DAMAGE DETECTION IN SIMPLE SUPPORTED PIPE CONVEYING FLUID USING AN ADDITIONAL MOVING MASS

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Abstract. This paper investigated the dynamic behavior a cracked simple supported pipe conveying fluid due to the application of an additional roving mass along the structure. First, a difference finite model of a simple supported Euler-Bernoulli beam conveying fluid experimental was developed to obtain natural frequencies of damped modal analysis. The results are compared to literature. In addition, the frequency shift technique is applied for an undamaged simple support pipe conveying fluid. Last, it was identified a cracked simple supported pipe using a frequency shift technique for different fluid velocity.

Keywords: Tube conveying fluid, crack detection, frequency shift.

1 Introduction

Previous damage detection in pipelines is extremally important to safe and satisfactory maintenance [1]. The fluid flowing inside a pipe acts as a tangential follower force. This gyroscopic fluid force modifies its dynamic characteristic [2–4].

Commonly, visual inspections are realized to investigate the presence of cracks but in some cases, this technique is unsuccessful, for instance, when the damage is in a place that is too hard to see. Thusly, many non-destructive methods have been applied to identification and localization of damages that prejudice the health of the structure. Generally, these techniques compare the intact and damaged response of the structures, which requires the identification of both beforehand. In most of the time, the intact response is impossibly obtained. Hence, researchers have been testing some techniques that use just the damaged response, such as Zhong et al. [5]. In the last decades, studies of identification of damage in structures using an auxiliary mass have been done, such as Zhong; Oyadiji [6], Cardoso et al. [7]. This technique consists of the application of an additional mass along the length of structure to magnify the effect of the discontinuities.

This paper investigated the dynamic behavior a cracked simple supported pipe conveying fluid due to the application of an additional roving mass along the structure. First, a difference finite model of a simple supported Euler-Bernoulli beam conveying fluid experimental was developed to obtain natural frequencies of damped modal analysis. The results are compared to literature. In addition, the frequency shift technique [5] is applied for an undamaged simple support pipe conveying fluid. Last, it was identified a cracked simple supported pipe using a frequency shift technique for different fluid velocity.

2 Roving Mass Technique on a Cracked Tube Conveying Fluid

The system is composed by on a cracked simply supported cracked simple supported pipe conveying fluid, as shown in Figure 1, with L as total length (geometry), U as fluid flow velocity U, m_f as fluid mass per unit length (fluid), EI as flexural rigidity, m as structure mass per unit length, x_c as crack position, x_a and m_a as position and mass of roving mass, respectively. Using the additional mass [5, 6], a roving mass m_a is positioned along of beam producing the frequency-shift graph (Figure 2).

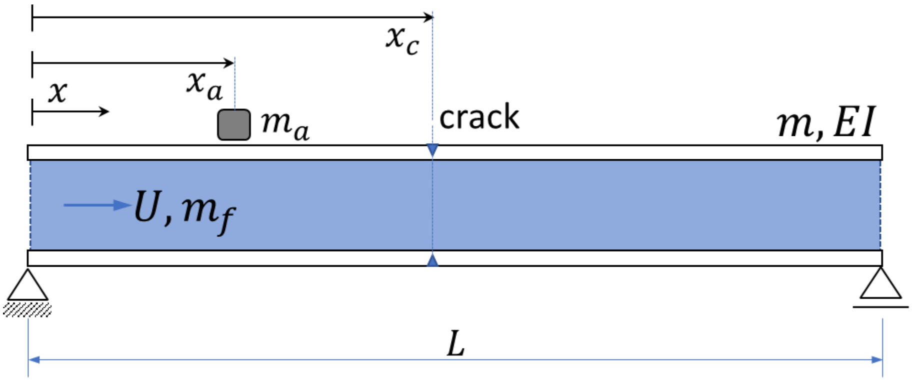
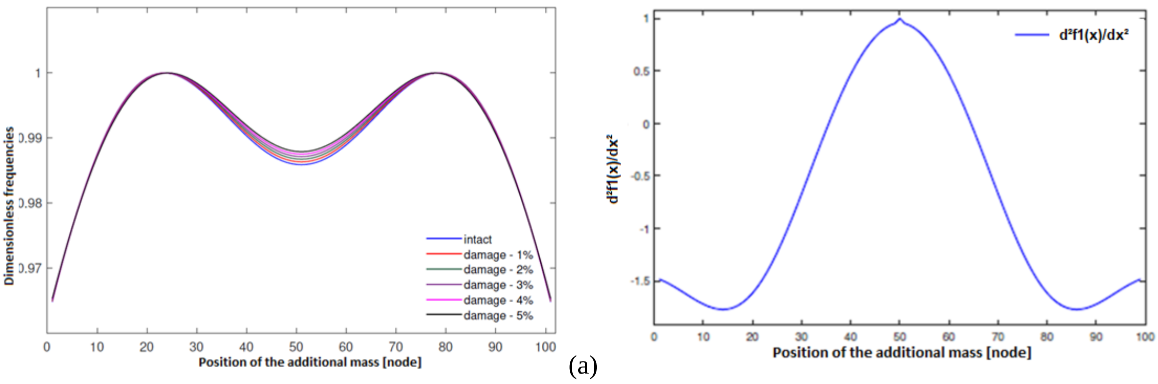


Figure 1. Geometry of a cracked simple supported pipe conveying fluid with a roving mass m_a.



(a) (b)

Figure 2. Fundamental frequency as function of additional mass m_a of a free-free beam with several mass damage ratio m_c/mL(a), and its second derivative for mass damage ratio $m_c/mL = 5\%$, (b) [7].

2.1 Mathematic Description of Tube Conveying Fluid

The linear formulation of pipe conveying fluid is a classical fluid-structure formulation well known until 60s [8, 9]. Païdoussis [10, 11] is a popular reference describing this equation of motion of tube conveying fluid by force balance. Considering incompressible fluid, pressures are measured in relation to atmospheric pressure p_o. and small displacements ($ds = dx$ referring to the equilibrium position), the fluid-structure dynamic equilibrium is done by:

$$EI\frac{\partial^4 w}{\partial x^4} + m_f U^2 \frac{\partial^2 w}{\partial x^2} + 2m_f U \frac{\partial^2 w}{\partial x \partial t} + [(m + m_f) + m_a \delta(x - x_c)] \frac{\partial^2 w}{\partial t^2} = 0 \tag{1}$$

2.2 Dimensionless Equation of Motion

The representation of the fundamental equations that govern the phenomenon in terms of dimensionless variables has been in common use [11]. And the dimensional approach makes possible a wide analysis due to the reduced parameters quantities to vary. Therefore, looking for dimensioning of Eq. (1) to apply the knowledge and techniques present in the literature, the following dimensioning terms are accepted $\xi = x/L$; $\eta = w/L$. By chain rule, the second derivative is obtained by,

$$\frac{\partial^2 w}{\partial x^2} = \frac{\partial}{\partial \xi}\left(\frac{\partial w}{\partial x}\right)\frac{\partial \xi}{\partial x} \quad \therefore \quad \frac{\partial^2 w}{\partial x^2} = \frac{1}{L}\frac{\partial^2 \eta}{\partial \xi^2} \tag{2}$$

And, then the others differential operators can be written as follows:

$$\frac{\partial^4 w}{\partial x^4} = \frac{1}{L^3}\frac{\partial^4 \eta}{\partial \xi^4}, \quad \frac{\partial w}{\partial x} = L\frac{\partial \eta}{\partial t}, \quad \& \quad \frac{\partial^2 w}{\partial t^2} = L\frac{\partial^2 \eta}{\partial t^2} \tag{3}$$

Replacing the dimensionless differential operator in (1), with some algebraic manipulations, it has:

$$\frac{\partial^4 \eta}{\partial \xi^4} + u^2\frac{\partial^2 \eta}{\partial \xi^2} + 2\beta^{\frac{1}{2}}u\frac{\partial^2 \eta}{\partial \xi \partial \tau} + \left[1 + \alpha\,\delta\left(\xi - \frac{x_c}{L}\right)\right]\frac{\partial^2 \eta}{\partial \tau^2} = 0 \tag{4}$$

*w*here, the dimensionless expression is:

$$\xi = \frac{x}{L}, \qquad \eta = \frac{w}{L}, \qquad \tau = \left(\frac{EI}{m_f + m}\right)^{\frac{1}{2}}\frac{t}{L^2}, \qquad u = \left(\frac{m_f}{EI}\right)^{\frac{1}{2}}UL, \qquad \beta = \frac{m_f}{m_f + m} \tag{5}$$

The present problem resumed by two dimensionless numbers: mass ratio β ($0 < \beta < 1$) and reduced flow velocity u. If Ω is the dimensionless frequency of movement related to circular frequency (ω), the equation is represented by:

$$\Omega = \left(\frac{M + m}{EI}\right)^{\frac{1}{2}}\omega L^2 \tag{6}$$

The dimensionless frequency Ω has a complex nature, characterizing the stability of the system, as a function of the imaginary point of $\Omega(\mathrm{Im}\,(\Omega))$, i.e, Im (Ω) being positive, an unstable spiral behavior is established; negative; stable spiral; and null, center.

3 Numerical Solution by Finite Difference

The discrete equation of motion of tube conveying fluid (4) using central finite difference technique in the space domain (ξ) is done. If there are cross derivate terms of η in relation to dimensionless space (ξ) and time (τ), this relation are used, for example,

$$\frac{\partial^2 \eta}{\partial \xi \partial \tau} = \frac{\partial}{\partial \xi}\left[\frac{\partial \eta}{\partial \tau}\right] = \frac{\partial \dot{\eta}}{\partial \xi} = \frac{1}{2\Delta\xi}\{\dot{\eta}_{i+1} - \dot{\eta}_{i-1}\} \tag{7}$$

Then, the equation of tube conveying fluid (4) take this finite difference discretized expression:

$$\frac{1}{\Delta\xi^4}\{\eta_{i+2} - 4\eta_{i+1} + 6\eta_1 - 4\eta_{i-1} + \eta_{i-2\}} + \frac{u^2}{\Delta\xi^2}\{\eta_{i+1} - 2\eta_i + \eta_{i-1}\} + \frac{2\beta^{1/2}u}{2\Delta\xi}\{\dot{\eta}_{i+1} - \dot{\eta}_{i-1}\} + \ddot{\eta}_1 = 0 \tag{8}$$

Or, reordering the displacement, velocity, and acceleration,

$$\ddot{\vec{\eta}}_i + \left\{\frac{2\beta^{1/2}u}{2\Delta\xi}\right\}\dot{\vec{\eta}}_i + \left\{u^2\frac{1}{\Delta\xi^2}\delta_\xi^2 + \frac{1}{\Delta\xi^4}\delta_\xi^4\right\}\vec{\eta}_i = 0 \tag{9}$$

where, ${\delta_\xi}^j$ is the central operator of finite differences in relation to space (ξ) and operator order j ($j = 1$ – first derivate, $j = 2$ – second derivate, as follow), whose representation is done by:

$$\delta_\xi \ldots = (\ldots)_{+1} - (\ldots)_{-1}\,, \tag{10}$$

$$\delta_\xi^2 \ldots = \delta_\xi\left(\delta_\xi \ldots\right) = (\ldots)_{+1} - 2(\ldots)_0 + (\ldots)_{-1}\ , \tag{11}$$

and,

$$\delta_\xi^4 \ldots = \delta_\xi^2\left(\delta_\xi^2 \ldots\right) = (\ldots)_{+2} - 4(\ldots)_{+1} + 6(\ldots)_0 - 4(\ldots)_{-1} + (\ldots)_{-2}\ . \tag{12}$$

For example, by applying the 4th order finite difference operator to η,

$$\delta_\xi^4 \eta_i = \{\eta_{i+2} - 4\eta_{i+1} + 6\eta_i - 4\eta_{i-1} + \eta_{i+2}\}\ . \tag{13}$$

Presenting Eq. (35) in a matrix form, the discretized equation of motion of the tube conducting fluid is:

$$\ddot{\vec{\eta}} + [C]\dot{\vec{\eta}} + [K]\vec{\eta} = \vec{0}\ . \tag{14}$$

where, $(\vec{\eta})^T = \{\eta_1\ \eta_2\ \eta_3 \ldots \eta_n\}$ is the displacement vector of structural discretized nodes; $[K] = \{(u^2/\Delta\xi^2)\ \delta_\xi^2 + (1/\Delta\xi^4)\ \delta_\xi^4 + (\gamma/2\Delta\xi)\ \delta_\xi\}$ is discretized stiffness matrix; $[C] = \{(2\beta^{1/2}u/2\Delta\xi)\ \delta_\xi^2\}$ is discretized damping matrix. Davoudi & Öchsner [12] and Silva & Soares [13] presents more details about finite difference technique.

3.1 Damped Eigensystem Solution

Assuming the tube conveying fluid solution as $\eta = N_o e^{i\Omega\tau}$, the equation of motion (14) takes following form:

$$\{-\Omega^2\ [\mathrm{I}] + \mathrm{i}\Omega\ [\mathrm{C}] + [\mathrm{K}]\}\ \vec{N}_o = \vec{0}\ . \tag{15}$$

To solve the damped eigenproblem above, the variable transformation $\vartheta = \dot{\eta}$ is carried out. Then (15) take this form, considering the identity $[I]\overrightarrow{\vartheta} = \overrightarrow{\eta}$, this expression taken this form:

$$\begin{bmatrix} [0] & [I] \\ -[K] & -[C] \end{bmatrix} \begin{bmatrix} \eta \\ \vartheta \end{bmatrix} = \begin{bmatrix} \dot{\eta} \\ \dot{\vartheta} \end{bmatrix}. \tag{16}$$

Then the matrix expression of the equation of motion of the tube conduction fluid.
In this format, the problem can be solved using integration methods in the time domain. The best adjusted techniques for the solution Eq. (46) are the 1st order differential equation solvers to solve initial value problems (PVI).
Admitting to a solution of the type below, in the frequency domain,

$$\vec{z} = \begin{bmatrix} \eta \\ \xi \end{bmatrix} = \begin{bmatrix} N_0 \\ \Xi_0 \end{bmatrix} e^{i\Omega\tau}\ . \tag{17}$$

Then, replacing (17) into state-space equation (16):

$$\begin{bmatrix} [0] & [I] \\ -[K] & -[C] \end{bmatrix} \begin{bmatrix} \eta \\ \xi \end{bmatrix} = \lambda \begin{bmatrix} \eta \\ \xi \end{bmatrix}. \tag{18}$$

It's a typical damped eigenproblem whose solution take form of complex frequencies $\lambda_i = \iota\Omega_\mathrm{i} = \iota\omega_{n,i} \pm \xi_i$, where, $\omega_{n,i}$ and ξ_i is the natural frequency and damping of i-th modal mode of this problem. For solution, it is recommended to reduce the matrix A in the "upper" form of Hessemberg and use the QR method to determine the eigenvalues.

4 Cracked Description

Yoon & Son [1] propose a stiffness damage as a torsional spring, Figure 3. For beam-like structures, this crack model approach (or discrete spring model) present a good agreement for the two first modal frequencies and for small damage ratio [14]. Then, supposing the boundary conditions of a cracked simply supported pipe for transversal deflection, bending moment, shear force and angular slope at crack ($\xi_c = x_c/L$), respectively, are done by:

$$\eta^+(\xi_c) = \eta^-(\xi_c), \qquad \frac{\partial^2\eta^+(\xi_c)}{\partial\xi^2} = \frac{\partial^2\eta^-(\xi_c)}{\partial\xi^2}, \qquad \frac{\partial^3\eta^+(\xi_c)}{\partial\xi^3} = \frac{\partial^3\eta^-(\xi_c)}{\partial\xi^3},$$
$$\frac{\partial\eta^+(\xi_c)}{\partial\xi} - \frac{\partial\eta^-(\xi_c)}{\partial\xi} = \frac{EI}{K_R}\frac{\partial^2\eta^+(\xi_c)}{\partial\xi^2} \tag{19}$$

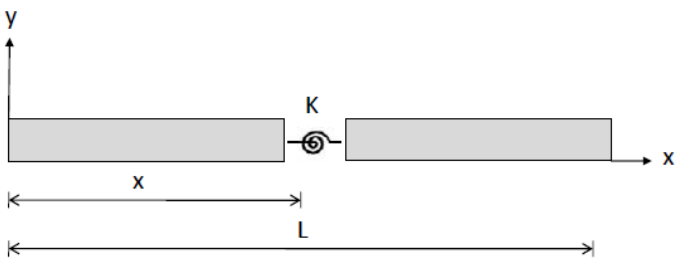


Figure 3. Stiffness damage modeled by a torsional spring [7].

where, the indices + and – represent the positions on right and left of crack position, and K_R is discrete torsional spring representing the open crack. For Mehrjoo, Khaji and Ghafory-Ashtiany [15], the equivalent spring stiffness for a single-sided open crack is expressed by:

$$K_R = \frac{1}{72\pi}\frac{Ewh^2}{f(\kappa)} \tag{20}$$

where crack-depth ratio $\kappa = d/h$ with crack depth d, beam depth h, and beam width w; E is the Young modulus; and

$$f(\kappa) = 0.6384\,\eta^2 - 1.0350\,\eta^3 + 3.7201\,\eta^4 - 5.1773\,\eta^5 + 7.5530\,\eta^6 - 7.3320\,\eta^7 + 2.4909\,\eta^8 \tag{21}$$

4.1 Finite Difference Cracked Modelling

Krishnan, George and Malathi [16] discuss about a consistent finite difference approach from a buckling load of stepped simply supported beam. This numerical approach to describe the bending displacement of two separate beam using four auxiliar fictitious nodes and four discretized boundary conditions, as described in Figure 4 :

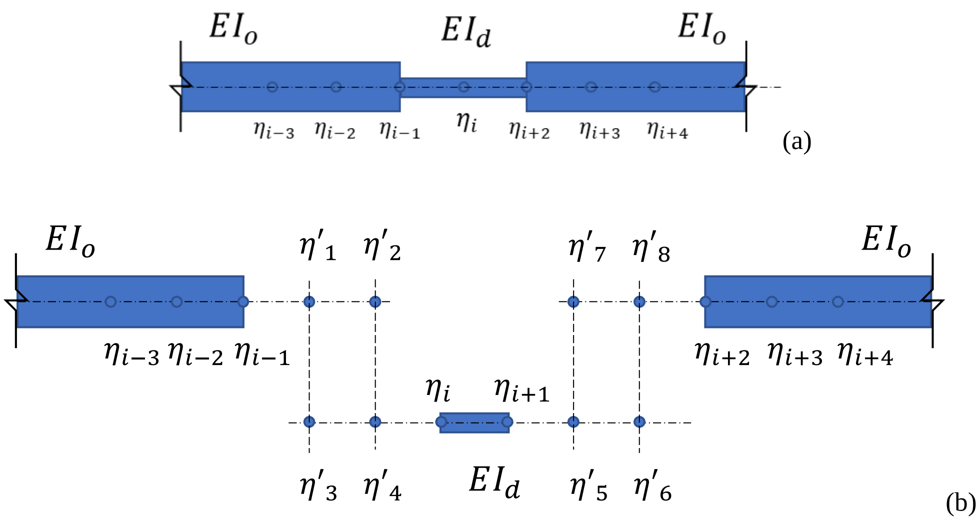


Figure 4. Finite difference approaches to model an open crack in a beam: (a) inconsistent continuous modeling, and (b) fictitious node approach of Krishnan George and Malathi [16]).

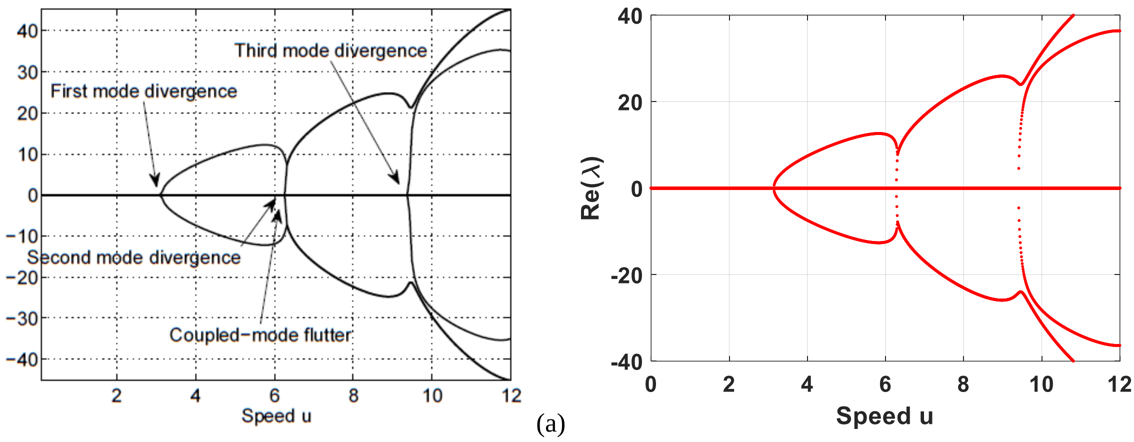
This approach can be used to describe a crack with hypothesis of torsional spring. Other approximation is approximation at crack point as a variation of a one line on linear system. This inconsistent approach did a substitution of intact bending stiffness by cracked one EI_d at crack node η_i. This simple (but inconsistent approach) is analyzed here.

5 Numerical Results

This preliminary work presents a validation of a simple supported tube conveying fluid (section 5.1) and an example of damage detection by a roving mass m_a for a constant flow fluid U (section 5.2).

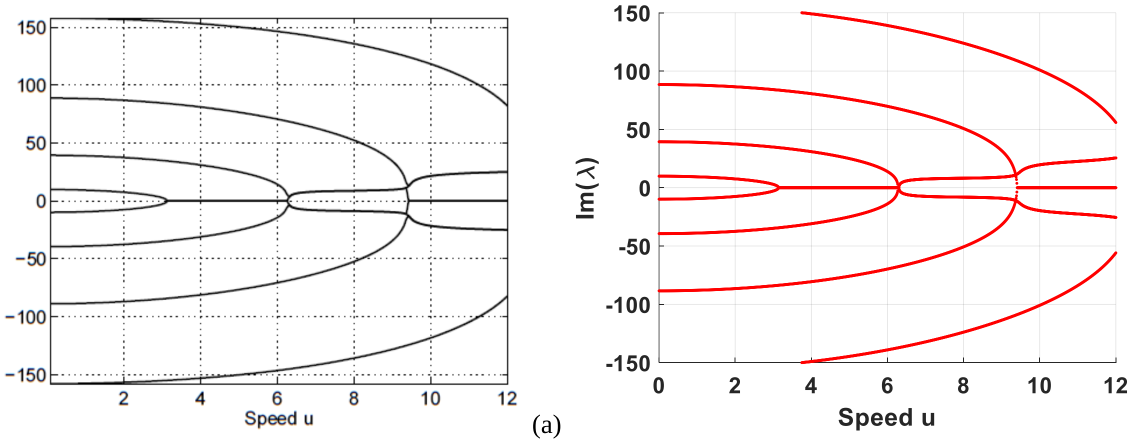
5.1 Validation – Simple Supported Tube Conveying Fluid

To validate the present modeling, an example of simple supported beam is comparted with literature example presented by Ritto et al [17]. After convergence checked, the presented result is done with 60 nodes. The first four frequencies are presented (i.e., eight eigenvalues) for this analysis with fluid mass ratio $\beta = 0.24$ and dimensionless flow speed $u \in [0,12]$. The stability chart is presented. Respectively, Figure 5 and Figure 6 show the real part and imaginary part of dimensionless frequency $\lambda\ (= \iota\Omega)$. This results present a good agreement with Ritto et al [17].



(a) (b)

Figure 5. Comparation between real part of eigenvalue λ as function of dimensionless flow speed u ($Re(\lambda) \times u$): (a) Ritto et al [17], and (b) present result.



(a) (b)

Figure 6. Comparation between imaginary part of eigenvalue λ as function of dimensionless flow speed u ($Im\ (\lambda) \times u$): (a) Ritto et al [17], and (b) present result.

5.2 Cracked Modelling

For a simple supported pipe with a mass ratio $\beta = 0.24$ modelled using 60 nodes. An open crack, in middle span ($x_c = L/2$), with a bending ratio $\alpha_{d,EI} = EI_o/EI_d = 22\%$ was proposed to being practical. The frequency shift of several roving mass $\alpha = m_a/m + m_f = [2\% \quad 4\% \quad 8\% \quad 16\% \quad 32\%]$ are presented in Figure 7 (first three added mass) and Figure 8 (last three added mass). The roving mass is responsible by a frequency discontinuity in frequency shift graph. It is necessary a high mass ratio to obtain a frequency increase of 1% that is difficult to be measured experimental.

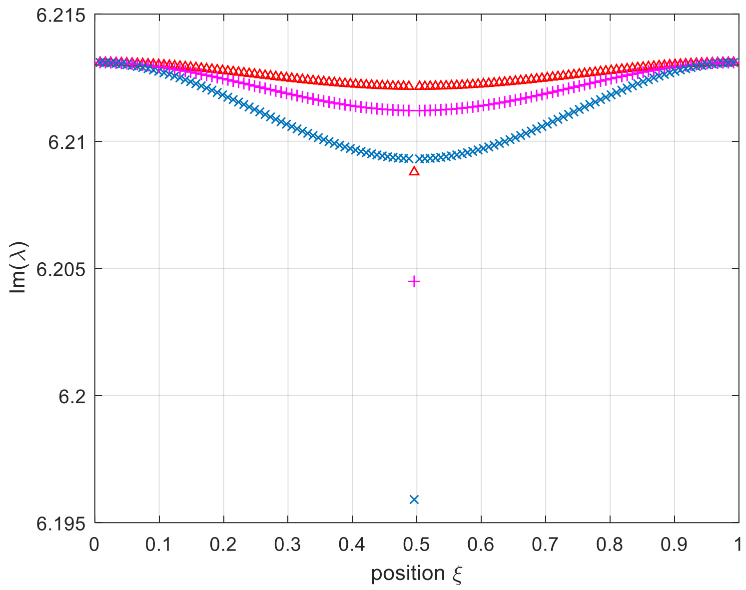


Figure 7. Frequency shift of a roving mass simple supported tube conveying fluid ($\beta = 0.24$) with several roving mass ratio α: (red triangle Δ) $\alpha = 2\%$, (pink plus $+$) $\alpha = 4\%$, (cyan cross $\times$) $\alpha = 8\%$.

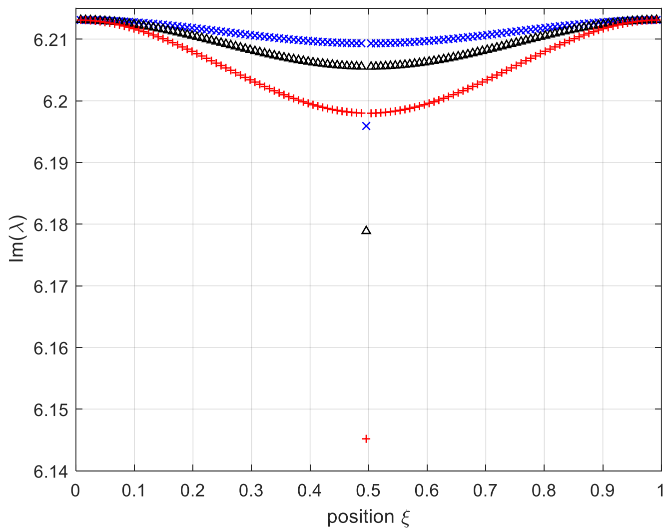


Figure 8. Frequency shift of a roving mass simple supported tube conveying fluid ($\beta = 0.24$) with several roving mass ratio α: (blue cross $\times$) $\alpha = 8\%$, (black triangle Δ) $\alpha = 16\%$, (red plus $+$) $\alpha = 32\%$.

6 Conclusion

In this paper, it was proposed a preliminary modeling of a dynamic behavior a cracked simple supported pipe conveying fluid due to the application of an additional roving mass along the structure. The numerical results are validate with literature [17]. At last, it was identified a cracked simple supported pipe using a frequency shift technique for different roving mass ratio. The roving mass is responsible by a frequency discontinuity in frequency shift graph but an increase of 1% frequency need the use of important mass ratio.

For the future work, a more precise model to describe the open crack in finite difference method. A more profound investigation are necessary to explore with results near to critical velocity, responsible for structural buckling (or divergence instability). And, at last, a study of breath cracks to precisely describe the nonlinear effects of defects in tube conveying fluids.

Acknowledgements. The authors of this research are grateful for the financial support of the Coordination for the Improvement of Higher Education Personnel (CAPES) and the Post-Graduate Decanate of the University of Brasília (DPG-UnB).

Authorship statement. First author (code programming and write text), second author (write text), third to fifth author (revision text)

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