

# UNVEILING PATTERNS IN THE INTERACTION BETWEEN RISERS AND SEABED

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**Abstract.** Risers play an important role in offshore production systems by conveying oil, gas and other fluids from subsea wells to floating production units (FPU) or vice versa. Free hanging (catenary) risers using flexible pipes have been the most conventional configuration in Brazilian production assets, but the participation of steel catenary risers has long been fostered by oil companies and their technological partners. The most challenging sections of free hanging risers are the vicinity of their top end and the touchdown zone (TDZ). This investigation focuses on the latter section, by examining the static interaction between riser and seabed, which is described by Skempton's backbone curve. Besides the external pressure and possible wall compression, the curvatures often impose the strongest constraints to the riser design space. Parametric studies using finite element methods, non-dimensional analysis and statistics seem to be promising route to assess the riser response. The results documented herein are limited to static response with no marine current or wave loads, focusing on the maximum curvature along the TDZ, and they supply zero-order approximations to studies which aim at describing the dynamic response using asymptotic approximation techniques.

**Keywords:** Riser-Soil Interaction, Finite Elements, Non-Dimensional Analysis.

## 1 Introduction

The interaction between marine risers and seabed is matter of great interest in ocean engineering. Risers are long pipes which provide hydraulic connection between subsea equipment and floating production units (FPUs), distinguished from flowlines because the former yield noticeable change of elevation to the flow, whereas the latter lay on the seabed. Risers may have several configurations and they can be made of different materials. The choice of the most suitable riser system depends on parameters such as water depth, FPU motions and its interfacing facilities, installation and pipelay resources, diameter, etc. Free-hanging risers, mostly made of flexible pipes, have been used in offshore Brazil. The oil companies have also encouraged the design and the use of steel catenary risers (SCRs), even though their engineering and installation have a number of challenging concerns, because SCRs may reduce overall costs and they are easier to manufacture, which may increase the competitiveness in bids.

Free-hanging risers undergo static loads derived from self-weight, buoyance, internal fluid weight and drag effects from waves, wind and currents; their configuration is also ruled by physical constraints such as the top-end stiffness, the seabed geometry, its stiffness and frictional properties. These risers usually undergo dynamic loads, which are ruled by top motion, inertia/vibrational effects and damping, although other components may exist (e.g. VIV).

The comparative dominance of loads or physical constraints at different riser sections makes convenient, for

engineering design purposes, to divide the free-hanging riser into three sections: (i) top end and interface with FPU; (ii) intermediate section; (iii) touchdown zone (TDZ) and instantaneous touchdown point (TDP). By segmenting the riser thus, the bending edge effects (see [1]) can be disregarded in intermediate section and the classic catenary theory can be used to predict the static riser configuration and static tension at any point therein. Because the dynamic response of free hanging risers is intricate, the usual method to predict it is numerical (finite element analysis). Nonetheless, some interesting studies (see [2]) have suggested that approximation techniques based on asymptotic expansions, in which zero-order solution is simply the static configuration, may yield reasonable estimates of the dynamic riser response.

In general, acknowledging that the combination of curvature and high tension would drive the riser structural integrity to hazardous states, the engineering design aims at minimizing the local curvature near the top end, either by stiffening (use of stress joints and bend stiffeners), by releasing the rotations (flex-joints) or by using special supporting devices (e.g.: stingers in pipelaying vessels). If these engineering means were not employed, the top angle and vessel motions would have to be limited in a way that may be unfeasible.

With regard to the touchdown zone, the designers have depended on the riser configuration (i.e.: top angle, TDP tension) and pipe properties (i.e.: weight, stiffness) to limit the curvature. Since it neglects the pipe stiffness, the catenary theory cannot provide any rough estimate of the TDP curvature. Indeed, the local curvature is ruled either by “catenary parameters”, pipe stiffness and soil stiffness, as shown by several studies using analytical techniques. PESCE (1997) develops asymptotic solutions for the riser equilibrium in TDZ supposing linear soil deformability and parametric studies in order to examine its effects. CROLL (2000) proposes an interactive process, which is both analytical and numerical, to assess the TDP curvature. PALMER (2008) proposes a simplified solution for soil indentation (which rules the local pipe curvature), as a function of the distributed reaction force exerted by the soil onto the riser. A trend towards better description of the seabed stiffness is noteworthy after the findings of STRIDE and CARISIMA JIPs (see THETHIS and MOROS, 2001[4]; [5]). As the consideration of nonlinear soil attributes such as plasticity and degradation make general numerical techniques (e.g. finite elements) more suitable than analytical methods, the latter have recently reduced their participation in the state of the art.

Two of the main advantages of analytical methods in riser analysis are the computational speed and easier cognition of the parameter-response relationships, which may be either obtained by fully analytical (e.g. deduction of partial derivatives) or partially numerical (parametric studies). The developed cognition of the parameter-response relationships has been a chief skill for riser designers, because of its value in discarding troublesome solutions (i.e. solutions whose drawbacks would be found by costly analyses) in early engineering phases and in accelerating the convergence to optimum solutions (despite optimization algorithms have progressively replaced the human role in design improvement). As analytical methods are abandoned due to their limitations in dealing with sophisticated soil models, costlier studies have to be used to map the parameter-response relationships that rule the riser mechanics.

Such mapping is the proposal of this research. It is performed by a few steps: (1) to run numerical simulations with different design parameters; (2) to find convenient ways to normalize the input data and the results (e.g. by using non-dimensional groups); (3) to identify trends and to look transformations (e.g. principal component analysis) that reinforce or reduce the relationships among groups of parameters or results; (4) to propose heuristic methods to predict key response parameters

## 2 Systems and models

The systems under study consist of free-hanging risers whose top angle ranges from 1 to 37° at water depths ranging from 250 to 3200 m. Although more universal approach was tried out (in order to depict flexible pipes), each riser is made of steel pipe, with constant diameter and thickness throughout its length. Apart from the quasi-static lifting of the riser end, no motion exists. No current or wave loads exist. Every riser is full of water. The seabed is flat; the soil is non-degradable.

The software ABAQUS/Standard is used to build and solve models, which are in-plane (2D). The risers are

modelled with 2-node Euler-Bernoulli beams, which achieved good computational performance. 3-node quadratic beam elements might allow the mesh to be coarser, but they were shown to hinder the performance of the contact algorithm. Hybrid formulation is not seen to be important for tensioned beams. The model length is defined to let the integral of the soil friction along the pipes section laid on the seabed balance the TDP tension. The mesh consists of 5000 elements, distributed in a way to be coarse in the section laid on the seabed and refined at the most curved sections.

The seabed was modelled with a flat analytical line, which is not rigid. The vertical soil stiffness is provided by defining the nonlinear tabular function of penetration displacement as a function of distributed contact force. Its values define Skempton's backbone curve, whose main parameter is the soil shear strength. Besides such vertical interaction, Coulomb-Amonton friction model with prescribed coefficients was employed to interaction in axial direction.

The riser is initially laid on the seabed. The first edge ("top node") is free to rotate and their vertical displacement is prescribed, whereas an encastre eliminates the degrees of freedom of the opposite end. The analysis is fully static. Distributed forces corresponding to self-weight, buoyancy and internal fluid weight are applied in the first step. Prescribed lifting displacement and horizontal component of tension are progressively applied in the second step. As the top end moves upwards and the horizontal component of tension increases, each intermediate solution is stored, since it describes a valid free-hanging configuration in equilibrium.

Despite exact solutions for the Clebsch-Kirchhoff's equations in 2D space are feasible only for simple and particular cases, approximate solutions can be obtained for static catenary risers in the vicinity of TDP. Depending on the proposed assumptions, the equation in the derivative of bending moment can be developed until the derivative of maximum curvature can be written as function of terms like  $s \frac{w}{EI}$  and  $k_0 \frac{T_0}{EI}$ , which give clues for the mutual dependencies among parameters.

### 3 Automation and sampling

Python scripts guess the parameters, build the models in ABAQUS, submit jobs, monitor their execution and get the key results just after each job is complete. The statistical study initially planned was huge, since it comprised an investigation with the variation of 10 design parameters. However, its feasibility depended on how quickly it was model simulation and the intended speed was not achieved unfortunately. Using the current procedure in ABAQUS/Standard, each model run in time between 60 to 4400 s (1018 s on average) in common dual-core notebook, during which about 15 catenary solutions are recorded. The post-processing script avoids recording solutions which are too close. Because of the rough performance, the sampling strategy was limited to 6 parameters and the other parameters were defined by the assumption that the pipe is made of steel and it is full of water. Therefore, in the current version, the sampling is defined by 6 nested loops of tabular core values for the parameters, to which random values are summed. These parameters are: (1) D/t ratio; (2) external diameter; (3) Coulomb friction coefficient; (4) water depth; (5) top angle; (6) undrained soil shear strength.

**Algorithm 1:** Part of python script: sampling the inputs parameters [5].

```
for aD/t in (25.0, 20.0, 15.0, 12.5, 10.0, 8.0):
  for aDe in (0.20, 0.25, 0.30, 0.35):
    for af_atr in (0.08, 0.13, 0.20):
      for aLDA in (300, 500, 900, 1300, 1800, 2300, 3000):
        for aAngTDegree in (2, 5, 7, 9.5, 12, 15, 20, 27, 35):
          for aSu0 in (500.0, 1000.0, 2000.0, 3000.0, 4000.0):
            Su0=aSu0+random.uniform(-0.1*aSu0, 0.2*aSu0)
            f_atr = af_atr + random.uniform(-0.02,0.03)
            AngTGraus= aAngTDegree +random.uniform(-1.0,2.)
```

```

AngT = AngTGraus/57.3
LDA = aLDA + random.uniform(-50.0, 200)
D/t = aD/t + random.uniform(-0.8,0.8)
De = aDe + random.uniform(-0.2,0.2)
t = De/D/t
caso = (De, t, f_atr, LDA, AngT, Su0)

```

The following results are acquired at intermediate and final simulation times: (1) coordinates of top node; (2) top angle; (3) TDP position or length laid on the seabed; (4) TDP tension; (5) TDP shear force; (6) TDP curvature; (7) top tension; (8) top shear force; (9) maximum curvature; (10) position of maximum curvature.

## 4 Pre-processing of simulation results

Normalization by creating non-dimensional groups is tried out in order to (1) determine the phenomena that are scalable; (2) reduce the number of variables (parameters) required to interpret the results. Indeed, the use of dimensionless group alone does not guarantee the easy cognition of physical laws in the riser response, because it may be ruled by a superposition of phenomena (i.e. beam bending, soil penetration, dissipation of energy, etc), which may be unsymmetrically affected by changes in scale. Nonetheless, some pathways can be examined.

Firstly, local bending is deemed to be a strong phenomenon with regard to the TDZ curvature. The “boundary layer” in which the bending effects are relevant is known to be proportional to a length given by  $l_f \equiv \sqrt{EI/T_0}$ , hence the dimensional analysis will hereinafter use it as characteristic size scale. Other characteristic scales are defined by the distributed weight force  $w$ , with units of mass per square of time, and, if necessary, the acceleration of gravity  $g$ , with units of length per square of time.

The following formulae define a first set of dimensionless numbers and the general rules for the composition of other dimensionless parameters:

$$\begin{aligned}
\tilde{s} &\equiv s/l_f, \tilde{L}_s \equiv L_s/l_f, \tilde{y}_t \equiv y_t/l_f, \tilde{D} \equiv D/l_f, \tilde{\kappa} \equiv \kappa \cdot l_f, \\
\tilde{T}_0 &\equiv T_0/(w \cdot l_f), \tilde{w} \equiv 1, \tilde{EI} \equiv EI/(w \cdot l_b^3), \tilde{EA} \equiv EA/(w \cdot l_b), \\
\tilde{S}_u &\equiv S_u \cdot l_f/w, \tilde{Q}_u \equiv Q_u/w
\end{aligned} \tag{1}$$

... where  $s$ = arc length position;  $L_s$ = suspended length;  $y_t$ = elevation of top node;  $D$ = pipe external diameter;  $\kappa$ = maximum curvature;  $T_0$ = TDP tension;  $w$ = distributed weight;  $S_u$ = undrained shear soil resistance;  $Q_u$ = soil load capacity;  $EI$ = pipe bending stiffness;  $EA$ = pipe axial stiffness. Other relevant dimensionless parameters are  $\mu$ = riser-seabed friction coefficient;  $\lambda_t \equiv \tan \theta_t$ = tangent of top angle (and its inverse  $1/\lambda_t$ , which is the non-dimensional curvature predicted by catenary equation as  $s \rightarrow 0$ ) and the diameter to thickness ratio  $D/t$ .

Despite the aforementioned features and expectations, the use of dimensionless numbers brought a didactic drawback to the statistical processing of the results of experiments. The method by which the sampling is performed (by means of nested loops) ensures that those six original parameters are mutually independent (correlation between any of them is close to zero as shown in Table 1, except to the effects of the removal of improper combinations of angle and water depth, which increase the correlation thereof). If three of them are replaced with their dimensionless versions, they will share characteristic scales, i.e. they will be multiplied or divided by the same factors (or combination thereof) and the correlation between them will not be zero any more (Table 2). It is said that they are not orthogonal. In order to mitigate it, the nested loops should have used dimensionless numbers instead; however, this pitfall was noticed too late.

Table 1. Correlation matrix of input parameters using their own dimension.

	$D_{ext}$	$y_{lda}$	$\theta_t$	$\mu$	$S_{u0}$
$D_{ext}$	1	-0.12871368	0.1178929	0.07192681	-0.12836268
$y_{lda}$	-0.12871368	1	-0.3881992	-0.01041626	-0.04081163
$\theta_t$	0.1178929	-0.3881992	1	0.08854680	0.13279087
$\mu$	0.07192681	-0.01041626	0.08854680	1	-0.04786095
$S_{u0}$	-0.12836268	-0.04081163	0.13279087	-0.04786095	1

Table 2. Correlation matrix of useful parameters in dimensionless form.

	$\tilde{D}_{ext}$	$\tilde{y}$	$\lambda_t \equiv \tan \theta_t$	$\mu$	$\tilde{S}_{u0}$
$\tilde{D}_{ext}$	1	0.67386168	0.58528062	0.11604512	-0.12727949
$\tilde{y}$	0.67386168	1	0.38194409	-0.02024184	0.39800335
$\lambda_t \equiv \tan \theta_t$	0.58528062	0.38194409	1	0.04305231	-0.09303051
$\mu$	0.11604512	-0.02024184	0.04305231	1	-0.02167299
$\tilde{S}_{u0}$	-0.12727949	0.39800335	-0.09303051	-0.02167299	1

In reality, there is not any golden rule in data science enforcing the researchers to use the exact set containing  $D/t$  ratio, non-dimensional diameter  $\tilde{D}_{ext}$ , Coulomb friction coefficient  $\mu$ ; non-dimensional elevation of top end  $\tilde{y}$ ; tangent of top angle  $\lambda_t$  and non-dimensional soil shear strength  $\tilde{S}_{u0}$  as “independent variables” in the interpretation of simulation results. Other composite parameters may should be used. For instance, the non-dimensional soil load capacity  $\tilde{Q}_u$  or stiffness  $\tilde{k}_s$  may be a parameter better than the non-dimensional soil shear strength; the non-dimensional horizontal component of tension  $\tilde{T}_0$  may replace  $D/t$  ratio. Such quest for the best parameters is a step in the job.

The lack of orthogonality in the set of parameters taken as “independent variables” may hinder the composition of heuristical laws to approximate the dependent parameters (i.e. the simulation results). Techniques such as principal component analysis (PCA) may be helpful to create alternative parameters, built from linear combinations among the original dimensionless parameters, in order that the “new” parameters form a basis closer to orthogonality condition.

By applying PCA, one may find that 99% of data variability that matters in the results can be retained in two linear functions of dimensionless horizontal tension  $\tilde{T}_0$ , dimensionless soil stiffness  $\tilde{k}_s$  and dimensionless maximum curvature  $\tilde{\kappa}_s$  after these parameters are centered (mean value shifted to zero) and scaled (standard deviation set to one). Their governing power is thus shown. Coherently, the dimensionless horizontal tension  $\tilde{T}_0$  and the dimensionless soil stiffness  $\tilde{k}_s$  are taken as the most prospective candidates for independent variables.

## 5 Correlations between independent variables and simulation results

The correlation between two variables can be quantified by the Pearson product-moment correlation coefficient

$r_{xy}$ , which measures a linear relation between them:

$$r_{xy} = \frac{\sum_{i=1}^N (\bar{x} - x_i)(\bar{y} - y_i)}{\sqrt{\sum_{i=1}^N (\bar{x} - x_i)^2} \sqrt{\sum_{i=1}^N (\bar{y} - y_i)^2}} \quad (2)$$

Figure 1 illustrates one of the methods used in search of the pairs of variables which get the strongest correlations. Although other values such as shear forces, instantaneous TDP curvatures, suspended lengths, first and second integrals (moments) of curvature over length were acquired in the simulations, the maximum curvature is ranked as the most important one in this investigation, because of its influence in riser design.

Different scales for length were tried out as characteristic lengths while making dimensionless groups. The suspended length  $l_s$  is shown to be the most suitable characteristic length to describe mutual relations between catenary parameters and more global results thereof, whereas the dimensionless groups focusing on the curvature at TDP have better representativeness if the characteristic bending length  $l_f \equiv \sqrt{EI/T_0}$  is used instead.

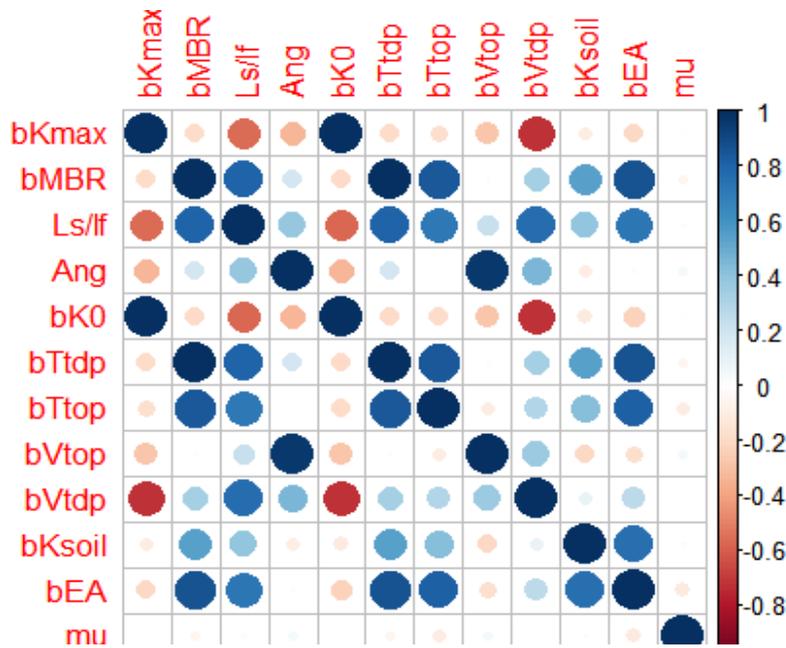


Figure 1. Correlation between local dimensionless maximum curvature and all local dimensionless parameters

The classic catenary formulations are known to be useless in estimation of the curvature at TDP, because it is ruled by local bending. According to that theory, the declination (referenced to vertical axis) of the cable (riser) can be assessed by  $\tan \theta(s) = T_x/T_y = \lambda_t \cdot l_s/s$ . If one insisted on calculating the curvature at TDP therefrom, he would try using:

$$\kappa(0) = \left. \frac{\partial \theta}{\partial s} \right|_{s \rightarrow 0} = -\frac{1}{\lambda_t \cdot l_s} \quad (3)$$

... where  $\lambda_t$  = tangent of top angle;  $l_s$  = suspended length. The correlation between curvatures and the inverse of tangent of top angle is thus a candidate for investigation. However, the correlation between maximum dimensionless curvature  $\tilde{\kappa}_{max}$  and  $\lambda_t$  is found to be very low, as shown in figure 2.

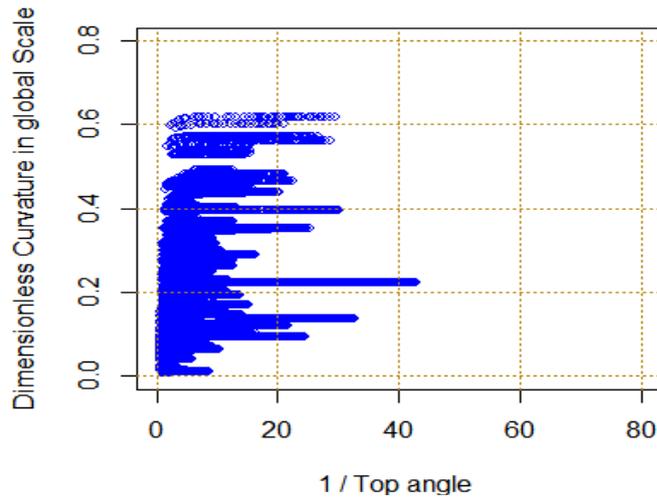


Figure 2. Plot of inverse tangent of top angle in maximum dimensionless

For each pair of parameters whose Pearson product-moment correlation coefficient exceeds 0.70, their XY-plots were examined. Because that correlation coefficient is limited to linear combinations, it cannot examine the goodness of relationships like inverse, power, trigonometric functions and so on. Therefore, composite variables which may better describe the riser response have to be built by hand and often by intuition. After a number of composite functions are conceived, the verification of the correlation coefficient can be performed. The reciprocal of the maximum curvature is one of these composite variables, which is named “minimum bending radius” (MBR for short).

The soil model and its parameters were expected to be relevant to the curvature at TDP. Many works such as PESCE 1997 and other have suggested that the local curvature is much affected thereby. Nonetheless, the correlation coefficient between maximum dimensionless curvature  $\tilde{\kappa}_{max}$  and the dimensionless soil stiffness  $\tilde{\kappa}_s$  is low, the correlation coefficient between dimensionless minimum bending radius  $\overline{MBR} \equiv 1/\tilde{\kappa}_{max}$  and that stiffness (or other soil parameters) is better though.

If no processing is performed, some relationships will go hidden among the results found in finite element analysis. Figure 3 show the plot of minimum bending radius in meter and tension at TDP in kN. No clear relationship is intuitive.

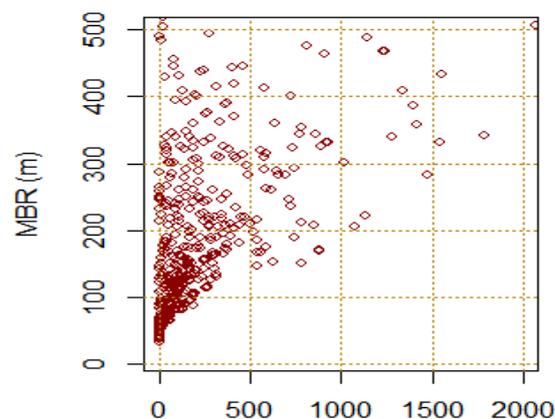


Figure 3. Relation between MBR and tension at TDP.

Indeed, after the due treatment is carried out to find dimensionless groups, the dimensionless curvature scaled by local bending length is evidenced to be very much affected by dimensionless tension at TDP.

Table 3 shows the correlation coefficient of some selected parameters and results.

Table 3. Correlation matrix of input parameters, curvature and minimum bending radius in dimensionless form.

	$\tilde{T}_0$	$\tilde{k}_s$	$\tilde{D}_{ext}$	$\tilde{L}_s$	$D/t$	$\lambda_t$	$\mu$
$\tilde{\kappa}_{max}$	-0.183107	-0.101714	0.67046	0.562981	0.134678	-0.337774	-0.000309
$\overline{MBR}$	0.999982	0.543048	0.27149	0.805664	-0.022229	0.182927	-0.055734

The strong correlation between the dimensionless minimum bending radius and dimensionless horizontal tension, shown in figure 4, exceeds initial expectations, since it is much higher than any other parameter including the soil stiffness, the soil friction and the soil stiffness. It suggests that the concerns on the riser-soil interaction, with respect to the curvature alone, may be exaggerated, because the curvature is ruled by tension more than any other parameter.

Nonetheless, precaution is recommended before any generalization, because the sample is not representative of the entire population of marine risers, it stand for those made of steel and full of water in absence of horizontal loads, within certain ranges of diameter, thickness and soil shear strength. In order to examine the dominance of tension in depth, the sample is split into set based on the bending radius (BR) values, and the correlations are revised, as shown in Table 4.

Table 4. Correlation matrix of dimensionless BR in different subintervals.

$\overline{MBR}$ intervals	Number of samples	$\tilde{T}_0$	$\tilde{k}_s$	$\tilde{D}_{ext}$
$0.916 < BR \leq 1.2$	65	0.9937322	-0.9721980	-0.9766317
$1.2 < BR \leq 1.8$	332	0.9955386	0.4640620	0.1627906
$1.8 < BR \leq 5.0$	1683	0.99937108	-0.0986709	0.7031990
$5.0 < BR \leq 2492$	8326	0.9999816	0.5446593	0.2488220

The dominance of tension on dimensionless MBR is maintained throughout the sample, but contributions of soil stiffness and pipe section are significant for small MBR. It has reasonable explanation, because TDZs with moderate curvatures yield smooth transitions at TDP and such smoothness does not require the soil parameters (other than geometry) to have any ruling role in local curvature. Typical steel catenary risers (SCRs) have dimensionless bending radii exceeding 2, whereas flexible pipes – which are not depicted in current sample – may easily have dimensionless BR as low as 1.

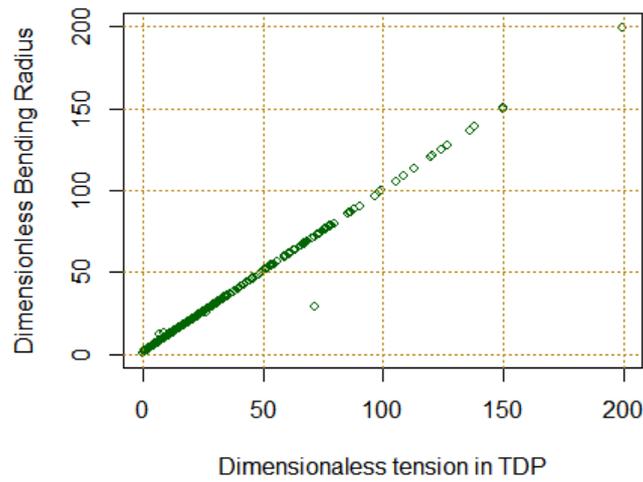
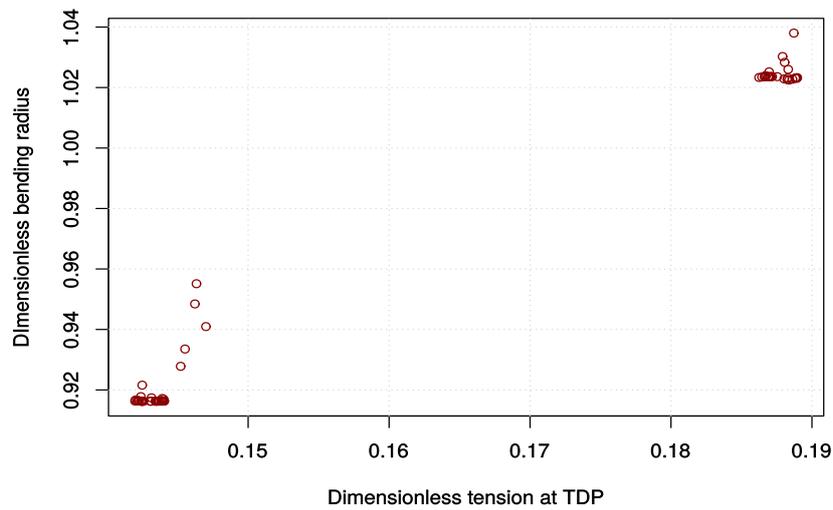


Figure 4. Relation between minimum bending radius and tension in TDP

Figure 5. Relationship between bending radius and tension at TDP for  $BR < 1.2$ .

The following regressions are good estimators for minimum bending radius:

$$\widetilde{MBR} \equiv \frac{1}{\widetilde{k}_{max}} = 1.107 + 0.9956 \widetilde{T}_0, \text{ with } r^2 = 1, \text{ for } \widetilde{MBR} > 1.2 \quad (4)$$

$$\widetilde{MBR} = 0.7131 + 1.719 \widetilde{T}_0 - 3.077 \times 10^{-4} \widetilde{k}_s, \text{ with } r^2 = 0.9954, \text{ for } \widetilde{MBR} \leq 1.2 \quad (5)$$

## 6 Concluding remarks

By running extensive numerical simulations and systematic application of statistical methods, this study examined the relationships between curvatures at TDP and many parameters, and it found thus that the reciprocal of curvature is parameter much better to be used in estimators (regressions). The minimum bending radii for practical SCR cases simulated herein are much more ruled by tension than any other parameters, although this

conclusion may have limited range of validity. The soil stiffness is shown to gain relevance as the curvature increases, along with the bending stiffness.

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