

# Suboptimal Control on Nonlinear Satellite Simulations using SDRE and H-infinity

Alessandro Gerlinger Romero<sup>1</sup>, Luiz Carlos Gadelha de Souza<sup>2</sup>

<sup>1</sup>*Space Mechanics and Control Division, National Institute for Space Research Sao Jos ˜ e dos Campos, S ´ ao Paulo, 12227-010, Brazil ˜ romgerale@yahoo.com.br* <sup>2</sup>*Federal University of ABC Sao Bernardo do Campo, 09210-580, Brazil ˜ luiz.gadelha@ufabc.edu.br*

Abstract. The control of a satellite can be designed with success by linear control theory if the satellite has slow angular motions. However, for fast maneuvers, the linearized models are not able to represent all the perturbations due to the effects of the nonlinear terms present in the dynamics which compromises the system's performance. Therefore, a nonlinear control technique yields better performance. Nonetheless, these nonlinear control techniques can be more sensitive to uncertainties. One candidate technique for the design of the satellite's control law under a fast maneuver is the State-Dependent Riccati Equation (SDRE). SDRE provides an effective algorithm for synthesizing nonlinear feedback control by allowing nonlinearities in the system states. The Brazilian National Institute for Space Research (INPE, in Portuguese) was demanded by the Brazilian government to build remotesensing satellites, such as the Amazonia-1 mission. In such missions, the satellite must be stabilized in three-axes so that the optical payload can point to the desired target. Although elsewhere the application of the SDRE technique has shown to yield better performance for the missions developed by INPE, a subsequent important question is whether such better performance is robust to uncertainties. In this paper, we investigate whether the application of the SDRE technique in the AOCS is robust stable to uncertainties in the missions developed by INPE. Moreover, in order to handle such uncertainty appropriately, we propose a combination of SDRE with H-infinity based on a left coprime factorization. In such a way that the attention is moved to the size of error signals and away from the size and bandwidth of selected closed-loop transfer function. The initial results showed that SDRE controller is robust to 5%, at least, variations in the inertia tensor of the satellite.

Keywords: Nonlinear, control, SDRE, H-infinity

## 1 Introduction

The design of a satellite attitude and orbit control subsystem (AOCS) that involves plant uncertainties, large angle maneuvers and fast attitude control following a stringent pointing, requires nonlinear control methods in order to satisfy performance and robustness requirements. An example is a typical mission of the Brazilian National Institute for Space Research (INPE), in which the AOCS must stabilize a satellite in three-axes so that the optical payload can point to the desired target with few arcsecs of pointing accuracy.

One candidate method for a nonlinear AOCS control law is the State-Dependent Riccati Equation (SDRE) method, originally proposed by [\[1\]](#page-6-0) and then explored in detail by [\[2](#page-6-1)[–4\]](#page-6-2). SDRE is based on the arrangement of the system model in a form known as state-dependent coefficient (SDC) matrices. Accordingly, a suboptimal control law is carried out by a real-time solution of an algebraic Riccati equation (ARE) using the SDC matrices by means of a numerical algorithm.

Elsewhere, we showed State-Dependent Riccati Equation (SDRE) is a feasible non-linear control technique that can be applied in satellites developed by INPE [\[5\]](#page-6-3). Moreover, we showed, through simulation using a Monte Carlo perturbation model, SDRE provides better performance than the PID controller, a linear control technique.

In this paper, we tackle the next fundamental problem: robustness. We evaluate robustness from two perspectives: (1) parametric uncertainty of the inertia tensor and (2) a uniform attitude probability distribution. Through the combination of these two perspectives, we grasp the robustness properties of SDRE in a broader sense. In order to handle the uncertainty appropriately, we combine SDRE with  $H_{\infty}$ .

SDRE was originally proposed by [\[1\]](#page-6-0) and then explored in detail by [\[4\]](#page-6-2). A good survey of the SDRE method can be found in [\[2\]](#page-6-1) and its systematic application to deal with a nonlinear plant in [\[3\]](#page-6-4). The SDRE method was applied by [\[5](#page-6-3)[–9\]](#page-6-5) for controlling a nonlinear system similar to the six-degree of freedom satellite model considered in this paper.

The application of SDRE method, and, consequently, the ARE problem that arises, have already been studied in the available literature, e.g., [\[10\]](#page-6-6) investigated the approaches for the ARE solving as well as the resource requirements for such online solving. Recently, [\[7\]](#page-6-7) proposed the usage of differential algebra to reduce the resource requirements for the real-time implementation of SDRE controllers. In fact, the intensive resource requirements for the online ARE solving is the major drawback of SDRE. Nonetheless, the SDRE method has three major advantages: (a) simplicity, (b) numerical tractability and (c) flexibility for the designer, being comparable to the flexibility in the LQR [\[7\]](#page-6-7).

SDRE method can be readily extended to nonlinear  $H_{\infty}$  [\[4\]](#page-6-2). The interest in  $H_{\infty}$  optimization for robust control of linear plants is mostly attributed to the influential work of [\[11\]](#page-6-8), in which the problem of sensitivity reduction by feedback is formulated as an optimization problem. Later, [\[12\]](#page-6-9) addressed the problem of robustly stabilizing a family of linear systems in the case where such family was characterized by  $H_{\infty}$  bounded perturbations of a normalized left coprime factorization of a nominal system.

The initial results showed that SDRE controller is robust to 5%, at least, variations in the inertia tensor of the satellite. This paper is organized as follows. In Section [2,](#page-1-0) the problem description is presented. In Section [3,](#page-3-0) the satellite physical modeling is reviewed. In Section [4,](#page-4-0) we explore the state-space model and the controllers. In Section [5,](#page-5-0) we share simulation results. Finally, the conclusions are shared in Section [6.](#page-5-1)

### <span id="page-1-0"></span>2 Problem Description

The SDRE technique entails factorization (that is, parametrization) of the nonlinear dynamics into the state vector and the product of a matrix-valued function that depends on the state itself. In doing so, SDRE brings the nonlinear system to a (nonunique) linear structure having SDC matrices given by Eq. [\(1\)](#page-1-1).

<span id="page-1-1"></span>
$$
\vec{x} = A(\vec{x})\vec{x} + B(\vec{x})\vec{u}
$$
  

$$
\vec{y} = C\vec{x}
$$
 (1)

where  $\vec{x} \in \mathbb{R}^n$  is the state vector and  $\vec{u} \in \mathbb{R}^m$  is the control vector. Notice that the SDC form has the same structure as a linear system, but with the system matrices,  $A$  and  $B$ , being functions of the state vector. The nonuniqueness of the SDC matrices creates extra degrees of freedom, which can be used to enhance controller performance, however, it poses challenges since not all SDC matrices fulfill the SDRE requirements, e.g., the pair (A,B) must be pointwise stabilizable.

The system model in Eq. [\(1\)](#page-1-1) is subject of the cost functional described in Eq. [\(2\)](#page-1-2).

<span id="page-1-2"></span>
$$
J(\vec{x_0}, \vec{u}) = \frac{1}{2} \int_0^\infty (\vec{x}^T Q(\vec{x}) \vec{x} + \vec{u}^T R(\vec{x}) \vec{u}) dt
$$
\n(2)

where  $Q(\vec{x}) \in \mathbb{R}^{n \times n}$  and  $R(\vec{x}) \in \mathbb{R}^{m \times m}$  are the state-dependent weighting matrices. In order to ensure local stability,  $Q(\vec{x})$  is required to be positive semi-definite for all  $\vec{x}$  and  $R(\vec{x})$  is required to be positive for all  $\vec{x}$  [\[10\]](#page-6-6).

The SDRE controller linearizes the plant about the current operating point and creates constant state space matrices so that the LQR method can be used. This process is repeated in all samplings steps, resulting in a pointwise linear model from a non-linear model, so that an ARE is solved and a control law is computed also in each step. Therefore, according to LQR theory and Eq. [\(1\)](#page-1-1) and [\(2\)](#page-1-2), the state-feedback control law in each sampling step is  $\vec{u} = -K(\vec{x})\vec{x}$  and the state-dependent gain  $K(\vec{x})$  is obtained by Eq. [\(3\)](#page-1-3) [\[3\]](#page-6-4).

<span id="page-1-3"></span>
$$
K(\vec{x}) = R^{-1}(\vec{x})B^{T}(\vec{x})P(\vec{x})
$$
\n(3)

where  $P(\vec{x})$  is the unique, symmetric, positive-definite solution of the algebraic state-dependent Riccati equation  $(SDRE)$  given by Eq.  $(4)$  [\[3\]](#page-6-4).

<span id="page-1-4"></span>
$$
P(\vec{x})A(\vec{x}) + A^T(\vec{x})P(\vec{x}) - P(\vec{x})B(\vec{x})R^{-1}(\vec{x})B^T(\vec{x})P(\vec{x}) + Q(\vec{x}) = 0
$$
\n(4)

Considering that Eq. [\(4\)](#page-1-4) is solved in each sampling step, it is reduced to an ARE. Finally, the conditions for the application of the SDRE technique in a given system model are [\[3\]](#page-6-4):

1.  $A(\vec{x}) \in C^1(\mathbb{R}^w)$ 

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<span id="page-2-1"></span>

Figure 1.  $H_{\infty}$  robust stabilization problem with left coprime factorization [\[13\]](#page-6-10).

- 2.  $B(\vec{x}), C(\vec{x}), Q(\vec{x}), R(\vec{x}) \in C^{0}(\mathbb{R}^{w})$
- 3.  $Q(\vec{x})$  is positive semi-definite and  $R(\vec{x})$  is positive definite
- 4.  $A(\vec{x})x \implies A(0)0 = 0$ , i.e., the origin is an equilibrium point
- 5.  $pair(A, B)$  is pointwise stabilizable (a sufficient test for stabilizability is to check the rank of controllability matrix)
- 6.  $pair(A, Q^{\frac{1}{2}})$  is pointwise detectable (a sufficient test for detectability is to check the rank of observability matrix)

#### <span id="page-2-3"></span>2.1 SDRE with  $H_{\infty}$

SDRE method can be readily extended to nonlinear  $H_{\infty}$  [\[4\]](#page-6-2). Consider the general nonlinear dynamics using SDC as:

$$
\begin{aligned}\n\dot{x} &= A(x)x + B_1(x)w + B_2(x)u \\
&z = C_1(x)x + D_{12}(x)u \\
&y = C_2(x)x + D_{21}(x)u\n\end{aligned} \tag{5}
$$

<span id="page-2-0"></span>where  $\vec{x} \in \mathbb{R}^n$  is the state vector,  $\vec{u} \in \mathbb{R}^m$  is the control vector,  $\vec{w} \in \mathbb{R}^m$  is the vector of exogenous signals (e.g., disturbances) and  $\vec{z} \in \mathbb{R}^n$  is the vector of "error" signal which is to be minimized in some sense to meet the control objectives. Furthermore, the additional functions are  $C^0(\mathbb{R}^w)$ .

Consider such a state-space model, Eq.  $(5)$ , described by a transfer function G. Now consider the stabilization of plant G which has a normalized left coprime factorization [\[12,](#page-6-9) [13\]](#page-6-10):

$$
G = M^{-1}N\tag{6}
$$

then a perturbed plant model  $G_p$  can be written as [\[13\]](#page-6-10):

$$
G_p = (M + \Delta_M)^{-1}(N + \Delta_N) \tag{7}
$$

where  $\Delta_M$ ,  $\Delta_N$  are stable unknown transfer functions which represent the uncertainty in the nominal plant G.

The objective of robust stabilization  $H_{\infty}$  is to stabilize not only the nominal plant G, but a family of perturbed plants defined by [\[12,](#page-6-9) [13\]](#page-6-10):

$$
G_p = \{ (M + \Delta_M)^{-1} (N + \Delta_N) :: ||[\Delta_M \Delta_N]||_{\infty} < \epsilon \}
$$
\n(8)

where  $\epsilon > 0$  is the stability margin. To maximize this stability margin is the problem of  $H_{\infty}$  robust stabilization of normalized coprime factor plant descriptions [\[12\]](#page-6-9). For the positive feedback of Fig. [1,](#page-2-1) the perturbed plant is robustly stabilizable if and only if the nominal feedback is stable and [\[12,](#page-6-9) [13\]](#page-6-10)

$$
\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} M^{-1} \|_{\infty} \le \epsilon^{-1}
$$
\n(9)

The maximum stability margin  $\epsilon$  and the corresponding minimum  $\gamma$  are given as [\[12\]](#page-6-9):

<span id="page-2-2"></span>
$$
\gamma_{min} = \epsilon_{max}^{-1} = (1 + \rho(XZ))^{\frac{1}{2}}
$$
\n(10)

where  $\rho$  denotes the spectral radius (maximum eigenvalue) and for the initial state-space realization Z and X are solutions of AREs.

<span id="page-3-1"></span> $Z$  and  $X$  are the solutions to the AREs [\[12,](#page-6-9) [13\]](#page-6-10):

$$
(A - BS^{-1}D^{T}C)Z + Z(A - BS^{-1}D^{T}C)^{T} - ZC^{T}R^{-1}CZ + BS^{-1}B^{T} = 0
$$
  

$$
(A - BS^{-1}D^{T}C)^{T}x + X(A - BS^{-1}D^{T}C) - XBS^{-1}B^{T}X + C^{T}R^{-1}C = 0
$$
  

$$
R = I + DD^{T}
$$
  

$$
S = I + D^{T}D
$$
 (11)

A controller which guarantees that [\[12,](#page-6-9) [13\]](#page-6-10):

$$
\| \begin{bmatrix} K \\ I \end{bmatrix} (I - GK)^{-1} M^{-1} \|_{\infty} \le \gamma
$$
 (12)

<span id="page-3-2"></span>for a specified  $\gamma > \gamma_{min}$ , is given by:

$$
K_{H_{\infty}} = \begin{bmatrix} A + BF + \gamma^2 (L^T)^{-1} Z C^T (C + DF) & \gamma^2 (L^T)^{-1} Z C^T \\ B^T X & -D^T \end{bmatrix}
$$
  
\n
$$
F = -S^{-1} (D^T C + B^T X)
$$
  
\n
$$
L = (1 - \gamma^2) I + X Z
$$
 (13)

Therefore, regarding the combination of SDRE and  $H_{\infty}$  the procedure to compute the controller that maximizes the stability margin for the perturbed plants in each step is:

- 1. Reconstruct the matrices using the SDC form;
- 2. Solve the two ARES of Eq. [\(11\)](#page-3-1) computing X and Z;
- 3. Compute  $\gamma_{min}$  using Eq. [\(10\)](#page-2-2);
- 4. Define a state-space model (A,B,C,D) using X, Z and a  $\gamma > \gamma_{min}$  by Eq. [\(13\)](#page-3-2);
- 5. Solve the third ARE that results from the state-space model described by Eq. [\(13\)](#page-3-2), which leads to  $P_{K_{H_{\infty}}}$  as the unique, symmetric, positive-definite solution of such ARE;
- 6. Compute the controller K for the original system using  $K(\vec{x}) = R^{-1}(\vec{x})B_2(\vec{x})P_{K_{H_{\infty}}}(\vec{x})$ .

It is known that if a controller can be found using that procedure, the exogenous signal will be locally attenuated by  $\gamma$  in each step [\[4,](#page-6-2) [12,](#page-6-9) [13\]](#page-6-10).

### <span id="page-3-0"></span>3 Satellite Physical Modeling

The focus is on a typical mission developed by INPE, in which the AOCS must stabilize a satellite in threeaxes so that the optical payload can point to the desired target. Next subsections explore the kinematics and the rotational dynamics of the satellite attitude available in the simulator.

#### 3.1 Kinematics

Given the ECI reference frame ( $\mathfrak{F}_i$ ) and the frame defined in the satellite with origin in its centre of mass (the body-fixed frame,  $\mathfrak{F}_b$ ), then a rotation  $R \in SO(3)$  ( $SO(3)$  is the set of all attitudes of a rigid body described by  $3 \times 3$  orthogonal matrices whose determinant is one) represented by a unit quaternion  $Q = [q_1 \ q_2 \ q_3 \ | \ q_4]^T$  can define the attitude of the satellite.

Defining the angular velocity  $\vec{\omega} = [\omega_1 \omega_2 \omega_3]^T$  of  $\mathfrak{F}_b$  with respect to  $\mathfrak{F}_i$  measured in the  $\mathfrak{F}_b$ , the kinematics can be described by Eq. [\(14\)](#page-3-3) [\[14\]](#page-6-11).

$$
\dot{Q} = \frac{1}{2}\Omega(\vec{\omega})Q
$$

$$
\Omega(\vec{\omega}) \triangleq \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}
$$
(14)

<span id="page-3-3"></span>*CILAMCE 2020*

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where the unit quaternion Q satisfies the following identity:  $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$ .

Eq. [\(14\)](#page-3-3) allows the prediction of the satellite's attitude if it is available the initial attitude and the history of the change in the angular velocity ( $\dot{Q} = F(\omega, t)$ ). Another possible derivation of the Eq. [\(14\)](#page-3-3) is using the vector g (Gibbs vector or Rodrigues parameter) as  $Q = [g^T|q_4]$ .

$$
\dot{Q} = -\frac{1}{2} \begin{bmatrix} \omega^{\times} \\ \omega^T \end{bmatrix} \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} + \frac{1}{2} q_4 \begin{bmatrix} 1_{3 \times 3} \\ 0 \end{bmatrix} \vec{\omega}
$$
 (15)

<span id="page-4-1"></span>where  $\omega^{\times}$  is the cross-product skew-symetric matrix of  $\vec{\omega}$  and 1 is the identity matrix. Note the Gibbs vector is geometrically singular since it is not defined for 180◦ of rotation [\[15\]](#page-6-12), nonetheless, the Eq. [\(15\)](#page-4-1) is global.

#### 3.2 Rotational Dynamics

The satellite has a set of 3 reaction wheels, each one aligned with its principal axes of inertia, moreover, such type of actuator, momentum exchange actuators, does not change the angular momentum of the satellite. Consequently, it is mandatory to model their influence in the satellite, in particular, the angular momentum of the satellite is defined by Eq. [\(16\)](#page-4-2).

<span id="page-4-2"></span>
$$
\vec{h} = (\vec{I} - \sum_{n=1}^{3} I_{n,s} a_n a_n^T) \vec{\omega} + \sum_{n=1}^{3} h_{w,n} \vec{a_n}
$$
\n(16)

where  $I_{n,s}$  is the inertia moment of the reaction wheels in their symmetry axis  $\vec{a_n}$ ,  $h_{w,n}$  is the angular momentum of the *n* reaction wheel about its centre of mass  $(h_{w,n} = I_{n,s} a_n^T \omega + I_{n,s} \omega_n)$  and  $\omega_n$  is the angular velocity of the n reaction wheel.

One can define  $I_b = \vec{I} - \sum_{n=1}^{3} I_{n,s} a_n a_n^T$ . Using  $I_b$ , the motion of the satellite is described by Eq. [\(17\)](#page-4-3).

$$
I_b \dot{\vec{\omega}}^b = g_{cm}^{\rightarrow} - \omega^{\times} (I_b \vec{\omega} + \sum_{n=1}^3 h_{w,n} \vec{a_n}) - \sum_{n=1}^3 g_n \vec{a_n}
$$
 (17)

<span id="page-4-3"></span>where  $g_{cm}$  is the net external torque and  $g_n$  are the torques generated by the reactions wheels  $(h_{w,n} = g_n)$ .

#### <span id="page-4-0"></span>4 Controller Design

Two dynamics states must be controlled: (1) the attitude (perhaps described by unit quaternions Q) and (2) its stability  $(Q)$ , in other words, the angular velocity  $\omega$  of the satellite). The following subsections explore the state-space modeling and the controllers'synthesis.

#### 4.1 Nonlinear Control based on State-Dependent Riccati Equation (SDRE) Controller

Assuming that there are no net external torques ( $g_{cm} = 0$ ), the state space model can be defined using Eq. [\(14\)](#page-3-3)  $(\Omega)$  and [\(17\)](#page-4-3), however, the SDC matrices do not fulfill the SDRE requirements, in particular, the pair (A,B) is not pointwise stabilizable.

An alternative option for the definition of the SDC matrices is to use Eq. [\(15\)](#page-4-1), which leads to Eq. [\(18\)](#page-4-4).

<span id="page-4-4"></span>
$$
\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{1}{2} \begin{bmatrix} \omega^\times \\ \omega^T \end{bmatrix} & 0 \\ 0 & 0 & -I_b^{-1} \omega^\times I_b + I_b^{-1} (\sum_{n=1}^3 h_{w,n} a_n)^\times \end{bmatrix} \begin{bmatrix} x_0 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ -I_b^{-1} \end{bmatrix} \begin{bmatrix} u_1 \end{bmatrix}
$$
\n(18)\n
$$
\begin{bmatrix} y \end{bmatrix} = 1 \begin{bmatrix} x_0 \\ x_2 \end{bmatrix}
$$

Eq. [\(18\)](#page-4-4) has been shown to satisfy SDRE conditions described in Section [2.](#page-1-0)

<span id="page-5-2"></span>

Table 1. Satellite characteristics, initial conditions and references.

#### 4.2 Nonlinear Control based on State-Dependent Riccati Equation (SDRE) with  $H_{\infty}$  Controller

Although the SDRE with  $H_{\infty}$  controller uses the Eq. [\(18\)](#page-4-4), it follows the procedure defined in Subsection [2.1.](#page-2-3) Such a procedure requires the solving of three AREs in each step, instead of one ARE as usual in the SDRE controller.

### <span id="page-5-0"></span>5 Simulation Results

A simulation was conducted with the full Monte Carlo perturbation model described as follows: (1) the initial Euler angles of the nonlinear spacecraft system are randomly selected using independent uniform distributions  $(minimum = -180°, maximum = 180°)$ ; (2) the initial angular velocity are randomly selected using independent uniform distributions (minimum = -0.01 rad/s, maximum = 0.01 rad/s), and (3) each element of the inertia tensor defined in Table [1](#page-5-2) is changed accordingly a normal distribution  $N(nominal, (nominal*0.016666)^2)$  $\sim$  so  $\pm$ 5% for three  $\sigma$  in each side of the Gaussian.

The Monte Carlo model ran 50 times. Such executions used simulation time 1500 seconds, fixed step 0.05 seconds, the data presented in Table [1](#page-5-2) and the controller defined by Eq. [\(18\)](#page-4-4) and [\(3\)](#page-1-3): SDRE+ $H_{\infty}$  controller  $(R = 1$  and  $Q = 1)$ . Fig. [2](#page-6-13) shows the simulation results, which are in accordance with Section [2.](#page-1-0)

## <span id="page-5-1"></span>6 Conclusion

The major contribution of the current paper is the extension of SDRE with  $H_{\infty}$  using exactly three AREs to find the sub-optimal controller, whereas the literature suggests the  $\gamma$ -iteration in each step in order to solve the general  $H_{\infty}$  problem [\[4\]](#page-6-2). Finally, the disturbances are locally attenuated by  $\gamma$  in each step.

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<span id="page-6-13"></span>

Figure 2. Simulation results for parametric uncertainty of SDRE Gibbs with  $H_{\infty}$ .

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