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Topology Optimization of Continuum Elastic Structures through the Progressive Directional Selection Method

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Abstract. The present work deals with a new approach for Topology Optimization (TO) of two-dimensional continuum elastic structures through the Progressive Directional Selection (PDS) method. To achieve the best topology of a structure, a typical goal is to define the best material distribution in a domain, considering an objective function and mechanical constraints. In general, most of the studies address the compliance minimization of structures. Numerical methods for TO of continuum structures have been investigated extensively. Most of those methods are based on finite element analysis, where the design domain is discretized into a fine mesh of elements. In such a setting, the optimization procedure is to find a structure's topology by determining for every point in the design domain if there should be material (solid element) or not (void element). To control the process of inserting or removing finite elements without forgotten the continuum representation, standard algorithms as Homogenization, Solid Isotropic Microstructure with Penalization (SIMP), and Evolutionary Structural Optimization (ESO) are applied in many studies. The latter approach is based on the simple concept that the optimal design can be achieved by gradually removing inefficient material (elements) from the design space. The ESO algorithms are easy to understand and implement. However, ESO is heuristic, and there is no proof that an optimum solution can be achieved by element elimination and admission. The original scheme is inefficient once it needs to find the best solution by comparing several intuitively generated solutions. To avoid this problem but taking advantage of ESO's simplicity, the PDS method, which is inspired by natural selection observed in biology, applies a strategy to minimize the strain energy of a discretized analyzed domain with a volume restriction. Based on the performance criteria adopted for the problem, the selected population is reached through an iterative process that converges when the optimal topology does not evolve anymore, i.e., there is no change in the final set of selected elements. An instructional problem that shows the essence of PDS for a rigid body problem and one example of topology optimization of a classical problem found in the literature are investigated.

Keywords: topology optimization, progressive directional selection method, continuum elastic structures.

1 Introduction

Human beings have always brought inspiration from nature to solve their common problems. In many fields of Science and Engineering, the act of paying attention to naturals phenomena, from the simplest to the most complex one, enabled the construction of ideas, which were later interpreted and modeled into relevant contributions to humanity, such as studies from Galileo Galilei, Isaac Newton, Leonhard Euler, and Albert Einstein.

Another remarkable work is “The origin of species” published by British naturalist Charles Darwin [1] on November 24, 1859. The Evolution Theory has radically changed the way nature is interpreted. The chapter III of his book shows that according to his theory, there is a struggle for survival in nature, but the one who survives is not necessarily the strongest, but the one that best adapts to the conditions of the environment in which he lives. Then the following chapter, Darwin introduces the main idea of Natural Selection. There are three forms of natural selection: directional, disruptive, and stabilizing, Ridley [2]. The first type is a natural selection mode in which a single phenotype is favored, causing the frequency of the allele to change continuously in one direction. The second one is a natural selection mode in which the extreme values of a characteristic are favored over the intermediate values. The stabilizing selection is characterized by the fact that genetic diversity decreases as the population stabilizes at a characteristic value.

It is natural to think that selection occurs at random due to various natural phenomena or even genetic mutations. However, natural selection can occur progressively and guided by agents that determine which characteristics are necessary for the selected individuals. An exciting study from Conover and Munch [3] shows a case of pink salmon (*Onchorhynchus gorbuscha*) in the Pacific Northwest that has been decreasing in size in recent years due to selective fishing. This case is an example of directional selection, which favors smaller individuals and will produce a decrease in the average body size if the character is inherited.

From a certain point of view, Topological Optimization methods are a process of evolution of a structure, where only the elements that contribute effectively are kept in the structural set. As a response to this evolution, it is expected to arrive at a structure that presents characteristics that most interest the designer. Numerical methods for topology optimization of continuum structures have been investigated extensively since Bendsøe and Kikuchi [4].

There are two main fields in structural Topology Optimization (TO), gradient-based, where mathematical models are derived from calculating the design variables' sensitivities, and non-gradient based, where the material is removed or added using a sensitivity function. Both fields have been investigated in detail over the last two decades, and there are already real-world structures designed using topology optimization, Steven and Xie [5]. However, unlike gradient-based methods, which have more complexity for computational implementation, as the Solid Isotropic Material with Penalization (SIMP), heuristic methods (not based on gradients) are a good alternative because of their simplicity, with results similar to those found by gradient-based methods, Munk *et al*. [6]. Most applications in structural topology optimization use the finite element method (FEM), but other numerical methods are also used, for example, boundary element methods (BEM) [7-8] and finite-volume theory (FVT) [9].

The SIMP algorithm has been the first to become efficient, robust, and widely used. As a result, SIMP optimizers have recently been introduced in some of the main finite element packages worldwide, Sigmund and Bendsøe [10]. The material properties are assumed to be constant within each element of the discretized domain of analysis, and the design variables are the relative densities of the elements. Thus, the elastic properties are modeled from the relative density of the material raised to a given power, to penalize the intermediate values for the relative densities of the material, Bendsøe [11], Zhou and Rozvany [12] and Mlejnek [13].

The ESO method initially proposed by Xie and Steven [14] is built on a pure heuristic principle that removing inefficient materials, and the structure evolves towards an optimum. Initially, ESO was implemented solely as a material removal method, which meant that removed parts could not be restored afterward. However, this led to convergence issues and mesh-dependence. The latter problems were overcome by extending the method called Bidirectional Evolutionary Structural Optimization (BESO) that allowed both material addition and removal, Huang and Xie [15]. However, the solution may worsen in terms of the objective function if the ESO/BESO technique continues with no stop or reach a local optimum, Rozvany [16]. Because the initial development of ESO methods is based on a heuristic concept and lacks theoretical rigor, most of the early work on ESO /BESO neglected significant numerical problems in TO, such as the existence of a solution, checker-board, mesh-dependency and local optimum, Xia *et al.* [17].

An interesting method called Genetic Evolutionary Structural Optimization (GESO) integrates the genetic algorithm (GA) with ESO to form a new algorithm, which takes advantage of the excellent behavior of the GA in searching for global optimums, Liu *et al.* [18]. It imposes an n bits length chromosome whose values of genes are all “1” to each element (n is selected arbitrarily), then removes elements whose values of genes are all ‘0’. However, GA solutions are computationally expensive in many cases and computationally prohibitive in some cases, Jakiela [19].

In this work, a new approach for Topology Optimization (TO) of two-dimensional continuum elastic structures through the Progressive Directional Selection (PDS) method is presented. The PDS is demonstrated on strain energy minimization problems. Two examples are analyzed to investigate the proposed method: an instructional problem that shows the essence of PDS on a rigid body and a bidimensional cantilever beam is analyzed applying finite-volume theory [20, 21] to define the optimum design by the PDS method.

2 Progressive Directional Selection Method

Traditional topological optimization methods generally seek the best design of a structure that produces the most rigid response with a given volume of material. In the traditional ESO method, a structure can be optimized by removing elements and, if the correct parameters are provided, the solution can be achieved. Although it is difficult to define these parameters, several prior analyzes need to occur until an engineer decides which solution to adopt. To overcome this problem, the Progressive Directional Selection method, inspired by Darwin's natural selection theory, specifically the directional selection, takes a discrete problem as a discretized structure in a "population" of structural elements. The selection can then be made progressively by eliminating the percentage of individuals who least contribute to the structure's stiffness.

In nature, when directional selection acts on a population, a specific characteristic can guarantee these individuals' survivor. Thus, the PDS method optimizes the structure by minimizing the objective function (compliance, strain energy, and von Mises stress) and defines which structural elements will remain at the end of the selection. The process is simple because, once the desired final volume of the structure is known, the main idea is to gradually remove the elements from an initial configuration, as many times as necessary, wherein each stage increases the number of removings and decreases the number of removed elements by removing, until verifying whether the process leads to the same solution.

For instance, suppose a floor of a building in which the designer needs to define the position of 10 columns, knowing that there are 20 possibilities. In this case, the initial population of 20 columns will go through progressive directional selection, where the individuals are the columns. Once the loads are applied to the structure, the first stage of selection is to remove the ten columns that contribute the least, at once. The number of elements removed is partitioning in the following stages to delete them progressively from the initial set. For example, the second stage is divided into two steps, first removing the first five columns that contribute the least, then applying the loads again to the remaining 15 elements, and then removing the last five columns, leaving the desired number of columns. In the third stage, there are three steps for removing the individuals. It can be done by removing the 4, 3, and 3 columns in each step, respectively. By the rearrange of internal forces, the final population of columns of each stage can be different. The PDS follows until the final population of the last stage of selection's structural elements is the same as the previous stages. In the next section, the primary procedure for implementing the PDS method will be presented in detail.

2.1 Numerical implementation of PDS

Based on the performance criteria adopted for the problem, the selected population is reached through an iterative process that converges when the optimal topology does not evolve anymore, i.e., there is no change in the final set of selected elements. The proposed PDS technique applies a strategy to minimize the objective function.

The optimization problem in their standard form can be expressed mathematically as:

$$
\begin{aligned}
&Minimize\ U = \frac{1}{2}\sum_{e=1}^{N} \boldsymbol{d}_e^t \boldsymbol{k}_e \boldsymbol{d}_e \\
&subject\ to\!: \boldsymbol{KD} = \boldsymbol{F} \\
&\frac{V}{V_0} = f
\end{aligned}
\tag{1}
$$

where U is the total strain energy of the population, $\boldsymbol{d}_e$ and $\boldsymbol{D}$ are respectively local and global displacement vector, $\boldsymbol{k}_e$ and $\boldsymbol{K}$ are respectively local and global stiffness matrix, $\boldsymbol{F}$ is the global force vector, N is the number of elements of the discretized design domain, V and V_0 are the material volume and design domain volume, respectively, and f is the prescribed volume fraction.

The procedures for running PDS are the following:

1. Initialize an original model (assemble the stiffness matrix and initial parameters) and determine boundary and loading conditions.
2. Assemble an array that identifies the elements.
3. Start the stage of the selection loop.

4. For the actual stage, specify the number of steps and the number of removed elements for each step.
5. Solve the problem and specify the optimization criterion, for example, strain energy.
6. Ranking individuals according to the optimization criterion.
7. For each step, remove the elements that contribute least to the structure.
8. Save the identities of the selected individuals.
9. Repeat the procedure steps from 4 to 8 until the current stage's identities are the same as one or more previous stages.

For rigid body problems, as beam plate support by springs, the removing procedure of the springs can be set equal to zero the spring constant K. However, in the case of continuous two-dimensional elastic structures, it is necessary to apply a penalty factor (PT) to the stiffness of the eliminated elements of the discretized analyzed domain, to avoid remeshing and singularity of the global stiffness matrix. In practice, values in the order of magnitude $PT = 10^{-6}$ are recommended.

Particular attention should be given to the classification of the individuals. To achieve a good ranking that overcomes some numeric precision problems, for each removal step, a selection tolerance (ST) is applied to the ranked array, which can add an element with a value of the objective function very close to the last selected element at the ranked array. This procedure directly interferes with the topology's symmetry, especially in analyzing two-dimensional continuous structures.

Another aspect of the procedure is to define the number of removed individuals (NR^{step}) in each removal step. For the *i-th* selection stage, NR^{step} is initially proportional to the final number of selected individuals (NS), which in the case of continuous problems, refers to the final volume desired for the structure. Thus, $NR^{step} = NS/i$, which for computational reasons to access an array by index, it must be an integer. When this does not happen, the NR^{step} must be adjusted, redistributing the decimal part among the other steps, as explained at the beginning of this chapter, in the example of the floor of a building, in the third stage of selection.

Another aspect of the procedure is to define the number of removed individuals (NR^{step}) in each removal step. The *i-th* selection stage is initially proportional to the final number of removed individuals (NS), which refers to the final volume desired for the structure in continuous problems. Thus, $NR^{step} = NS/i$, which for computational reasons to access an array by index must be an integer. When this does not occur, the NR^{step} must be adjusted, redistributing the decimal part among the other steps, as explained at the beginning of this chapter, in the example of a building's floor, in the third stage selection. The problem is treated as essentially it is, a discrete problem, differently of the approaches based on the concept of material density and penalization technique.

3 Examples and Discussion

3.1 Rigid Beam on Springs

The first example, shown in Figure 1, is a rigid beam supported by springs and is subject to a triangular distributed load $q_1 = 0$ e $q_2 = 250N/m$. The spring constants' values are obtained from a normal distribution, with mean $\mu_K = 10N/m$ and standard deviation $\sigma_K = 1N/m$. The beam length is 10, and the thickness is neglected. The number of the original population is $N = 1000$ springs, and two cases of selections are verified for 400 and 300 selected individuals. Two convergence criteria are adopted to define an algorithm stop strategy. Criterion C2 was implemented for the repetition of selected springs in 2 consecutive stages and criterion C3 for the repetition of the final set of selected springs in 3 consecutive stages.

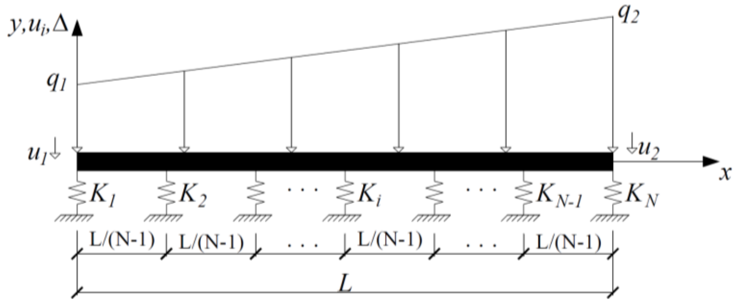


Figure 1. Rigid beam supported by springs.

The optimization criteria are the minimization of the elastic potential energy (EPE) of the springs. The EPE of the *i-th* spring can be expressed as follows:

$$U_i = \frac{1}{2} \cdot K_i \cdot \Delta_i^2 \, , \tag{2}$$

where K_i and Δ_i are the spring constant and the elongation of the *i-th* spring, respectively. The Δ_i can be defined in terms of degrees of freedom u_1 and u_2:

$$\Delta_i = \left(\frac{i-1}{N-1} - 1\right) u_1 - \left(\frac{i-1}{N-1}\right) u_2, \tag{3}$$

Figure 2 shows the PDS computing results for the rigid beam. Fig. 2 (a) and (c) present the histograms of the original population and selected 400 and 300 springs with C2 criteria. The histogram of the selected population indicates a behavior like that found in the biology literature regarding directional natural selection, where the normal probability density function shifts from the initial situation. Furthermore, in Fig. 2 (b) and (d), the mean and standard deviation evolution are shown with their minimization, which indicates the apparent trend of convergence.

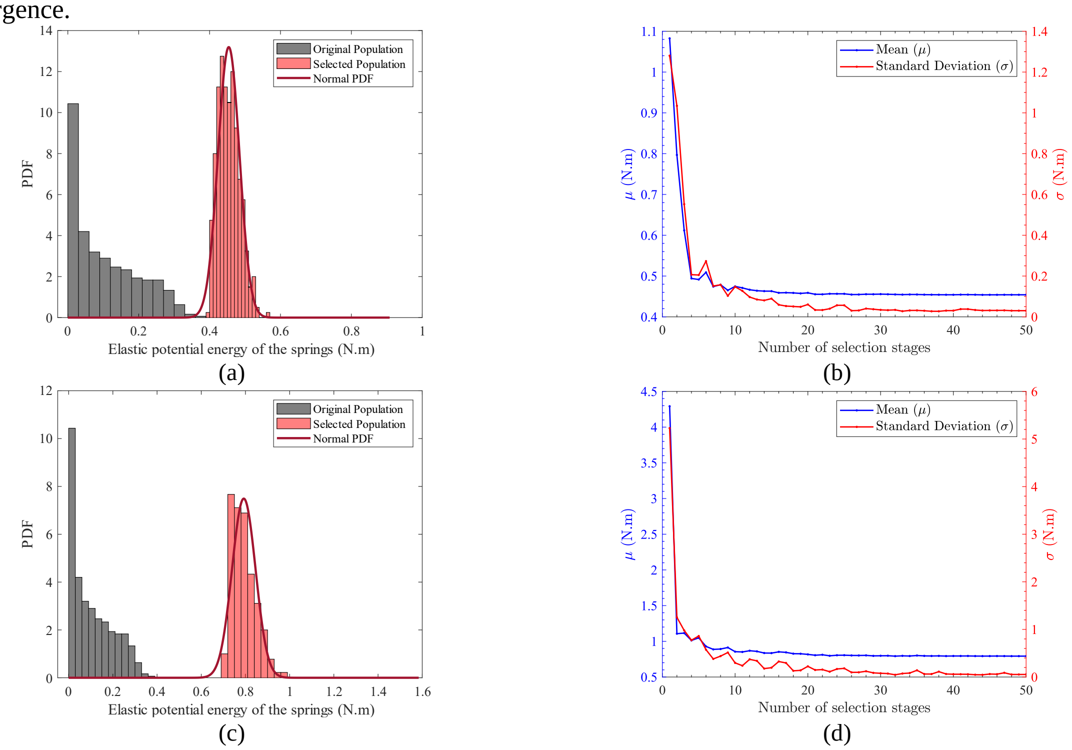
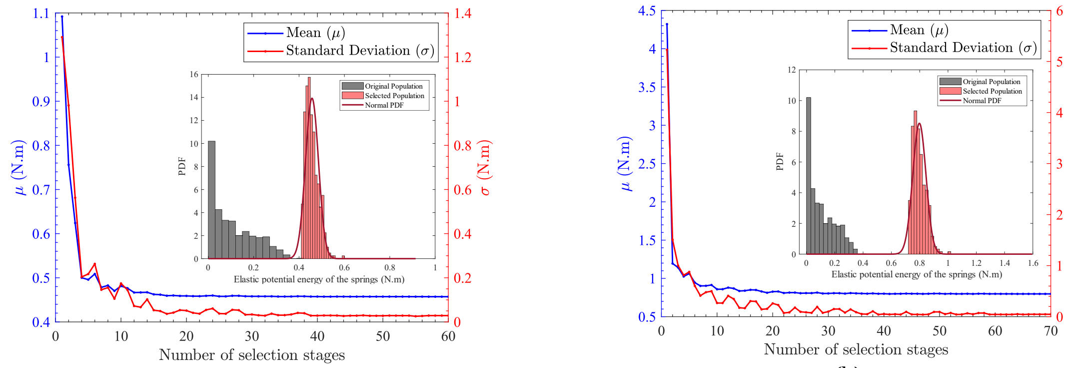


Figure 2. PDS computing results for C2.

For the C3 criteria, the original population's histograms, and selected selection of 400 and 300 springs are similar to the previous analyses. Fig. 3 (a) and (b) present the mean and standard deviation evolution. There is an increase in the selection stages, from 50 to 60 and 70 for the 400 and 300 spring selections, respectively. This increase indicates that despite the C2 criterion showing an apparent convergence trend, applying C3 improves the solution's reliability, once similar mean values are found and smaller standard deviation, which means the springs are more uniformly solicited, as shown in Table 1.

Table 1. Comparison of results for the adopted convergence criteria.

Number selected springs	EPE Mean (μ)		EPE Standard deviation (σ)		Number of stages	
	C2	C3	C2	C3	C2	C3
300	0.7925	0.7977	0.0533	0.0468	50	70
400	0.4541	0.4571	0.0302	0.0286	50	60

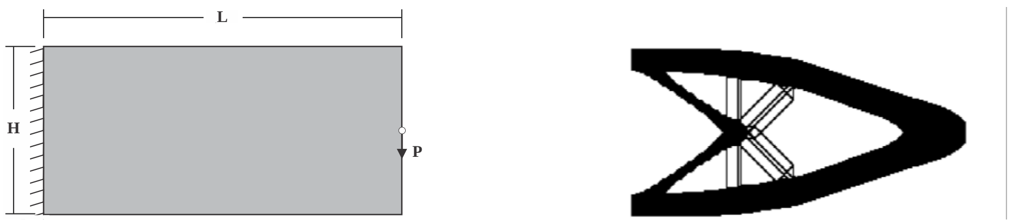


(a) (b)

Figure 3. PDS computing results for C3.

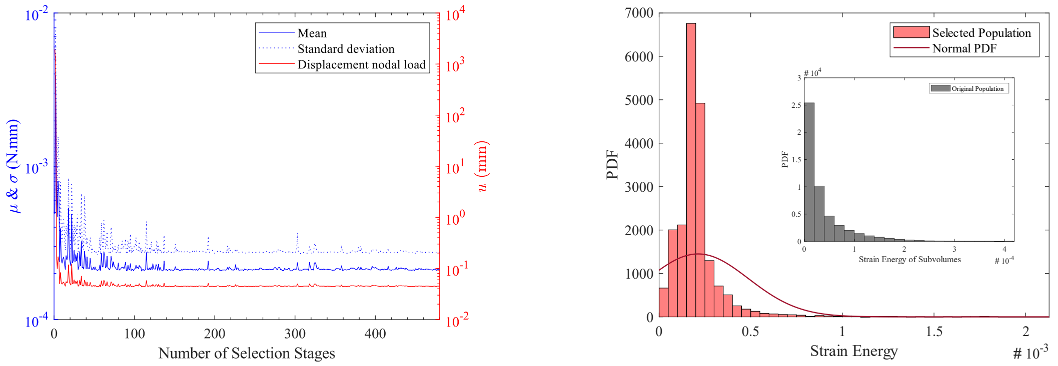
3.2 Cantilever beam

The second example is a cantilever beam fixed in the left border and a concentrated load in the middle of the right border, under the given boundary conditions shown in Figure 4 (a). The proposed optimization problem consists of minimizing the strain energy of the subvolumes. Also, this problem's objective is to find the stiffest structure with a given volume of 40% of the design domain volume. The convergence criterium used is implemented to repeat the set of selected subvolumes in 3 consecutive stages. The dimensions for the design domain are $H = 400\,mm$, $B = 800\,mm$ and thickness $t = 10\,mm$. Young's modulus $E = 200 \times 10^3\,MPa$ and Poisson's ratio $\upsilon = 0.3$ are assumed. The penalty factor is $PT = 10^{-6}$ and the adopted selection tolerance is $ST = 10^{-6}$. The structure is analyzed with a mesh of 230×115 quadrilateral subvolumes. In this case, each subvolume in the mesh is treated as an individual from the initial population of 26,450 subvolumes.



(a) (b)

Figure 4. Cantilever beam.



(a) (b)

Figure 5. PDS computing results for the cantilever beam.

The formulation of the finite-volume theory employed in this example has its bases on the zeroth-order Cartesian formulation for bidimensional structures of the finite-volume theory presented by Cavalcante and Pindera [20, 21]. Figure 4 (b) shows the optimal topology for the cantilever beam. This optimal topology is similar to several results found in the literature for this type of problem – [5], [9], [14], and [17] – indicating the potential of the PDS. The mesh dependency is noticed, with slander bars in the optimal topology, which can be treated by some filtering technique. As in the previous example, the method's convergence is remarkable, indicating the evolution of the mean and the standard deviation to the minimization, Figure 5. The histogram of the selected population shifted as expected.

4 Conclusions

A new approach for Topology Optimization (TO) for two-dimensional continuum elastic structures through the Progressive Directional Selection (PDS) method has been presented in this paper. Numerical examples considered herein have shown that optimal structures can be achieved by the proposed method. However, it is important to continue investigating the PDS method's application in other topological optimization problems and thoroughly comparing it with results from other structures found in the literature.

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References

[1] C. Darwin and W. F. Bynum. *The origin of species by means of natural selection: or, the preservation of favored races in the struggle for life*. New York: AL Burt, 2009.
[2] M. Ridley. Evolution. Wiley-Blackwell, 2003.
[3] D. O. Conover and S. R. Munch. "Sustaining fishery yields over evolutionary time scales". *Science*, v. 297, n. 5578, p. 94–96, 2002.
[4] M. P. Bendsøe and N. Kikuchi. "Generating optimal topologies in structural design using a homogenization method". Computer Methods in Applied Mechanics and Engineering. v. 71, pp. 197–224, 1988.
[5] G. P. Steven and Y.M. Xie, "Evolutionary Topology Optimization of Continuum Structures: Methods and Applications", John Wiley & Sons, 2010.
[6] D. J. Munk, G. A. Vio and G. P. Steven. "Topology and shape optimization methods using evolutionary algorithms: a review". Structural and Multidisciplinary Optimization, v. 52, n. 3, pp. 613–631, 2015.
[7] L. C. Neches, A. P. Cisilino. Topology optimization of 2D elastic structures using boundary elements. "Engineering Analysis with Boundary Elements", 32(7), 533–544, 2008
[8] H. L. Simonetti, V. S. Almeida, V. M. A. Leitão and F. D. A. das Neves. "Application of evolutionary structural optimization based on the boundary element method using initial stress fields". Structures, v. 20, pp. 550–559, 2019.
[9] M. V. O. Araujo, E. N. Lages and M. A. Cavalcante, "Checkerboard free topology optimization for compliance minimization applying the finite-volume theory". Mechanics Research Communications, p. 103581, 2020.
[10] M.P. Bendsøe and O. Sigmund. Topology Optimization: Theory, Methods and Applications, Springer–Verlag Berlin Heidelberg, 2003.
[11] M. P. Bendsøe. "Optimal Shape design as a material distribution problem". Structural Optimization, v. 1, pp. 193–202, 1989.
[12] M. Zhou and G. I. N. Rozvany, "The COC algorithm, part II: Topological, geometry and generalized shape optimization". Computer Methods in Applied Mechanics and Engineering, Essen, v. 89, pp. 197–224, 1991.
[13] H. P. Mlejnek, "Some aspects of the genesis of structures". Structural and Multidisciplinary Optimization, v. 5, pp. 64–69, 1992.
[14] G. P. Steven and Y.M. Xie, "A Simple Evolutionary Procedure for Structural Optimization", Computers & Structures, v. 49, n. 5, pp. 885–401, 1993.
[15] X. Huang, Y.M. Xie, "Convergent and mesh-independent solutions for the bi-directional evolutionary structural optimization method". Finite Elements in Analysis and Design, v. 43, n. 14, pp. 1039–1049, 2007.
[16] G. I. N. Rozvany, "Stress ratio and compliance based methods in topology optimization–a critical review". Structural and Multidisciplinary Optimization, v. 21, n. 2, 109–119, 2001.
[17] L. Xia, Q. Xia, X. Huang and Y. M. Xie, "Bi-directional evolutionary structural optimization on advanced structures and materials: a comprehensive review". Archives of Computational Methods in Engineering, v. 25, n. 2, pp. 437–478, 2018.
[18] X. Liu, W. J. Yi, Q. S. Li and P. S. Shen, "Genetic evolutionary structural optimization". Journal of constructional steel research, v. 64, n. 3, pp. 305–311, 2008.
[19] M. J. Jakiela, C. Chapman, J. Duda, A. Adewuya, and K. Saitou, "Continuum structural topology design with genetic algorithms". Computer Methods in Applied Mechanics and Engineering, v. 186, n. 2–4, pp. 339–356, 2000.
[20] M. A. A. Cavalcante and M. J. Pindera. Generalized finite-volume theory for elastic stress analysis in solid mechanics – part I: framework. Journal of Applied Mechanics, vol. 79, n. 5, pp. 051006, 2012.
[21] M. A. A. Cavalcante and M. J. Pindera. Generalized finite-volume theory for elastic stress analysis in solid mechanics – part II: results. Journal of Applied Mechanics, vol. 79, n. 5, pp. 051007, 2012.