Dynamic Effects of Moving Loads on Structures

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Abstract. This study presents a computational analysis of the dynamic behavior of a cantilever steel beam subjected to a moving mass and corresponding load that travels along its entire length. It aims to understand the behavior of such a structure and to determine dynamic impacts on displacements. Present methods for structural analysis usually consider acting forces as static. However, structures subjected to masses whose position change considerably in time need to be more carefully considered. It is essential to ensure the safety of the structure, the proper functioning of equipment and the comfort and safety of users. Consideration of the dynamic behavior of a structure allows to evaluate the results with greater reliability, since it brings the model closer to reality, as the actual masses and corresponding loads vary with time and, therefore, produce dynamic effects. Advances in computational power and in the development of computational methods allow the design of these structures to include dynamic analysis, which promotes more realistic results in engineering design. Here, a discretized model was developed, based on the Finite Element Method, using numerical integration by Newmark's Method for the solution of the nonlinear 2nd order ordinary differential equations and obtaining the displacements of the structure in the time domain, to evaluate its behavior due to moving masses and loads.

Keywords: Moving loads. Structural Dynamics. Newmark's method.

1 Introduction

Several real systems present longitudinal displacement of loas in structures, which can cause dynamic stresses, such as bridges where vehicles transit, cranes, overhead cranes, projectiles and rockets. Zhao, Hu and Heijden [1] identify the classic example of the mobile load problem as the idealization of a vehicle-bridge system, where the vehicle is treated as a mobile force or load.

Fryba [2] and Rao [3] present the analytical solutions to the problem of moving loads presenting the general conditions for calculating beams with different boundary conditions.

Siddiqui, Golnaraghi and Heppler [4] studied the dynamic behavior of flexible cantilever beam carrying a moving mass. Zhao, Hu and Heijden [1] present a study of a cantilever beam with variable section, submitted to the moving mass moving from the fixed end to the free end of the structure, for application in naval and defense industry. Reis and Pala [5] analyze the response of a cracked cantilever beam subject to moving loads. Esen [6] studies the transverse and lateral responses of thin beams subjected to mobile loads with variable acceleration using finite element method.

This work presents a case study, in which a cantilever beam is subjected to a moving mass in longitudinal motion, and aims to find the transverse responses of the structure.

2 Modelling

The studied system is shown in Figure 1. It consists of a cantilever beam, with length L, modulus of elasticity E, cross section area A, moment of inertia about the y-axis I and density ρ. Along the beam, the moving mass M travels with velocity $v(t)$.

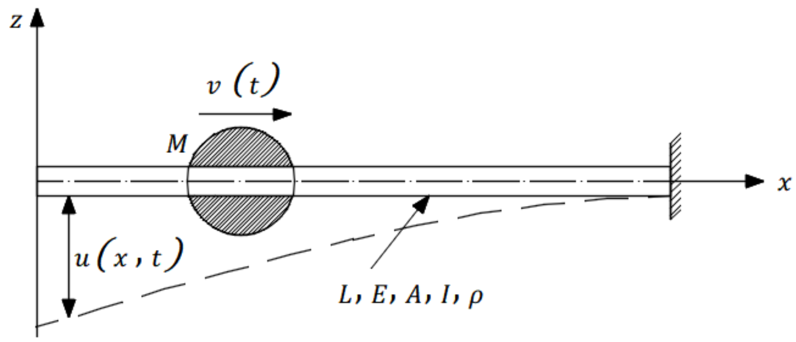

Figure 1. Cantilever beam subjected to a moving mass system.

The model is then discretized and analyzed using the finite element method where the real structure is represented as a model consisted of several elements, as shown in Figure 2, with several degrees of freedom. Each element have mass, stifness and corresponding damping, leading to the generation of matrices of mass, stiffness and damping of each element.

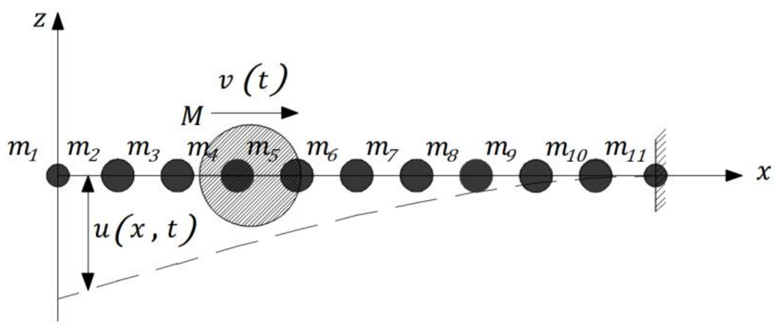

Figure 2. Discretized model

The local matrices are then converted to global matrices, and the equation of the system's motion is considered as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = p(t) \tag{1}$$

where $[M]$ is the mass matrix, $[C]$ the damping matrix and $[K]$ the stiffness matrix of the system. Mazzilli et al. [7] report that it is a sufficient condition for the damping to be of the proportional type such that the damping matrix $[C]$ is a linear combination of the mass and stiffness matrices, expressed by

$$[C] = \sum_b a_b[M]([M]^{-1}[K])^b \tag{2}$$

where the particular case of the Rayleigh damping can be considered

$$[C] = a_0[M] + a_1[K] \tag{3}$$

where the factors a_0 e a_1 are obtained imposing damping rates ξ arbitrarily adopted for two chosen modes, finding the solution for the system

$$\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix} = \frac{1}{2}\begin{bmatrix}\frac{1}{\omega_i} & \omega_i\\ \frac{1}{\omega_j} & \omega_j\end{bmatrix}\begin{bmatrix}a_0\\ a_1\end{bmatrix} \tag{4}$$

To solve the problem of moving loads, the moving mass M is considered changing position between the nodes, changing the mass matrix $[M]$ at each time step, depending on the constant speed $v(t)$, solving the nonlinear problem in a linearized form.

In an analytical study, for a cantilever beam with the free left end, the boundary conditions are

$$\ddot{u}(0,t) = 0; \qquad \dddot{u}(0,t) = 0; \qquad u(l,t) = 0; \qquad \dot{u}(l,t) = 0; \tag{5}$$

Consequently, for the vibration modes, the displacement boundary conditions can also be found,

$$\Phi_i''(0) = 0; \qquad \Phi_i'''(0) = 0; \qquad \Phi_i(l) = 0; \qquad \Phi_i'(l) = 0; \tag{6}$$

Fryba[2] points as a way to find the vibration modes in beams with any boundary conditions,

$$\Phi_i(x) = sin\frac{\lambda_i x}{L} + A_i\,cos\frac{\lambda_i x}{L} + B_i\,sinh\frac{\lambda_i x}{L} + C_i\,cosh\frac{\lambda_i x}{L} \tag{7}$$

To obtain the coefficients $\lambda_i\ A_i\ B_i\ C_i$, for the boundary conditions expressed in the Equations 6, Fryba [2] points out the following equations

$$1 + cos\,\lambda_i * cosh\,\lambda_i = 0; \tag{8}$$

$$A_i = C_i = \frac{\sin\lambda_i + \sinh\lambda_i}{\cos\lambda_i + \cosh\lambda_i}; \tag{9}$$

$$B_i = 1; \tag{10}$$

Finding the roots of Equation 8, and applying on the Equations 9 and 10 we obtain the constants shown in Table 1.

Table 1. Constants

i	λ	A	B	C
1	1,875	-1,362	-1,000	-1,362
2	4,694	-0,982	-1,000	-0,982
3	7,855	-1,001	-1,000	-1,001
4	10,996	-1,000	-1,000	-1,000

Applying them in Equation 7, it is possible to find the vibration modes of the system as presented in Figure 3.

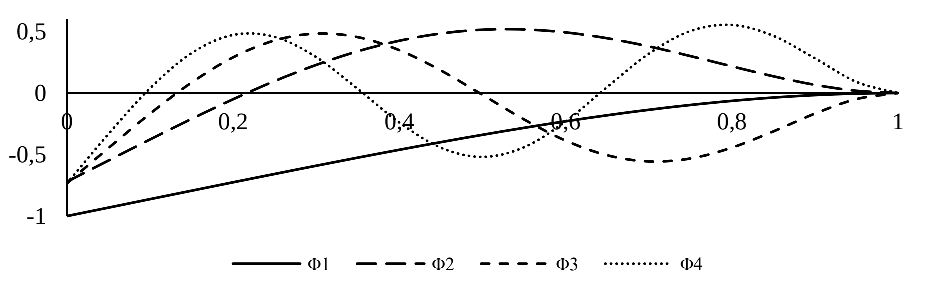

Figure 3. Modes of vibration of a beam with free left end and embedded right end

Fryba [2] points out that to obtain the natural frequencies of the structure one should compute

$$\omega_i^2 = \frac{\lambda_i^4 EI}{L^4 \mu} \tag{11}$$

where μ corresponds to the mass per unit length of the structure.

In order to obtain system responses, it is necessary to integrate the equations of motion, so the Newmark method is used for direct integration, Toledo [8] reports that displacements and velocities are developed in Taylor series, with the rest being calculated approximately according to free parameters that are fixed later. Brasil and Silva [9] report that, given the vectors of displacements, velocities and accelerations, their values are determined in an instant $t + \Delta t$.

$$u_{t+\Delta t} = u_t + \Delta u \tag{12}$$

$$\dot{u}_{t+\Delta t} = b_0 \Delta u - b_2 \dot{u}_t - b_3 \ddot{u}_t \tag{13}$$

$$\ddot{u}_{t+\Delta t} = b_1 \Delta u - b_4 \dot{u}_t - b_5 \ddot{u}_t \tag{14}$$

The coefficients $b_0, b_1, b_2, b_3, b_4, b_5$ are chosen in order to approximate the variation of the vectors, obtaining a system of algebraic equations that allows to find the increments of displacements in the step,

$$\hat{K} \Delta u = \hat{p}_{t+\Delta t} \tag{15}$$

with equivalent stiffness

$$\hat{K} = b_1 M + b_0 C + K \tag{16}$$

and equivalent step load

$$\hat{p}_{t+\Delta t} = p_{t+\Delta t} + M(b_2 \dot{u}_t + b_3 \ddot{u}_t) + C(b_4 \dot{u}_t + b_5 \ddot{u}_t) - K u_t \tag{17}$$

from which the displacement velocities and accelerations of the next step are determined.

3 Discussion and Results

For the study, a model was analyzed using Mathworks Matlab® to find the solutions, consisting of a cantilever steel I-section beam, with length $L = 5{,}00m$, modulus of elasticity $E = 2{,}1x10^{11}$, cross section area $A = 8{,}2 * 10^{-3} m^2$, moment of inertia about y-axis $I = 3{,}42 * 10^{-4}\, m^4$ and density $\rho = 7850\, kg/m^3$. The moving mass $M = 2370\, kg$ moves at a constant speed. We analysed the structure's response to different speeds between $v(t) = 1{,}00\, m/s$ and $v(t) = 1000{,}00\, m/s$.

The model is then discretized to perform computational analysis using the finite element method in a model with 10 elements and 11 nodes, each node with two degrees of freedom, relative to vertical displacement and rotation, and axial efforts can be disregarded.

Stiffness matrices and mass matrices are then generated for each moving mass position.

Analyzing the structure in the condition of undamped free vibrations, it is then possible to find the natural frequencies for mobile load in any of the nodes of the discretized model, as seen in Table 2.

Table 2. Natural frequencies in different positions of the mobile load

	Node 1	Node 2	Node 3	Node 4	Node 5	Node 6	Node 7	Node 8	Node 9	Node 10
1st natural frequency (rad/s)	26,55	30,90	36,53	44,00	54,19	68,45	88,67	115,39	115,39	147,93

Analytically, it is possible to find the natural frequency of the unloaded system to be 148,57 rad/s, very close to the 1st frequency found numerically when the moving mass approaches the fixed end, at the position of node 10.

Using the Newmark Method, the responses of the motion equation for each position of the mobile load are then calculated, as a function of the displacement speed, for different constant speeds between $1{,}00\, m/s$ and $1000{,}00\, m/s$, obtaining the results shown in Figure 4.

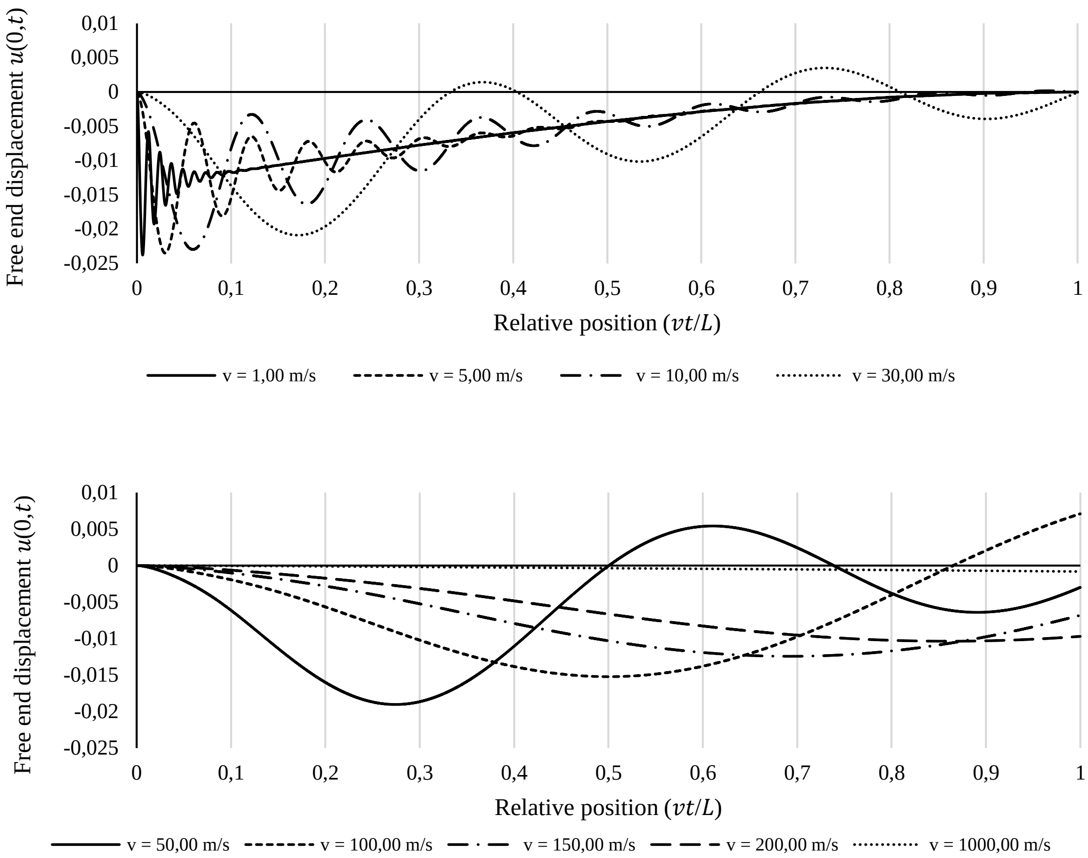

Figure 4. Response of cantilever beam with free left end to moving load at different speeds

4 Conclusions

It is possible to observe that due to the change in the position of the mass, the structure responds with different natural frequencies, approaching the natural frequency for the unloaded case as the loading approaches the fixed end.

When subjected to low speeds, the structure achieves greater absolute displacements, but dissipates energy and dampens vibrations before the load reaches the fixed end, when subjected to increased speed, the structure dissipates less energy and tends to propagate vibrations in order to reach inverse displacements, and at very high speeds, the structure does not reach large displacements and vibrations.

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