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Adaptive Universal Integral Regulator For Flight Control - A Preliminary Study

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Abstract. This article proposes a new treatment for the design of the Universal Integral Regulator (UIR). The UIR is a non-linear control technique based on sliding mode control with the inclusion of a Conditional Integrator (CI). The main tuned gain of this control law is constant in the original approach, in the present work we propose to make this gain dependent on the tracking error (adaptive gain). The performance of this change will be demonstrated in a flight control case, performance indexes will allow to quantitatively compare its performance with the original one. Simulations results show a superior performance of the UIR after the proposed gain modification, called at this work as Adaptive UIR (AUIR)

Keywords: Universal Integral Regulator, Adaptive UIR, Flight Control, Adaptive Gain

1 Introduction

This work addresses a preliminary study about a modification of a control theory known as Universal Integral Regulator (UIR) developed by [1]. This control technique was the result of a series of papers as [2] and [3]. The application of UIR in the present work is related to the flight control system of an aircraft, then, it is worth mentioning that the UIR performance has been proved in several works in this area as in [4], [5],[6] and recently in [7].

Some features make this control technique an important tool to improve the performance of the dynamic response of non-linear systems. One of this features is its non-linear nature, the UIR is based on the non-linear Sliding Mode Control (SMC) control law but include a saturator instead the original on/off switching approach of SMC in order to reduce the chattering phenomenon at the control output. The only necessary model knowledge to design it is the relative degree (ρ) of the system to be controlled, that is, the number of derivatives of the output to be controlled necessary to find a direct relation with a control input predefined [7]. Another important feature, is the inclusion of a Conditional Integrator (CI), that avoid the wind-up integrator problem caused by conventional integrators. The CI improve the transient response of the system and guarantee zero tracking error at the same time.

The original UIR has itself an adaptive characteristic. Its saturator change its structure when closer to equilibrium, this is, when the sliding surface is greater than a certain value (this value is called boundary layer), the behaviour of the of the SMC plus CI is like an on/off switch, but near to zero error the saturation approach is activated in order to reduce chattering. In [8], a modification was done to the UIR, called at this work as Adaptive UIR (AUIR). It was assumed that the main gain of the controller does not depends on the internal dynamic of the system in order to simplify the analytical computing of the controller parameters but at expense of guaranteeing only semiglobal stability (and not global as in the original UIR).

In the present work, it is proposed a modification of the main gain of the UIR control law, in order to make this gain a directly dependent variable of the tracking error, following a known linear function. It is expected to reduce the control demand closed to the equilibrium and improve the tracking response of the flight control states of an aircraft. The use of UIR controller in the present work is, in part, because its easy implementation and tune of its parameters. Another reason is the ”non-necessity” of exact model knowledge, very useful feature when the entire model of the aircraft is not available and its capacity of rejecting disturbances.

2 Universal Integral Regulator Controller Design

The Universal Integral Regulator (UIR) such as developed by [1], is addressed for the output regulation of MIMO (Multi-Input Multi-Output) non-linear systems. One of the limitations of this approach is that the reference must be asymptotically constant. In order to simplify the explanation, in the present work the system to be adopted is a second order MIMO system in the affine in the input canonical form as in Eq. 1.

$$\begin{cases} \dot{x_1} = x_2 \\ \dot{x_2} = f(x_1, x_2) + g(x_1, x_2)u \\ y = x_1 \end{cases} \tag{1}$$

Where $x = \{x_1, x_2\}$ belongs the the R^n space and is called the state vector, $y \in R^n$ is the output vector, $u \in R^n$ the control input and $f(x_1, x_2) \in R^n$ and $g(x_1, x_2) \in R^{n\times n}$ are continuous functions. Let is define $y_{ref} = x_{1ref}$ as the reference function, then the tracking error is defined as $e_1(t) = y - y_{ref}$, this is, the difference between the actual and the reference output.

The control problem will be to determine the value of the control input u in order to guarantee the asymptotic convergence of the tracking error $e_1(t)$ to zero. To do this, it is necessary to transform the system to the normal form, this is done by computing several times a Lie Derivative of the output vector y until the control input u to appear, this procedure allows to separate the internal dynamic of the system of the "external" one. In the case of the system defined in Eq. 1, the external dynamic of the system is as described in Eq. 2.

$$\begin{cases} \dot{e_1} = e_2 \\ \dot{e_2} = f(e_1, e_2) + g(e_1, e_2)u \end{cases} \tag{2}$$

Then, in order to create the SMC based of the UIR technique, we write the sliding function as in Eq. 3.

$$s = k_0\sigma + k_1 e_1 + e_2 \tag{3}$$

$k_1 \in R^{n\times n}$ is selected such that $k_1 + s$ is Hurwitz, this is done in order to guarantee convergence of the sliding surface s. As mentioned before, the UIR include a Conditional Integrator (CI), in Eq. 3 the variable σ represents the output of that CI, and the CI itself is defined as:

$$\dot{\sigma} = -k_0\sigma + \mu sat(s/\mu) \tag{4}$$

In Equation 4, the variable μ is called boundary layer, and the function where it is inside is called saturation function, this function is expressed as:

$$sat(s/\mu) = \begin{cases} sign(s) & if \quad |s| \geq \mu \\ s/\mu & if \quad |s| < \mu \end{cases} \tag{5}$$

Finally, the UIR control law is defined as:

$$u = -K(\cdot)sat(s/\mu) \tag{6}$$

According to [1] the controller parameters to be tuned are K and μ. μ should be chosen sufficiently small in order to recover the performance of the ideal SMC (switching sliding surface) and the minimum value of gain K can be determined through an analytical stability demonstration as done in [7] and [8] or simply be defined as the maximum physical allowable value.

3 Adaptive UIR - Methodology

In this section we address the modification done to the UIR in order to improve its performance, we also will describe the performance indexes used to compare the response of both controller (with and without the modification).

As shown in Eq. 6, the gain K of the original UIR is a constant. Its minimum value can be determined with precision only if the mathematical model of the system is well known. Inspired in the works of [9],[10] and [11], where a gain change with tracking error. In the present work is proposed the following linear function (Eq. 7) to the gain K such that it depends on the error.

$$K(e_1) = a|e_1| + b \tag{7}$$

Where the real parameters a and b are found by a trial and error process. The a and b parameters are defined as slope and zero-error-gain respectively.

The results are compared using two indicators or performance indexes, the accumulated error (AE) and the control demand (CD). They are defined in this work as follows:

$$AE = \sum_{t=0}^{T} e_1(t) \tag{8}$$

$$CD = \sum_{t=0}^{T} u(t) - u_{eq} \tag{9}$$

Where T is the simulation time and u_{eq} the equilibrium control input, this last parameter is constant and depend on the initial equilibrium conditions.

4 Model And Numerical Simulations

4.1 Aircraft model

In this section, we briefly present the model of the non-linear aircraft, Mirage III aircraft, adopted in this work, which will serve as a plant for the application of Universal Integral Regulator (UIR) and Adaptive Universal Integrative Regulator (AUIR) control techniques.

The equations that describe the movement of the aircraft are based on Newton's second law, whose features are [12]: *i)* the rate of change of the aircraft's linear moment is equal to the sum of the acting forces, in which they are, the inertial, propulsive and aerodynamic forces and *ii)* the rate of change of the aircraft's angular momentum is equal to the moments acting around the aircraft's center of gravity (CG), which are produced by the aerodynamic, propulsive forces and the kinematic equations.

The model studied is a three-dimensional aircraft with six degrees of freedom whose equations of motion are formulated using the body's axis system as a reference and assuming flat ground [13], [12]. It is a rigid and fixed-wing aircraft. The mass of the aircraft is considered time-invariant due to the short simulation times and the stability coefficients and derivatives are assumed to be constants.

The set of non-linear differential equations that represents the movements of the aircraft under study will be described below. The movements to be considered in this work are the longitudinal, lateral and directional movements, the latter two are always coupled. The longitudinal movement is characterized as a flight with levelled wings with the velocity vector always contained in the plane of symmetry of the aircraft, the degrees of freedom (DOF) are three: translation in the x and z axes and rotation around the y axis, as shown in Fig. 1. The latero-directional movements include translation along the y-axis and rotations around the x and z axis.

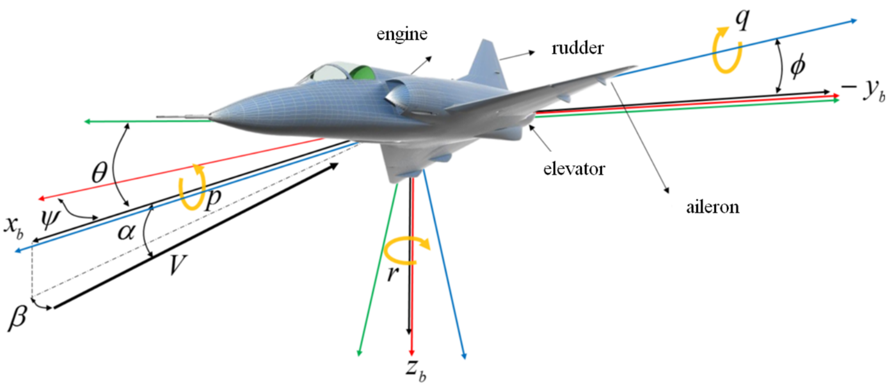


Figure 1. Aircraft reference system and orientation
Adapted from [14]

This aircraft model is composed by twelve equations that describe its behaviour, three translational speed equations, three angular speed equations, three aircraft kinematics equations and three navigation equations. The three translational speed equations are presented in Eq. 10. The three equations of angular velocities are presented in Eq. 11 and the three equations of the aircraft kinematics are presented in Eq. 12.

$$\begin{aligned}
\dot{u} &= m^{-1}\left(F_x + Tcos\alpha_f\right) - gsin\phi + rv - qw \\
\dot{v} &= m^{-1}F_y + gsin\phi\, cos\theta + pw - ru \\
\dot{w} &= m^{-1}(F_z + Tsin\alpha_f) + gcos\phi\, cos\theta + qu - pv
\end{aligned} \tag{10}$$

$$\begin{aligned}
\dot{p} &= (c_1 r + c_2 p)\, q + c_3 L + c_4 N \\
\dot{q} &= c_5 pr - c_6\left(p^2 - r^2\right) + c_7 M \\
\dot{r} &= (c_8 p - r)q + c_4 L + c_9 N
\end{aligned} \tag{11}$$

$$\begin{aligned}
\dot{\phi} &= p + qsen\phi\, tan\theta + rcos\phi\, tan\theta \\
\dot{\theta} &= qcos\phi - rsen\phi \\
\dot{\psi} &= (qsen\phi + rcos\phi)sec\theta
\end{aligned} \tag{12}$$

It should be noted that in this work the three navigation equations will be used, the negative sign is due to the fact that the zb axis is considered positive downwards. Soon the equations will be as in Eq. 13.

$$\begin{aligned}
\dot{x} &= ucos\phi\, cos\psi + v\left(sen\phi sen\theta cos\psi - cos\phi sen\psi\right) + w\left(cos\phi sen\theta cos\psi + sen\phi sen\psi\right) \\
\dot{y} &= ucos\theta\, sen\phi + v\left(sen\phi sen\theta sen\psi + cos\phi cos\psi\right) + w\left(cos\phi sen\theta sen\psi - sen\phi cos\psi\right) \\
\dot{z} &= usen\theta - vsen\phi\, cos\theta - wcos\phi\, cos\theta
\end{aligned} \tag{13}$$

Finally, the model presented throughout this section has similar characteristics to the Mirage III aircraft, used in [12]. Then, the data was extracted directly from it. The dimension, weight and inertia values are presented at Table 1 and stability derivatives at Table 2.

Table 1. Aircraft dimensions, weights and inertia properties
Source: [12]

Item: symbol [unit]	**Value**	**Item: symbol [unit]**	**Value**
Wing surface: S $[m^2]$	36	Moment of inertia: I_{zz} $[kg \cdot m^2]$	6×10^4
Aerodynamic average chord: c $[m]$	5.25	Moment of inertia: I_{xz} $[kg \cdot m^2]$	1.8×10^5
Aircraft mass: m $[kg]$	7400	Moment of inertia: I_{xy} $[kg \cdot m^2]$	0
Moment of inertia: I_{xx} $[kg \cdot m^2]$	9×10^4	Moment of inertia: I_{yz} $[kg \cdot m^2]$	0
Moment of inertia: I_{yy} $[kg \cdot m^2]$	5.4×10^4	Wingspan b $[m]$	5.25

Table 2. Aircraft stability and control derivatives
Source: [12]

Derivative	**Value**	**Derivative**	**Value**	**Derivative**	**Value**	**Derivative**	**Value**
C_{L_0}	0	C_{m_α}	-0.17	$C_{y_{\delta a}}$	0.01	$C_{n_{\delta l}}$	-0.085
C_{L_α}	2.204	C_{m_q}	-0.4	$C_{L_{\delta r}}$	0.075	C_{l_β}	-0.05
C_{L_q}	0	$C_{m_{\delta p}}$	-0.45	C_{n_β}	0.150	C_{l_p}	-0.25
$C_{L_{\delta p}}$	0.7	C_{y_β}	-0.6	C_{n_p}	0.055	C_{l_r}	0.06
C_{D_0}	0.015	C_{y_p}	0	C_{n_r}	-0.7	$C_{l_{\delta a}}$	-0.30
C_{m_0}	0	C_{y_r}	0	$C_{n_{\delta a}}$	0	$C_{l_{\delta l}}$	0.019

4.2 Numerical simulations and results

As mentioned before, a modification was made to the UIR in order to improve its performance. To do this, the main gain (K) of the original UIR controller in Eq. 6 was assumed to be dependent on the tracking error e_1 by Eq. 7. This section show the results of both controllers (with and without the modification) in a an aricraft tracking problem. The initial/equilibrium flight conditions of the aircraft for the numerical simulations are: *(i)* aircraft total speed $V_{eq} = 250 m/s$, *(ii)* aircraft altitude $H_{eq} = 5000 m$ and *(iii)* Mach number $M = 0.78$.

As stated earlier, in order to quantitatively compare the quality of the tracking provided by the two control techniques to be implemented, two performance indices were proposed in this work, an index called Accumulated Error (AE) defined by Eq. 8 and the Control Demand (CD) defined in Eq. 9. The first makes the sum of the absolute value of the tracking error over the simulation time and the second similarly accumulates the absolute value of the control, these estimates are important, because they not only compare the amplitude of the control surface used, but also, the time it is demanded during the maneuver.

Velocity and altitude control

[15] and [16] presented non-linear control laws for altitude and velocity of a hypersonic aircraft. In these references, the elevator and traction were used to control altitude and velocity respectively, and the control technique used is variable structure control, also known as Sliding mode control. The altitude and velocity controllers in this work adopted the same control inputs. The UIR controllers were designed as described from Eq. 3 to 6 and using the relative degree of the system ($\rho = 3$) to design the sliding surface in Eq. 3.

The elevator deflection (δ_p) was used exclusively to control velocity (V), that is, keep the equilibrium velocity ($V = 250 m/s$) constant while using the fuel lever (δ_π) a change of $50 m$ in altitude (H) is performed. The expressions of the control laws, with the values of the gains used and the thickness of the boundary layer are presented in Eq. 14.

$$
\begin{aligned}
\delta_\pi^H &= -0.6\ .\ sat\left(\frac{0.29\sigma_H + 1e_H + 4.2\dot{e}_H + 1\ddot{e}_H}{49}\right) \\
\delta_p^V &= -0.002\ .\ sat\left(\frac{9\sigma_V + 1e_V + 2\dot{e}_V + 1\ddot{e}_V}{3.3}\right)
\end{aligned}
\tag{14}
$$

Equation 14 shows the UIR controller without the modifications. Eq. 15, on the other hand, presents the values of the parameters and gains after the modification of the UIR proposed in this work, that is, a control law AUIR.

$$
\begin{aligned}
\delta_\pi^H &= [(-0.02\ .\ |e_1|) - 0.6]\ .\ sat\left(\frac{0.29\sigma_H + 1e_H + 4.2\dot{e}_H + 1\ddot{e}_H}{49}\right) \\
\delta_p^V &= [(-0.003\ .\ |e_1|) - 0.002]\ .\ sat\left(\frac{9\sigma_V + 1e_V + 2\dot{e}_V + 1\ddot{e}_V}{3.3}\right)
\end{aligned}
\tag{15}
$$

Figures **??**(a) and **??**(b) show the response of the total velocity and altitude of the aircraft during the commanded maneuver under the effect of both controllers (UIR and AUIR).

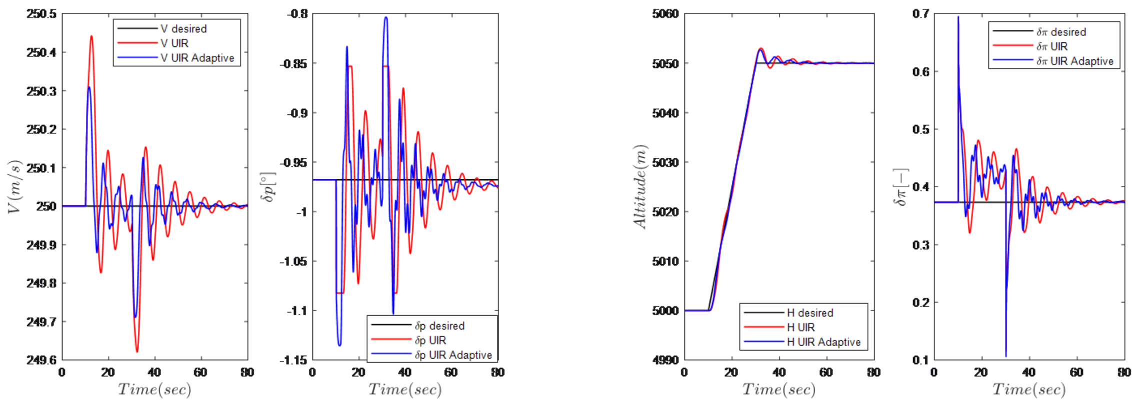
Performance indexes Accumulated Error (AE) and Control Demand (CD) for velocity (V) and altitude (H) controllers are summarized in Table 3. It is worth mentioning that the AUIR provides a reduction of the AE

Table 3. Performance indexes to velocity and altitude control problem

	H-UIR	**H-AUIR**	**H reduction**	**V-UIR**	**V-AUIR**	**V reduction**
AE	735.4777	501.5394	31.8%	93.3932	52.2365	44.06%
CD	42.9388	36.4408	15.13%	51.2672	39.4424	23.06%

Aircraft lateral position control

The control problem for the aircraft lateral position proposed in this work is to command the aircraft y position an offset of $20m$ to the left using aileron deflection (δ_a) as control input. controller was designed through



(a) (b)

Figure 2. (a) Aircraft total velocity response, (b) Aircraft altitude ramp change response

the knowledge of the relative degree, with ($\rho = 4$). The expressions of the control laws, with the values of the gains used and the thickness of the boundary layer are presented in Eq. 16 for original UIR and Eq. 17 for AUIR.

$$\delta_a = 0.005 \;.\; sat\left(\frac{0.01\sigma_y + 1.1e_y + 2.2\dot{e}_y + 3.1\ddot{e}_y + \dddot{e}_y}{5}\right) \tag{16}$$

$$\delta_a = [(0.0005 \;.\; |e_1|) + 0.005] \;\; .\; sat\left(\frac{0.01\sigma_y + 1.1e_y + 2.2\dot{e}_y + 3.1\ddot{e}_y + \dddot{e}_y}{5}\right) \tag{17}$$

The corresponding response of the aircraft lateral position is shown in Fig. 3. It can be noted the responses are similar in behaviour, the, in order to quantitatively compared its performance, Table 4 presents the performance index to each case.

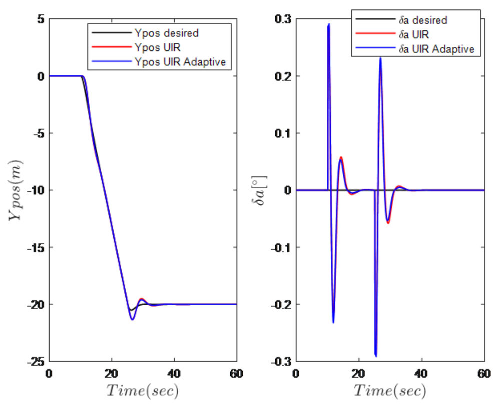


Figure 3. Aircraft lateral position ”y” response

Table 4. Performance indexes to lateral position control problem

	Y-UIR	Y-AUIR	Y reduction
AE	93.7512	77.2328	17.62%
CD	43.0365	40.5146	5.86%

Aircraft yaw angle control

The first two cases presented the application of the AUIR to the control of two kind of dynamics, longitudinal, controlling total velocity and altitude and lateral controlling the lateral y position. In this case, the directional dynamic state variable ψ, also known as yaw or heading angle is tested. The control problem is to command a doublet of $130°$ using aileron deflection (δ_a) as control input. The controller was designed tacking into account the relative degree of the system to be controlled ($\rho = 3$). The UIR and AUIR controllers and its parameters were found, as mentioned in previous section, by a trial and error process, the structure of such controllers is presented in Eq. 18 and 19 respectively.

$$\delta_a = -0.01 \ . \ sat\left(\frac{0.00001\sigma_\psi + 1.3e_\psi + 22.9\dot{e}_\psi + \ddot{e}_\psi}{4.8}\right) \tag{18}$$

$$\delta_a = [(-0.006 \ . \ |e_1|) - 0.01] \ . \ sat\left(\frac{0.00001\sigma_\psi + 1.3e_\psi + 22.9\dot{e}_\psi + \ddot{e}_\psi}{4.8}\right) \tag{19}$$

The response of the aircraft to the commanded doublet is shown in Fig. 4. It can be easily observed that the AUIR had a better performance and small tracking error than the original UIR, even demanding less control during the simulation time. Table 5 presents the quantitative estimation of this improvement.

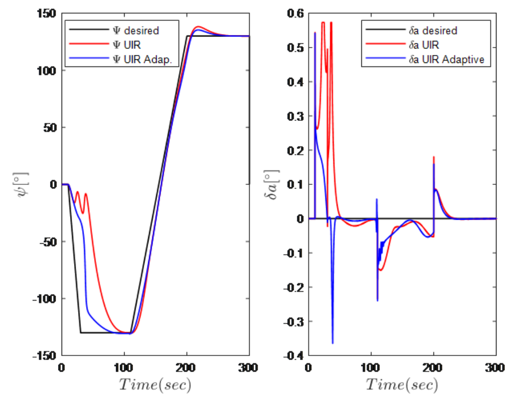


Figure 4. Aircraft total velocity response

Table 5. Performance indexes to lateral position control problem

	ψ-UIR	**ψ-AUIR**	**ψ reduction**
AE	176.58	105.46	40.27%
CD	318.7801	160.2027	49.745%

5 Conclusions

In this work, a preliminary study based only in numerical simulations was done regarding to a modification to the Universal Integral Regulator control law. Three cases associated to longitudinal, lateral and directional dynamics of an aircraft were adopted in order to compare the response of the aircraft dynamics under the effect of UIR and the proposed modification called AUIR. Qualitatively speaking results showed an improvement on performance of the controller for all tracking problems proposed. Two performance indicators were proposed in order to quantitatively compared the controllers, the accumulated error (AE) and the Control Demand (CD) performance indexes. It was demonstrated that the use of the UIR with the proposed modification can be used for the control of the altitude, velocity, lateral position and yaw angle of an aircraft. Showing a reduction between $17 - 44\%$ in AE, that is, more precision of the maneuver commanded and between $5 - 49\%$ of reduction in CD, in other words, less effort and wear of controls.

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