Theoretical study of Sommerfeld effect in flexible rotor shaft on structural model with two degrees of freedom

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Abstract. This paper address nonlinear dynamical analysis of a foundation structure for a flexible rotor machine. The residual unbalance will always be present in rotating equipment systems. This aspect is often not considered during the design steps, although all real motors are non-ideal power sources. Thus, the interest in further studying the machine-foundation and soil-foundation interactions.
Our mathematical model considers this system as non-ideal, subjected to the Sommerfeld effect, which may manifest close to the low power machine's resonance, with possible jumps from lower to higher frequency rotation regimes without intermediate stable solutions. The model proposed is defined by two degrees of freedom, vertical and horizontal translations, and an additional one associated with the rotation of the rotor shaft (intrinsic to the so-called non-ideal systems).
The mathematical model that describes the system's motion is derived via Lagrange equations. The solution of this system of differential equations can in principle be solved analytically, but this can be difficult or even impossible in some cases, particularly when these equations are nonlinear, such as the proposed model. The numerical solution adopted here will be implemented in the Matlab®. This paper aims to analyze a little studied problem of practical importance.

Keywords: machine-foundation interaction, Sommerfeld Effect, non-ideal systems, soil-structure interaction, elastic half-space theory.

1 Introduction

For many systems, disregard the influence of structural motion on their excitation source is an acceptable simplification, but for many others it is not. Sommerfeld (1904) was the first to study the phenomenon of this interaction, later called the Sommerfeld Effect, making an experiment of an elastic base excited by an unbalanced machine. A few years later this experiment was replicated by Kononenko and Korablev [1], which had their work re-analyzed by Nayfeh and Mook [2].

Non-ideal systems are those in which the structural motion influences its source of excitation. These systems can be linear, or nonlinear, regardless of its excitation. In general, its power supply is limited, moving even further away from the ideal system the greater the machine-structure interaction is. Mathematically it is imperative to exist an equation that describes the dynamics of the motor. Therefore, an additional degree of freedom is required to model non-ideal systems, Nayfeh and Mook [2].

In this work we develop a non-ideal system model of a machine with a unbalanced flexible rotor. This non-ideal system is composed of a rigid foundation supported directly on the ground, for which Petrobras N-1848 [3], recommends considering the elastic theory of half-space for the foundation. Criteria and analysis of this type can be found in Brasil [4] and Bachmann [5].

From 2003 to 2017, according to theoretical reviews by Balthazar et al. [6] [7], there are new phenomena and emerging areas of structures that support machine with unbalanced rotor. Balthazar's review presented, for example, results for the Sommerfeld effect, the phenomenon of saturation and the influence of fractional damping. This kind of review is necessary and relevant to deepen the scientific researches in this field.

This work aims to present an ongoing research of the machine-foundation and soil-foundation interactions. The mathematical development of the proposed non-ideal system will be done via Lagrange equations. In section 2, the physical model representing the foundation structure and its driver source, the machine, will be presented. Next, section 3 presents the mathematical model composed of equations that describe the displacements and velocities of the physical model, as well as the formulas for the calculation of stiffness and damping according to the elastic theory of half-space (taken from Petrobras standard). The mathematical model describing the motion of the system proposed in section 4 is obtained using Lagrange's equations. Section 5 will present the final discussion and conclusions.

2 Physical model

The proposed physical model considers the machine and structure interaction and consists of three degrees of freedom, the two translations (vertical and horizontal) and one associated with the rotation of this axis. This additional degree of freedom is typical of the so-called non-ideal systems, as can be seen in Figure 1.

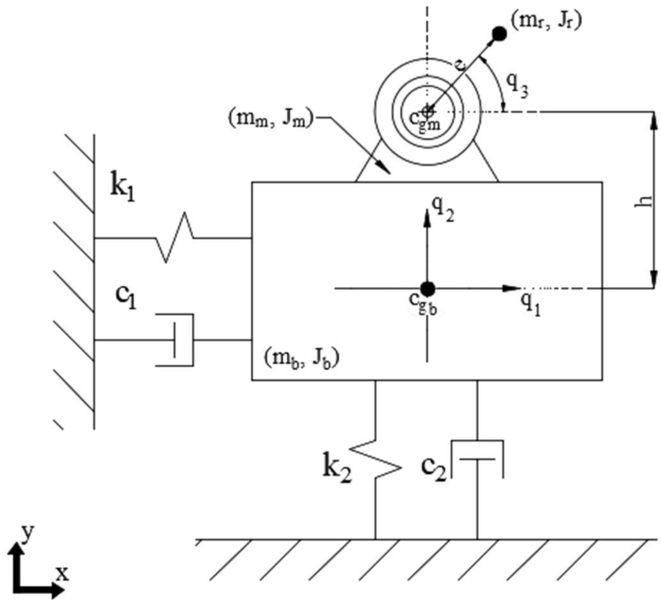

Figure 1 - model of a foundation structure with unbalanced excitation source

The damping $C_i \quad (i = 1, 2)$ and stiffness $K_i \quad (i = 1, 2)$ of the model are soil parameters, supposed elastic, homogeneous and isotropic, according to Boussinesq (1885, apud Petrobras N- 1848 [3]).

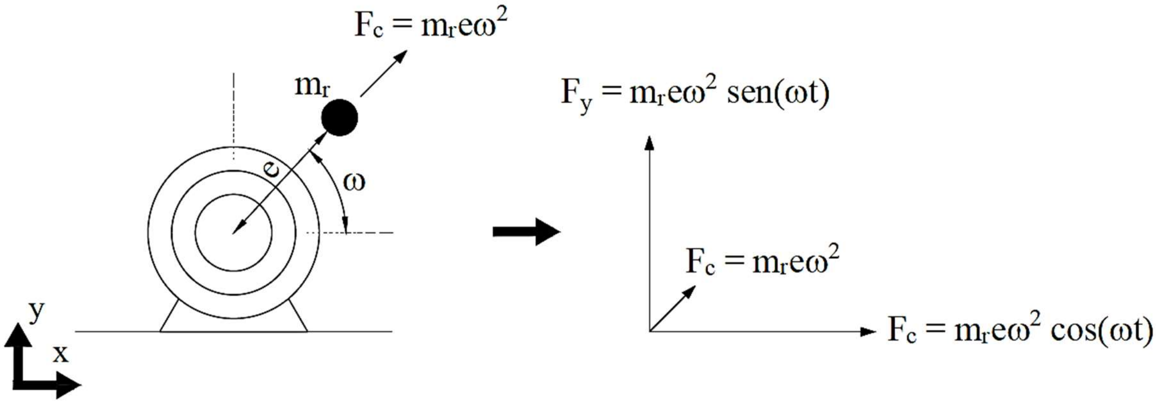

Figure 2 - decomposition of the centrifugal force

Figure 2 shows the unbalanced forces acting in the $x - y$ system.

3 Mathematical model

Parameters q_1 and q_2 are, respectively, the generalized coordinates related the horizontal and vertical motions, and the q_3 parameter is the angular velocity of the motor shaft. The eccentricity (e) is obtained through the quality of the balance of rotating machines with flexible shaft and the height (h) between the motor shaft and the foundation axes (Figure 1).

3.1 Unbalanced mass (M_r)

The force exercised in the unbalanced mass is also obtained in the Figures 1 and 2:

$$x_r = q_1 + e\,cos(q_3) \qquad y_r = q_2 + e\ \ sin(q_3) + h \tag{1}$$

$$\dot{x}_r = \dot{q}_1 - \dot{q}_3 e\,sin(q_3) \qquad \dot{y}_r = \dot{q}_2 + \ \dot{q}_3 e\,cos(q_3) \tag{2}$$

3.2 Rotor mass (M_m)

The machine presented in the Figure 1 will have the influence of the forces:

$$x_m = q_1 \qquad y_m = q_2 + h \tag{3}$$

$$\dot{x}_m = \dot{q}_1 \qquad \dot{y}_m = \dot{q}_2 \tag{4}$$

3.3 Mass of the foundation (M_b)

The foundation will have the influence similar to the forces in the machine:

$$x_b = q_1 \qquad y_b = q_2 \tag{5}$$

$$\dot{x}_b = \dot{q}_1 \qquad \dot{y}_b = \dot{q}_2 \tag{6}$$

3.4 Kinetic Energy (*T*)

The energy of the motion is obtained as followed:

$$T=\frac{1}{2}\left\{\begin{matrix} M_r[(\dot{q}_1\text{-}\dot{q}_3 e\sin(q_3))^2+(\dot{q}_2+\dot{q}_3 e\cos(q_3))^2]+M_m\left(\dot{q}_1{}^2+\dot{q}_2{}^2\right)+M_b\left(\dot{q}_1{}^2+\dot{q}_2{}^2\right)+ \\ +J_r\dot{q}_3^2 \end{matrix}\right\} \quad (7)$$

In which J_r is the machine rotor's moment of inertia.

3.5 Strain energy (*U*)

In this case the deformation energy can be obtained by:

$$U=\frac{1}{2}(K_1 q_1^2+K_2 q_2^2) \quad (8)$$

In which $K_i \quad (i = 1,\ 2)$ are the soil stiffness coefficients.

3.6 Total Potential Energy (*V*)

The energy consumption will be determined by:

$$V=U-W \quad (9)$$

$$W=-g[M_r(q_2+e\,sin(\,q_3)+h)+M_m(q_2+h)+M_b(q_2)] \quad (10)$$

In which g it is the acceleration due to gravity.

3.7 Elastic half-space theory

Table1provides constants of stiffness and damping of a foundation that is directly supported on the ground, according to the degrees of freedom of the model considered here. The auxiliary coefficient (mass ratio) is a dimensionless parameter created to assist in the intermediate calculations of the table, Petrobras N-1848 (2011).

Table1- adaptation, damping and stiffness by Petrobras N-1848 (2011)

Vibration model	Razão de massa (auxiliary coefficient)	Damping factor	Stiffness constant
Translation in X	$B_x=\frac{(7-8v)}{32(1-v)}\frac{m}{\rho\, r_x^3}$	$C_1=\frac{0{,}2875}{\sqrt{B_x}}$	$K_1=\frac{32(1-v)}{7-8v}G\, r_x$
Translation in Y	$B_y=\frac{(1-v)}{4}\frac{m}{\rho\, r_y^3}$	$C_2=\frac{0{,}425}{\sqrt{B_y}}$	$K_2=\frac{4\,G\,r_y}{3(1-v)}$

In which $G,\ \rho,\ v$ and $r_i\ (i = x,\ y)$ are respectively, soil shear modulus, soil specific mass, soil's Poisson's ratio and radii equivalent to the area of contact between the foundation and the ground for the purpose of translation of the foundation-machine assembly.

4 Lagrange equations

The first equation of Lagrange can be presented as:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right)-\frac{\partial L}{\partial q_i}=N_i \quad (i=1,2,3) \tag{11}$$

In which $L = T - V$ is the Lagrangean function and N_i are the non-conservative generalized loads.

4.1 Motion equations

4.1.1 First degree of freedom

For the first degree of freedom the equation of motion is:

$$(M_r + M_m + M_b)\ddot{q}_1 + C_1\dot{q}_1 + K_1 q_1 = M_r e(\ddot{q}_3\, sin(\, q_3) + \dot{q}_3\, cos(\, q_3)) \tag{12}$$

4.1.2 Second degree of freedom

For the second degree of freedom the equation of motion is:

$$\begin{gathered}(M_r + M_m + M_b)\ddot{q}_2 + C_2\dot{q}_2 + K_2 q_2 = -M_r e(\ddot{q}_3\, cos(\, q_3) - \dot{q}_3\, sin(\, q_3)) + \\ -g(M_r + M_m + M_b)\end{gathered} \tag{13}$$

4.1.3 Third degree of freedom

For the third degree of freedom the equation of motion is:

$$\begin{gathered}(M_r e^2 + J_r)\ddot{q}_3 = \Im\dot{q}_4 + M_r e[(\ddot{q}_1 + \dot{q}_2)\, sin\, q_3 + (\dot{q}_1 - \ddot{q}_2)\, cos\, q_3] \\ -g M_r e\, cos(\, q_3)\end{gathered} \tag{14}$$

In which $\Im\dot{q}_4$ is the net torque of the rotor.

4.2 Isolation from accelerations

Re-writing the equations of motion (12 to 14) in a matrix:

$$\begin{bmatrix}(M_r + M_m + M_b) & 0 & -M_r e\, sin(\, q_3) \\ 0 & (M_r + M_m + M_b) & M_r e\, cos(\, q_3) \\ -M_r e\, sin(\, q_3) & M_r e\, cos(\, q_3) & M_r e^2 + J_r\end{bmatrix}\begin{Bmatrix}\ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3\end{Bmatrix} =$$

$$= \begin{Bmatrix} -K_1 q_1 - C_1\dot{q}_1 + M_r e(\dot{q}_3\, cos(\, q_3)) \\ -K_2 q_2 - C_2\dot{q}_2 + M_r e(\dot{q}_3\, sin(\, q_3)) - g(M_r + M_m + M_b) \\ \Im\dot{q}_4 + M_r e(\dot{q}_1\, cos(\, q_3) + \dot{q}_2\, sin(\, q_3) - g M_r e\, cos(\, q_3) \end{Bmatrix} \tag{15}$$

Equations of this type are difficult to solve, so is convenient to transform the second order differential equation system into a first order differential equation system (16 and 17).

$$x_1 = q_1 \quad x_2 = \dot{q}_1 \quad x_3 = q_2 \quad x_4 = \dot{q}_2 \quad x_5 = q_3 \quad x_6 = \dot{q}_3 \quad x_7 = t \tag{16}$$

$$\dot{x}_1 = x_2 \quad \dot{x}_2 = \ddot{q}_1 \quad \dot{x}_3 = x_4 \quad \dot{x}_4 = \ddot{q}_2 \quad \dot{x}_5 = x_6 \quad \dot{x}_6 = \ddot{q}_3 \quad \dot{x}_7 = 1 \tag{17}$$

5 Discussion and conclusions

Studies of models considering non-ideal systems are important for a better practice of engineering, but are not usually done, being replaced by approximations. The system of differential equations of the model studied here is coupled, nonlinear and of second order, quite difficult to solve analytically. So, to obtain the numerical solution is necessary to use a numerical method.

Among the possible solver methods to this model, the Runge-Kutta method will be used in the next stages of development. In this system the soil was modeled by the elastic half-space theory (according to the Petrobras N-1848 standard of rigid foundations) to obtain stiffness and damping (two parameters that are fundamental in the modeling results).

As explained before, as this is a nonlinear mathematical model, it is convenient to solve it with a numerical method. In the future, the solution for this model will be developed in the Matlab® program using the ode45 function, that uses the combination of fourth and fifth order Runge-Kutta methods. The implementation of the mathematical model requires the transformation of the of second-order differential equations systems (15) into a first order system (16,17) to achieve the objective that is to assess if the modeling of this type of foundation as non-ideal systems can be important in practice.

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