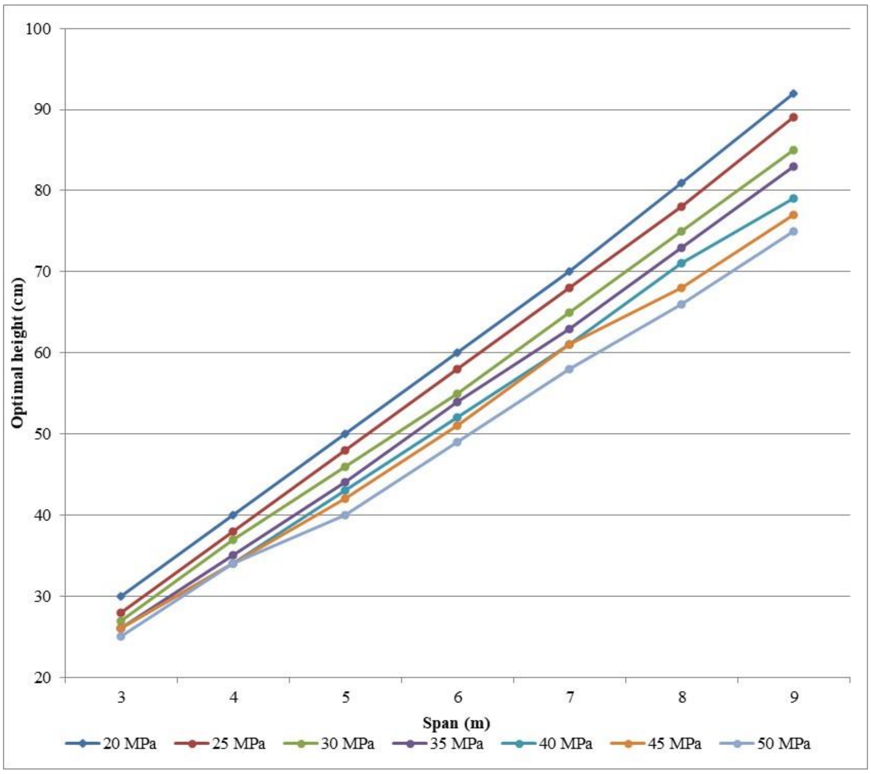


Figure 3. Height optimization as a function of *Fck* and span.

Margarido [8] proposes that during the pre-design of beams, their height can be considered at 10% of the span length. Therefore, it appears that the value suggested by the author is close to the value ranges found, for most of the concrete resistance classes.

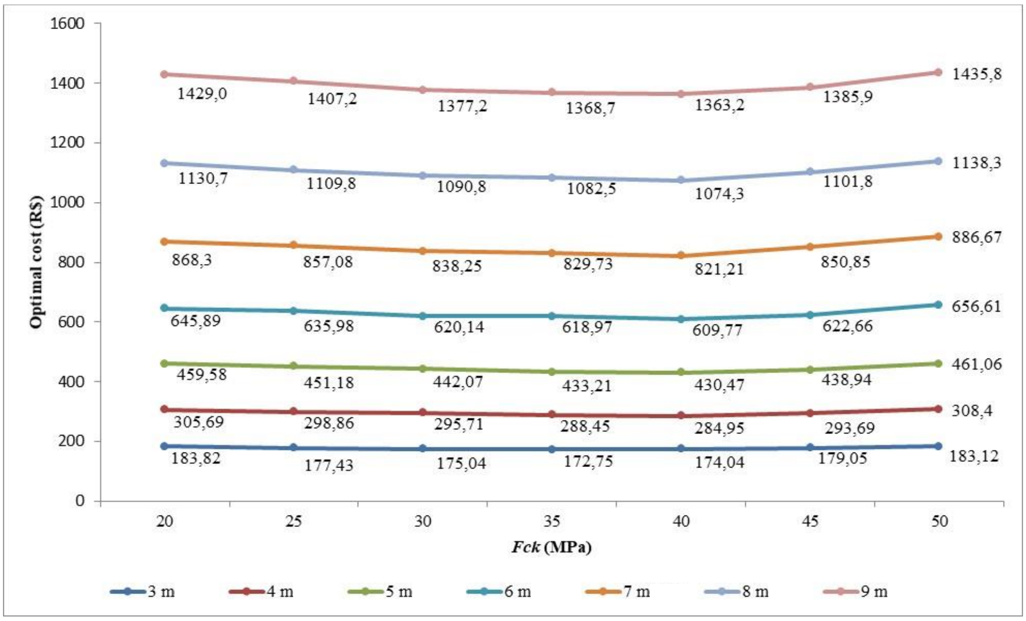


Figure 4. Optimal beam cost as a function of *Fck* and span.

The results presented in Figure 4 demonstrate again that the intermediate resistance classes of concrete present more economical values in relation to the final cost of the beam, with the resistance class of 40 MPa once more expressing in most results being the most favorable, as seen in items 3.1 and 3.2, returning the lowest value in the final cost of the beams for different spans. Although the C40 class presents itself as the most suitable, it should not be considered as an example of greater economic advantage, since in different regions or periods of time, there may be a variation in the cost of this material in relation to the other concretes of different resistances, resulting in in the variation of the most economical material to be used in the project.

## 4 Conclusions

Through the analysis of the optimum results it was found that the beam with the smallest cross-sectional dimensions will not always return the lowest cost values in terms of materials used, because with the reduction of the height of the beam more steel is required in the final cost resulting into an increase in the final cost of the structural element. However, beams with smaller transverse dimensions imply a reduction in the surface area. Which ultimately impacts on a smaller area for the use of formwork, a major input in the composition of the cost of the beams, with participation of up to 53% of the final cost of the beam. Which highlights the importance of price research, correct execution and reuse of this material.

The increase in the resistance of the concrete made it possible to reduce the optimum height of the beam, and in most cases the choice of concrete with greater resistance, especially classes C35 and C40, influences lower costs for the project. According to the results obtained, class C40 presented the lowest costs for the problems studied in the most varied degrees of applied distributed load and free spans. However, the study referred to an analysis, with specific parameters and input prices defined for a particular region and date, consequently attention is needed in choosing which class of concrete to be used in a project, since the parameters mentioned may be different in other regions.

The optimum height of reinforced concrete beams can be analyzed according to the span to be overcome and the active loading. The optimum relationship between the height of the beam and the span that leads to the minimum cost is between 8% to 10% of the span length, for a load of 12 kN / m and different classes of concrete.

**Authorship statement**

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