XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Spectral characterization of the electron drift instability in a Hall thruster

Rodrigo A. F. Alves [1], Rodrigo A. Miranda[1],

[1] *UnB-Gama Campus and Plasma Physics Laboratory, University of Brasilia (UnB), Brasilia-DF, Brazil.*
rodrigoalkimim@gmail.com, rmiracer@gmail.com

Abstract. The Hall thruster is an electric propulsion device commonly used for station-keeping purposes in satellites and in deep-space missions since 1971. This device generates thrust from the interaction between a plasma and electromagnetic fields. Several aspects of this interaction and the thruster operation are still not well understood. Recently, numerical simulations and experimental measurements have shown the occurence of a high-frequency, low-wavenumber instability known as the $\mathbf{E} \times \mathbf{B}$ electron drift instability. This instability modifies the electron cross-field mobility in the plasma and can affect the thruster efficiency.

In this study we perform numerical simulations of a SPT-100 Hall thruster using the particle-in-cell method. We describe a two-dimensional model in cylindrical coordinates in which the axial and azimuthal directions are kept, neglecting variations in the radial direction. Our numerical simulations show that the instability induces a large-amplitude wave in the azimuthal electric field and the ion density, as expected. We proceed by characterizing the frequency and wavenumber of this instability using a spectral analysis, and compute power-laws in the resulting power spectra. Our results demonstrate that the plasma in a Hall thruster displays a turbulent behavior with an energy cascade induced by the $\mathbf{E} \times \mathbf{B}$ electron drift instability. We also show that the microturbulence is characterized by an inertial subrange which extends to scales smaller than 1 mm.

Keywords: Hall thruster, Plasmas, Particle-in-cell simulation, Electron drift instability, turbulence

1 Introduction

The Hall thruster is a technology mainly used for orbit-correction of satellites and propulsion for deep-space probes. This device generates thrust from the interaction between a plasma and electromagnetic fields. Hall thrusters have been employed in space technologies since 1960, however several aspects are still not well understood. Recently it was demonstrated that an instability known as $\mathbf{E} \times \mathbf{B}$ electron drift instability (EDI) arises in Hall thrusters in numerical simulations [1], as well as experimental measurements[2]. This instability is characterized by a high frequency and low wave number, and can modify the mobility of electrons in the plasma reducing the efficiency of the device.

In this work we focus on the microturbulence due to the $\mathbf{E} \times \mathbf{B}$ EDI. We perform numerical simulations of the Hall thruster SPT-100, using the particle-in-cell method. The device is described using a two-dimensional model in cylindrical coordinates, keeping the axial and azimuthal variations and neglecting changes in the radial direction. Our numerical simulations show that the instability induces a large-amplitude wave in the azimuthal electric field and in the ion density. We also demonstrate that the EDI-induced turbulence is characterized by an inertial subrange with a power-law $\sim -5/3$.

2 Methodology

The plasma dynamics in a Hall thruster is best described using a Particle-in-Cell (PIC) approach. Three-dimensional self-consistent simulations are extremely difficult to solve and can take a large computational time [3]. For this reason most numerical studies define a one- or two-dimensional model by neglecting one spatial direction. For the present study a two-dimensional domain is defined by keeping the axial and azimuthal directions, and neglecting variations in the radial direction. This is justified because the $\mathbf{E} \times \mathbf{B}$ EDI occurs in the azimuthal direction, and ions are accelerated in the axial direction. Models that include the radial direction are important to understand wall-particle interactions and wall erosion [4] which are outside the scope of this paper.

A Cartesian system was adopted by neglecting the curvature of the Hall thruster. This is shown in figure 1. Periodic boundary conditions are imposed in the azimuthal direction. In the axial axis, the boundary conditions are adopted as follows. Particles that cross the boundaries at $z = 0$ and $z = 4cm$ are removed from the system. Dirichlet boundary conditions were imposed to the electrostatic potential, namely, a potential of 300 V at the anode ($z = 0$) and 0 V at the cathode ($z = 4cm$).

We use the XOOPIC code [5] in which the plasma charged particles are modelled using the PIC method. In this method the charged particles are represented by computational particles, and the electromagnetic field are computed self-consistently [6]. There are several techniques that can be applied to reduce the computational effort, for example, reduction of the proton-electron mass ratio, and modification of the vacuum permittivity. However the interpretation of the results can become difficult. We avoid to apply any of these simplifications and choose to use argon gas as the propellant due to the reduced ion mass in comparison to xenon gas.

The dotted line in Figure 1 at $z = 2.5cm$ is the exhaust plane which is the limit between the channel region ($0 \leq z \leq 0.025m$) and the external side ($z \geq 0.025m$). The exhaust plane coincides with the maximum value of the magnetic field $B_0 = 140$ gauss.

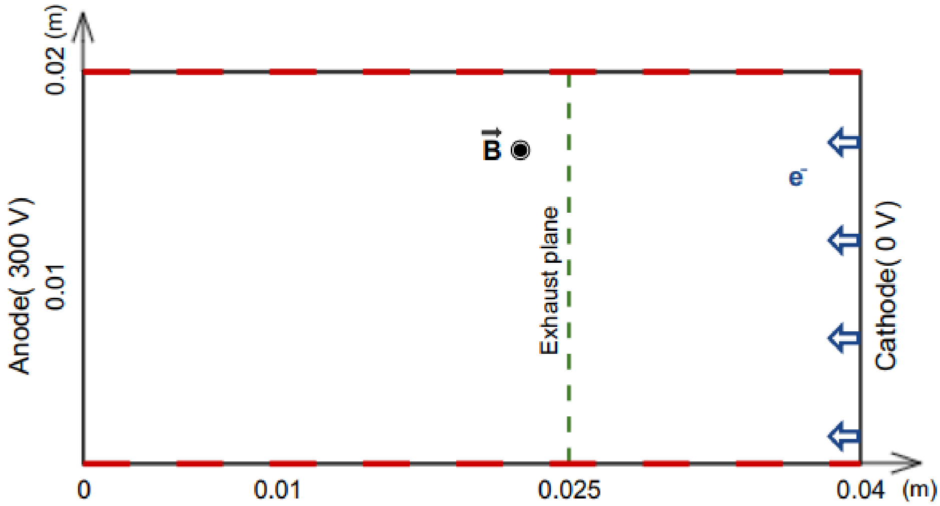

Figure 1. Axial-azimuthal model of a Hall thruster. The dashed red line represents periodic boundary conditions.

At each time step, new electrons are injected through the cathode plane. The number of injected electrons is set to generate a current of $4.5A$. Its energy distribution is chosen from a Maxwellian energy distribution with a temperature of 1 eV. The time step must be less than the cyclotron frequency of the electron which is given by

$$\omega_{ce} = \frac{eB_0}{m_e}, \tag{1}$$

where B_0 is the modulus of the magnetic field at the exhaust plane (17 mT), e and m_e are the electron charge and mass, respectively. The cyclotron period is given by

$$T_e = \frac{2\pi}{\omega_{ce}}. \tag{2}$$

At the exhaust plane one has $T_e = 2.55 \times 10^{-9}$ s. Thus, the time step was set to $\Delta_t = 10^{-11}$ s.

We assume that the magnetic field is purely radial, and displays a Gaussian shape as a function of z following previous works [1, 4, 7–9]. The maximum value of the radial magnetic field B_0 occurs at the exhaust plane and decreases with the distance from this plane. The magnetic field vector is thus represented by

$$\mathbf{B}(z) = \begin{cases} B_0 \exp\left(-\frac{(z-z_B)^2}{\sigma_{B1}^2}\right)\hat{\mathbf{r}} & 0 < z < z_B, \\ B_0 \exp\left(-\frac{(z-z_B)^2}{\sigma_{B2}^2}\right)\hat{\mathbf{r}} & z_B < z. \end{cases} \tag{3}$$

where $z_B = 0.025$, $\sigma_{B1}^2 = 6.4 \times 10^{-5}$ and $\sigma_{B2}^2 = 4 \times 10^{-4}$.

The distribution of the neutral gas within the acceleration region varies according to the axial direction and is given by

$$n_N(z) = n_0 \left(1 - \tanh\left(\frac{z - z_N}{\sigma_N^2}\right)\right), \tag{4}$$

where $n_0 = 3 \times 10^{19}$, z_N = 0.02 and σ_n^2 = 0.004.

The Fourier transform analysis method allows to relate a function of time $f(t)$ with a function in the frequency domain $F(\omega)$. The functions $f(t)$ and $F(\omega)$ can be regarded as two different representations of the same function [10].

$$\begin{cases} F(\omega) = \int_{-\infty}^{\infty} f(t) \exp^{i\omega t} dt, \\ f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) \exp^{-i\omega t} d\omega \end{cases} \tag{5}$$

If t is measured in seconds, then ω in the above equation is in cycles per second (s^{-1}) or Hertz (Hz). The Fourier analysis can also be applied to other quantities. If f is a function of the position x (e.g., x is in meters), then F will be a function of the inverse of spatial scale $F(k)$, where $k = \frac{1}{\lambda}$ is called the wavenumber. In this paper we use the fast Fourier transform code implemented in the MATLAB software (version R2018a).

3 Results

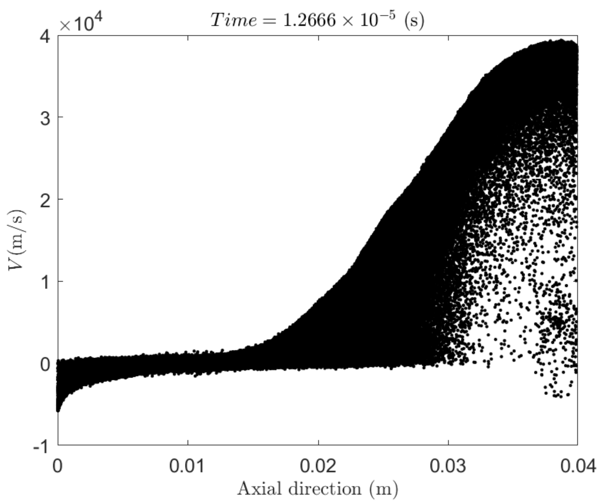

Figure 2. The axial velocity of ions as a function of the axial position z.

The PIC simulations are executed until $t = 0.12666\mu s$ (i.e., after 1266600 time steps). At that time the simulation reaches a steady-state in which the ions created by collisions between electrons and the neutral gas are accelerated towards the exit region. Figure 2 shows a scatter plot of the axial velocity of ion particles as a function of the axial position. This figure reproduces the typical profile of the internal ion flux of a Hall thruster [4, 11]. For example, for $z > 0.02$ m the ions have a positive axial velocity which increases with the distance from the exhaust plane. This indicates that the ions are being accelerated due to the electrostatic potential.

Figure 3 shows the azimuthal electric field E_ϕ and ion density n_i using a discrete color scale which highlights the electron drift instability developing at the exhaust plane. In this simulation the propellant gas is set to represent argon gas. The electron drift instability can be elucidated as elongated wave-like structures appearing at $z = 0.025$. The vertical dashed line indicates the position of the selected data for the spectral analysis.

The upper panel of Fig. 4 shows the power spectrum of the azimuthal component of the electric field. This figure was computed as an average of 12 steady-state power spectra. The main peak at $k \approx 10^3 m^{-1}$ indicates that the characteristic wavelength is $\approx 1mm$, in agreement with previous results [1, 8]. The lower panel of Fig. 4 shows the compensated power spectrum, which was obtained multiplying the power spectrum to $k^{+5/3}$. The dashed lines indicate the wavenumber interval in which the compensated power spectrum is nearly horizontal. This technique allows to clearly elucidate that the power spectrum displays a power-law behavior with a spectral law -1.74 ± 0.087. This value is close to the $-5/3 \sim -1.67$ power-law behavior characteristic of turbulence in neutral fluids and plasmas [12, 13].

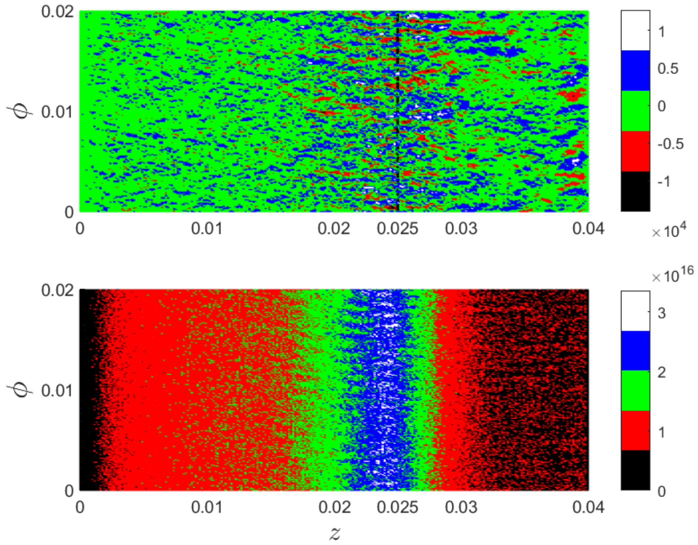

Figure 3. Azimuthal electric field (upper panel) and ion density (lower panel) showing the onset of the electron drift instability with argon as the propellant. The vertical dashed line indicates the position of the selected data for the spectral analysis.

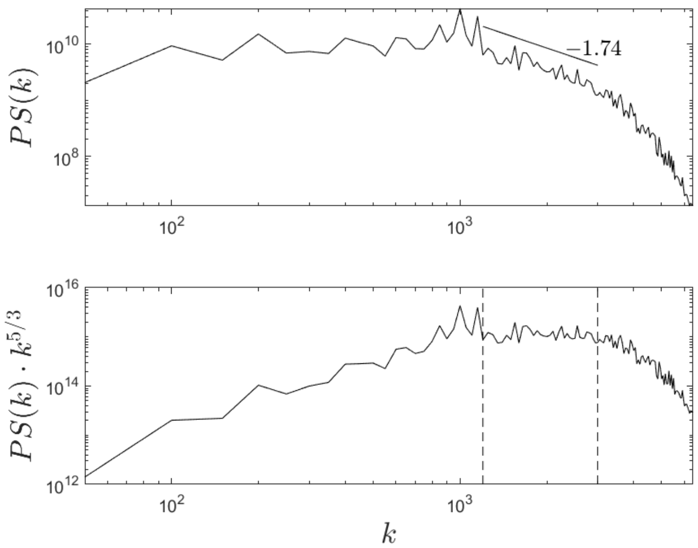

Figure 4. Upper panel: The power spectrum of the azimuthal component of the electric field, in log-log scale. The straight line represents a power law ~ 1.74. Lower panel: The compensated power spectrum. The vertical dashed lines indicates the interval in which the power-law was computed.

4 Conclusions

We presented numerical simulations of a SPT-100 Hall thruster using a two-dimensional model in cylindrical coordinates in which the axial and azimuthal directions are kept, neglecting variations in the radial direction. This geometry allows to study the $\mathbf{E} \times \mathbf{B}$ electron drift instability, which is responsible for the observed anomalous propagation of electrons across the magnetic field. Our numerical simulations show that the instability induces a large-amplitude wave in the azimuthal electric field and the ion density. The frequency and wavenumber of this instability were characterizing by a Fourier analysis. Our results show that the plasma displays a turbulent behavior induced by this instability, and the resulting microturbulence is characterized by an inertial subrange with a power-law behavior which extends to scales smaller than 1 mm. Our results can contribute to the understanding of this instability that can affect the efficiency of Hall thrusters.

Acknowledgements

The authors are grateful to one reviewer for his/her comment. RAFA acknowledges support from DPP/UnB (Edital 11/2019) and PROAP/CAPES (Brazil). RAM acknowledges support from FAP DF (Brazil) under grant 180/2020.

References

[1] Adam, J., Héron, A., & Laval, G., 2004. Study of stationary plasma thrusters using two-dimensional fully kinetic simulations. *Physics of Plasmas*, vol. 11, n. 1, pp. 295–305.

[2] Tsikata, S., Lemoine, N., Pisarev, V., & Gresillon, D., 2009. Dispersion relations of electron density fluctuations in a Hall thruster plasma, observed by collective light scattering. *Physics of Plasmas*, vol. 16, n. 3, pp. 033506.

[3] Boeuf, J.-P., 2017. Tutorial: Physics and modeling of hall thrusters. *Journal of Applied Physics*, vol. 121, n. 1, pp. 011101.

[4] Braga, L. L. & Miranda, R. A., 2019. Particle-in-cell numerical simulation of the PHall-IIc Hall thruster. In *Journal of Physics: Conference Series*, volume 1365, pp. 012013. IOP Publishing.

[5] Verboncoeur, J. P., Langdon, A. B., & Gladd, N., 1995. An object-oriented electromagnetic pic code. *Computer Physics Communications*, vol. 87, n. 1-2, pp. 199–211.

[6] Birdsall, C. K. & Langdon, A. B., 2004. *Plasma physics via computer simulation*. CRC press.

[7] Boeuf, J.-P. & Garrigues, L., 2018. E$\times$ b electron drift instability in Hall thrusters: Particle-in-cell simulations vs. theory. *Physics of Plasmas*, vol. 25, n. 6, pp. 061204.

[8] Coche, P. & Garrigues, L., 2014. A two-dimensional (azimuthal-axial) particle-in-cell model of a Hall thruster. *Physics of Plasmas*, vol. 21, n. 2, pp. 023503.

[9] Miranda, R. A., Martins, A. A., & Ferreira, J. L., 2017. Particle-in-cell numerical simulations of a cylindrical Hall thruster with permanent magnets. In *Journal of Physics. Conference Series (Online)*, volume 911.

[10] Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P., 2007. *Numerical recipes 3rd edition: The art of scientific computing*. Cambridge university press.

[11] Taccogna, F., Longo, S., Capitelli, M., & Schneider, R., 2005. Plasma flow in a hall thruster. *Physics of plasmas*, vol. 12, n. 4, pp. 043502.

[12] Frisch, U., 1995. *Turbulence: the legacy of AN Kolmogorov*. Cambridge University press.

[13] Biskamp, D., 2003. *Magnetohydrodynamic turbulence*. Cambridge University Press.