XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

SSI-COV method applied to ambient vibrations of a concrete block of a dam: Uncertainty of modal parameters

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Abstract.
Due to advances and automation in operational modal analysis (OMA) methods, it is possible to extract modal parameters (natural frequencies, damping ratios and vibration modes) in a quick and easy way solely from data collected during operation. However, one of the shortcomings of these methods lies in the assessment of the accuracy of the information obtained. In this work, the uncertainty in the identified modal data is quantified by calculating confidence intervals using the Bootstrap technique. The modal parameters of a concrete block of a dam are identified through the application of the SSI-COV method to acceleration measurements from a triaxial sensor installed in the block. Then, the bootstrap technique is applied and a comparison is made between three common methods of resampling the time series based on nonrandom as well as random block lengths.

Keywords: concrete block, modal parameters, Bootstrap

1 Introduction

The process of identifying modal parameters involves measuring and acquiring data, system identification and the estimation of the modal parameters from the described identification system. Each of these steps is subject to error, which causes the identified modal parameters to exhibit a degree of uncertainty. According to Reynders et al. [1], the sources of error that lead to these uncertainties are: The finite number of data, which causes that the covariance matrices not to be calculed exactly, non-stationary, coloured and deterministic loads, system non-linearities, instrumentation noise and errors induced by the identification algorithm. Some of the purposes of modal parameter identification are calibration and updating of numerical finite element models and damage detection, then, in these cases, estimating these errors in the modal parameters is of utmost importance, either to more accurately predict the dynamic response of the structure or to assess whether changes in a set of estimated modal parameters are due to changes caused by damage or by these errors inherent in the identification process.

The uncertainties in modal parameters can be quantified through the approach bootstrap. This technique consists of repeatedly sampling random datasets from the observed data and repeat the parameter estimation routine for each individual bootstrap dataset (Majid et al. [2], Farrar et al. [3], Hastie et al. [4]). When the observed data depend on each other as is the case of acceleration time series, resampling should be carried out through data blocks and not through individual observations. Some of the block bootstrap methods are: Moving blocks, blocks of blocks and stationary bootstrap. The first two methods resample blocks of observations with a nonrandom block length, while the last one uses a random block length and hence, has a slightly more complicated structure. In this article, the uncertainties in the modal parameters (frequencies and damping ratios) of a concrete block of a hollow gravity type dam are determined through the application of the three block bootstrap methods mentioned above. The SSI-COV algorithm used for modal identification is outlined below, in section 3 briefly explains the block bootstrap methods, in section 4 is shown the procedure to determine the confidence intervals, in section 5 the procedure is implemented and the results are shown and in section 6 the conclusions are presented.

2 Covariance-driven stochastic subspace method (SSI-COV)

The starting point of the SSI-COV method is to evaluate the output covariance matrices Λ_l for time lags between Δt and $(2i-1)\Delta t$ or equally for $l = 1, 2, ..., 2i-1$. Then, these output covariance matrices are gathered in a block Toeplitz matrix $T_{1|i} \in \mathbb{R}^{n_y i X n_y i}$.

$$T_{1|i} = \begin{bmatrix} [\Lambda_i] & [\Lambda_{i-1}] & \cdots & [\Lambda_1] \\ [\Lambda_{i+1}] & [\Lambda_i] & \cdots & [\Lambda_2] \\ \cdots & \cdots & \cdots & \cdots \\ [\Lambda_{2i-1}] & [\Lambda_{2i-2}] & \cdots & [\Lambda_i] \end{bmatrix}. \tag{1}$$

In reality a finite number N of data is available, so, the output covariance matrices can be evaluated separately from the following equation:

$$\Lambda_l = \frac{1}{N-l} \sum_{k=0}^{N-l-1} y_{k+l} y_k^T, \tag{2}$$

however, this expression can be very time-consuming, therefore, another available alternative is to use a high-speed fast Fourier transform (FFT) based approach.

The Toeplitz matrix can be factorized into the extended observability matrix $O_i \in \mathbb{R}^{n_y i X 2n_m}$ and the reversed extended stochastic controllability matrix $\Gamma_i \in \mathbb{R}^{2n_m X n_y i}$, as show bellow:

$$T_{1|i} = O_i \Gamma_i = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^{i-1} \end{bmatrix} \begin{bmatrix} A^{i-1}G & \cdots & AG & G \end{bmatrix}, \tag{3}$$

thus, the Toeplitz matrix is the result of the product of a matrix with $2n_m$ columns by a matrix with $2n_m$ rows ($2n_m$ is the dimension of the state-space model), hence, the rank of $T_{1|i}$ is $2n_m$, if $2n_m < n_y i$. The rank of $T_{1|i}$ is not less than $2n_m$ due to the noises in the observed data. Generally, singular values caused by noise are much lower than those caused by true data. To reduce the effects of noise, it is used the truncated decomposition of singular values, which converts the singular values caused by noise into zeros. This is a common method used in signal processing. Then, $T_{1|i}$ can be written as a decomposition of singular values in the following way:

$$T_{1|i} = USV^T = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} S_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}^T = U_1 S_1 V_1^T, \tag{4}$$

where $U \in \mathbb{R}^{n_y i X n_y i}$ and $V \in \mathbb{R}^{n_y i X n_y i}$ are orthonormal matrices, with $U^T U = UU^T = I \in \mathbb{R}^{n_y i X n_y i}$ and $V^T V = VV^T = I \in \mathbb{R}^{n_y i X n_y i}$. $S \in \mathbb{R}^{n_y i X n_y i}$ is a diagonal matrix containing positive singular values in descending order. The comparison of eq. (3) and eq. (4) shows that the observability and the controllability matrices can be calculated from the outputs of the SVD using the following partition of the singular values matrix:

$$O_i = U_1 S_1^{1/2}, \tag{5}$$

$$\Gamma_i = S_1^{1/2} V_1^T. \tag{6}$$

With the observability O_i and controllability Γ_i matrices already calculated, computing the system matrices A and C is simple. Matrix C equals to the first n_y rows of O_i. There are two ways to identify the state transition matrix A: by exploiting the shift structure of the extended observability matrix proposed by Kung [5] , as follows:

$$\begin{bmatrix} CA \\ CA^2 \\ \cdots \\ CA^i \end{bmatrix} = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^{i-1} \end{bmatrix} A \iff A = \begin{bmatrix} C \\ CA \\ \cdots \\ CA^{i-1} \end{bmatrix}^{\dagger} \begin{bmatrix} CA \\ CA^2 \\ \cdots \\ CA^i \end{bmatrix} = \underline{O_i}^{\dagger} \, \overline{O_i}, \tag{7}$$

where $(\bullet)^\dagger$ represents the Moore-Penrose pseudo-inverse of a matrix. $\underline{O_i}$ contains the first $n_y(i-1)$ rows of O_i and $\overline{O_i}$ contains the last $n_y(i-1)$ rows of O_i. Alternatively, Zeiger and Mcewen [6] suggests that the matrix A could also be computed from the decomposition property of a shifted block Toeplitz matrix:

$$T_{2|i+1} = O_i A \Gamma_i, \tag{8}$$

where the shifted matrix $T_{2|i+1}$ is composed of output matrices covariances Λ_l for $l = 2, 3, ..., 2i$. Introducing eq. (5) and eq. (6) into eq. (8) and solving for A gives:

$$A = O_i^\dagger T_{2|i+1} \Gamma_i^\dagger = S_1^{-1/2} U_1^T T_{2|i+1} V_1 S_1^{-1/2}. \tag{9}$$

From matrices A and C the modal parameters are extracted. The decomposition of the state transition matrix A is given by:

$$\underset{2n_m x 2n_m}{A} = \sum_{k=0}^{2n_m} \psi_k \lambda_k \psi_k^{-1}, \tag{10}$$

where ψ_k and λ_k are the discrete-time eigenvectors and eigevalues respectively. As A was obtained from the discretization of the continuous-time matrix A_c, the equivalent continuos-time eigenproperties can be computed as follows:

$$\psi_{ck} = \psi_k, \;\; \lambda_{ck} = \frac{ln\left(\lambda_k\right)}{\Delta t}. \tag{11}$$

Natural frequencies and damping ratios are calculated from:

$$f_k = \frac{|\lambda_{ck}|}{2\pi} \;\; [Hz], \tag{12}$$

$$\xi_k = -\frac{Re\left(\lambda_{ck}\right)}{|\lambda_{ck}|} * 100 \;\; [\%], \tag{13}$$

where $Re\left(\bullet\right)$ represents the real part of a complex number. Finally, the observed modes can be computed through the combination of eigenvectors of matrix A with matrix C:

$$\phi_k = C\psi_k \tag{14}$$

3 The bootstrap method

The bootstrap method was developed by Efron [7]. Supose the random variable y is the outcome of some stochastic process with unknown probability distribution F and n independent measurements, collected in the sample $Y = \{y_1, y_2, ..., y_n\}$, are available to estimate a parameter of interest $s(Y)$. The bootstrap method consist in create additional collections of data, denoted $Y^{(b)*} = \{y_1^{*(b)}, y_2^{*(b)}, ..., y_n^{*(b)}\}$, as a randomized or resampled version of the original sample $Y = \{y_1, y_2, ..., y_n\}$. Once these additional samples are formed, the usual sample statistics can be applied. The basic assumptions in this method are: The measured outcomes of the random variable y collected in $Y = \{y_1, y_2, ..., y_n\}$ must be independent and the measured outcomes y_i must be representative of the random source. When working with time series, the first assumption is violated due to the time series $Y = \{y_1, y_2, ..., y_n\}$ is a collection of serially dependent measurements, hence, the application of the resampling process with the individual outcomes will break up the covariance structure of the time series. Then, it is possible to extent the method to time series through the resample blocks of data rather than individual observations. Three methods of block bootstrap are shown below.

3.1 The moving block bootstrap (MB)

The steps to apply the moving block bootstrap are given by Künsch [8] and can be summarized as follows:
1. Break the time series $Y = \{y_1, y_2, ..., y_n\}$ into $n - l + 1$ overlapping blocks $B_i = \{y_i, y_{i+1}, ..., y_{i+l-1}\}$ of length l for $i = 1, 2, ..., n - l + 1$. Then, form the collection $B = \{B_1, B_2, ..., B_{n-l+1}\}$.
2. Resample $k = n/l$ blocks B_i with replacement to form B bootstrap time series replica by collecting the k resampled B_i. For example, the b^{th} bootstrapped time series may be: $Y^{(b)*} = \{B_1, B_5, ..., B_{n-l+1}, ..., B_1\}$.
3. Compute the bootstrap replica of the statistic of interest, $s^{(b)*}(Y) \equiv s\left(Y^{(b)*}\right)$. The estimator $s^{(b)*}(Y)$ could be for instance the sample correlation estimate $\Lambda_{Y^{(b)*}}(r)$ of the bootstrapped time series $Y^{(b)*}$, where r denotes the lag.
4. Compute the sample statistic of interest over the ensemble of the B generated bootstrap replica $s^*(Y)$.

3.2 The block of blocks bootstrap (BB)

The procedure for applying the block of blocks bootstrap was proposed by Politis [9] and it is described below:
1. Determine the maximum lag r_{max} of interest up to which the bootstrapped correlations function are computed. This value determine the length of the m-tuples Y_i and is $m = r_{max} + 1$.
2. Form the possible $n - m + 1$ overlapping, consecutive m-tuples Y_i for the measured responses.
3. Form the auto-correlation function of each m-tuple Λ_{Y_i}.
4. Compute the average over l consecutive auto-correlation functions $B_{i,Y} = l^{-1} \sum_i^{i+l-1} \Lambda_{Y_i}$ and get the collection $B_Y = \{B_{1,Y}, B_{2,Y}, ..., B_{n-m-l+2,Y}\}$. l takes the role of the block length.
5. Resample $k = \left[\frac{n-m+1}{l}\right] + 1$ auto-correlation functions $B_{i,Y}$ from this collection to form the bootstrapped auto-correlation fuctions $\Lambda_{Y^{(b)*}} = k^{-1} \sum_{j=1}^{k} B_{j,Y}$.
6. Repeat the process to obtain the desired B bootstrap replica.

3.3 The stationary bootstrap (SB)

To alleviate the effects of joining independent blocks, the stationary method was suggested by Politis [10]. Let $B_{i,b} = \{Y_i, Y_{i+1}, ..., Y_{i+b-1}\}$ be the block consisting of b observations starting from Y_i. When $j > N$, Y_j is defined to be Y_i, where $i = j(modN)$ and $Y_0 = Y_N$. Let p be a fixed number in $[0, 1]$. Independent of $Y_1, ..., Y_N$, let $L_1, L_2, ...$ be a sequence of independent and identically distributed random variables having the geometric distribution, so that the probability of the event $L_i = d$ is $(1-p)^{d-1} p$ for $d = 1, 2, ...$ Independent of the Y_i and the L_i, let $I_1, I_2, ...$ be a sequence of independent and identically distributed variables which have the discrete uniform distribution on $1, ...N$. Now, a pseudo time series is generated in the following way. Sample a sequence of blocks of random length by the prescription $B_{I_1,L_1}, B_{I_2,L_2}....$ The first L_1 observations in the pseudo series are determined by the fisrt block B_{I_1,L_1} of observations $Y_{I_1}, ..., Y_{I_1+L_1-1}$ the next L_2 observations in the pseudo time series are the observations in the second sampled block B_{I_2,L_2}, namely $Y_{I_2}, ..., Y_{I_2+L_2-1}$. The process is stopped once N observations in the pseudo time series have been generated and then, one can compute the quantity of interest $\Lambda_{Y^{(b)*}}(r)$.

4 Confidence intervals of the modes with Bootstraps

To apply the Bootstrap technique in operational modal analysis, it is assumed that there is a single set of simultaneous responses measured at different locations along the structure to determine the modal parameters of the system. If it were possible to obtain a set B of modal parameters (i.e. repeat B times the modal test) for B large enough, the statistics of these modal parameters such as mean, standard deviation and confidence intervals could be determined. Repeating a measurement test a large number of times implies that the time series must be long enough for an acceptable identification of the modal parameters, therefore, these time limitations make this repetition process impractical. Then, the Bootstrap technique allows to simulate response data from the only available set of measurements. According to Giampellegrini [11], once response data is simulated, it is then possible to get a collection of B* sets of bootstrapped modal parameters by application of a curve-fit algorithm to the bootstrapped time-series of correlation functions (for the SSI-COV method), from, which the statistics of the system's model can be determined. This process is shown in Fig. 1. Finally, the 95% confidence intervals for each vibration mode are estimated by choosing the 2.5-percentile and 97.5-percentile of each bootstrap mode population.

5 Implementation and results

In this article we will study the structure used in Ardila et al. [12], which consists of a concrete block of a hollow gravity dam. This concrete block is equipped with a 3-channel sensor that is set-up to acquire acceleration signals at a sampling frequency of 200 Hz. Acceleration time series of 19/05/2019 were collected from 10:00 a.m. to 10:30 a.m. with a total of 360000 samples in each direction (Longitudinal, transversal and vertical). These acceleration data were decimated in order to obtain a final sampling frequency of 40 [Hz] i. e, a total of 72000 samples. The SSI-COV method was applied to the time series with a model order of 30. The results of the natural frequencies and damping rates of the first five vibration modes are presented in Table 1. To determine the confidence intervals for these identified parameters, the three block resampling methods shown in Section 3 were

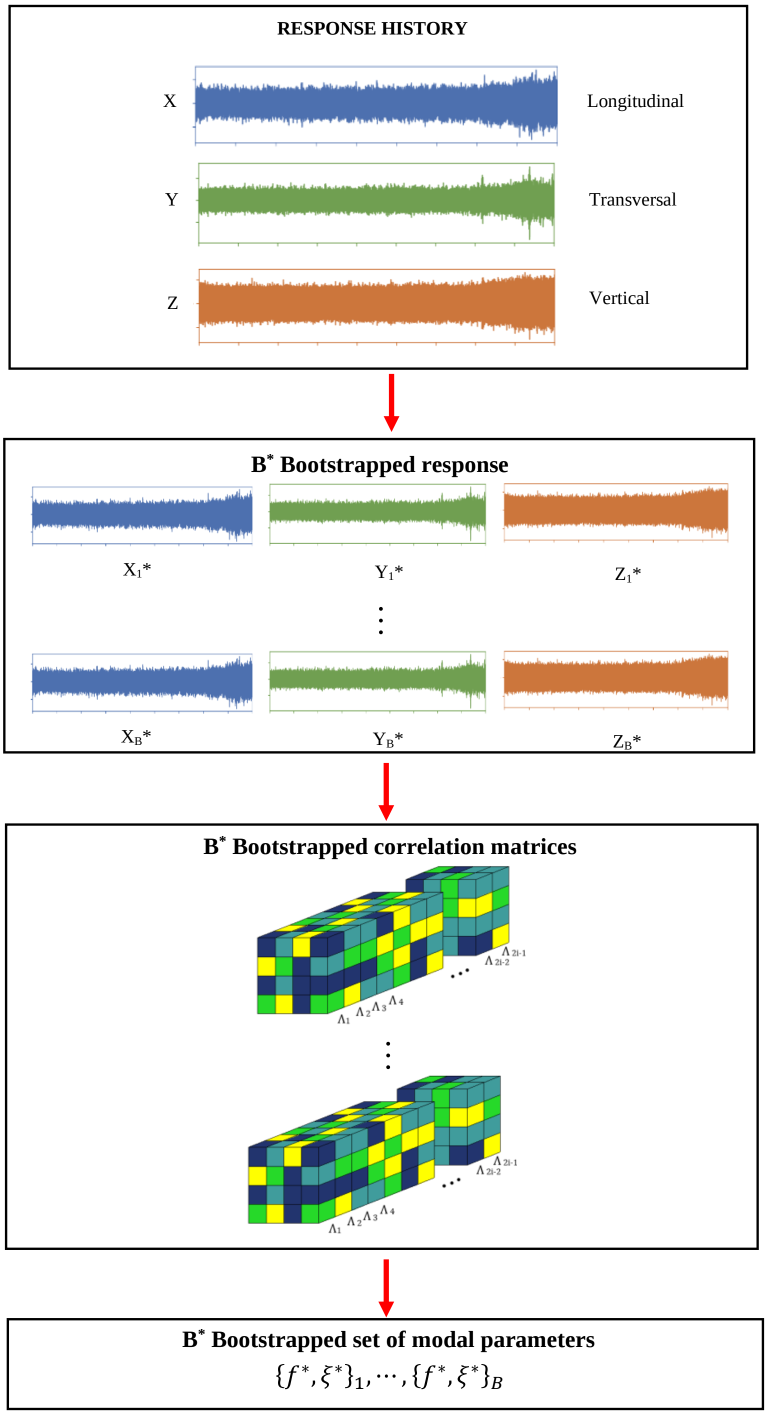


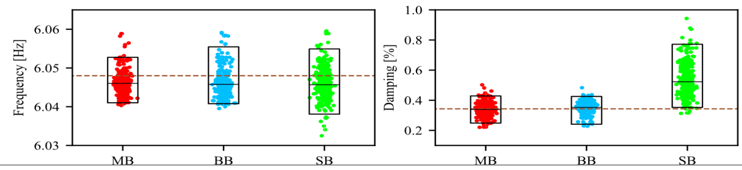
Figure 1. Bootstrap procedure.

applied. For the Moving Block and Block of Blocks methods, the time series of 72000 samples was divided into 12 equal consecutive blocks and a length of the block l equal to 3 i.e. 18000 samples was selected as the optimal. 300 response series were generated for each case and the sets of modal parameters obtained were grouped according to the euclidean distance $d_{i,j} = \sqrt{f_i - f_j}$.

The results are presented in Fig. 2. The 95% confidence intervals are shown. According to Bajric et al. [13], damping ratios are the modal parameter most sensitive to changes in correlation functions. It is also known that acceleration time series of real large structures are non-stationary. When the time series are stationary, all the points in each block have the same distribution and consequently the correlation function of each block will be representative of the entire random process.

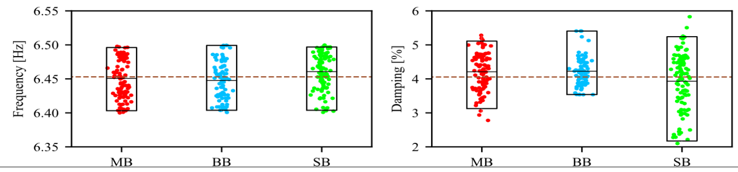
Mode	Frequency [Hz]	Damping ratio [%]
1	6,048	0,343
2	6,453	4,056
3	6,718	0,616
4	7,567	0,623
5	8,172	1,424

Table 1. Modal parameters identified in the concrete block



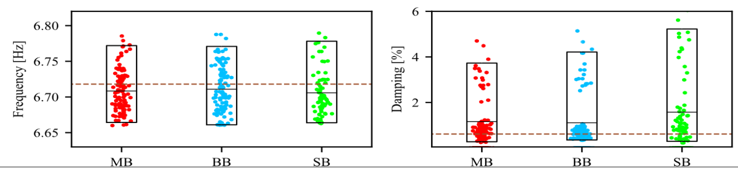
Mode 1

	MB	BB	SB	MB	BB	SB
Lower limit	6.041	6.041	6.038	0.248	0.241	0.353
Mean	6.046	6.046	6.046	0.341	0.35	0.538
Upper limit	6.053	6.055	6.055	0.429	0.425	0.772



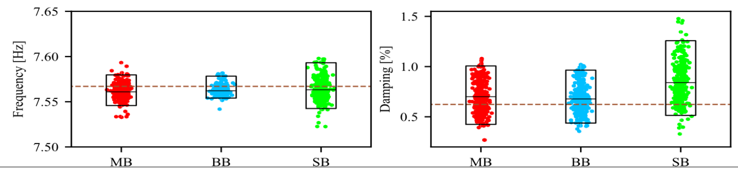
Mode 2

	MB	BB	SB	MB	BB	SB
Lower limit	6.403	6.404	6.404	3.126	3.539	2.172
Mean	6.451	6.448	6.461	4.209	4.225	3.932
Upper limit	6.496	6.499	6.497	5.112	5.407	5.241



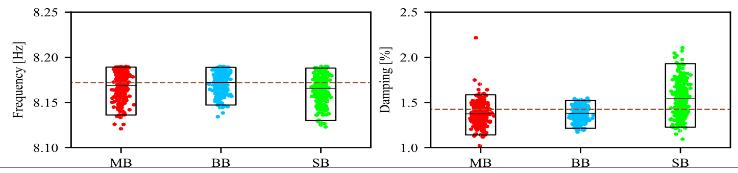
Mode 3

	MB	BB	SB	MB	BB	SB
Lower limit	6.664	6.661	6.664	0.277	0.356	0.296
Mean	6.708	6.711	6.706	1.159	1.11	1.577
Upper limit	6.772	6.771	6.778	3.73	4.216	5.227



Mode 4

	MB	BB	SB	MB	BB	SB
Lower limit	7.546	7.554	7.543	0.425	0.437	0.514
Mean	7.561	7.562	7.563	0.7	0.678	0.84
Upper limit	7.58	7.578	7.593	1.007	0.965	1.258



Mode 5

	MB	BB	SB	MB	BB	SB
Lower limit	8.136	8.147	8.13	1.142	1.215	1.227
Mean	8.169	8.172	8.166	1.375	1.38	1.541
Upper limit	8.189	8.189	8.188	1.586	1.523	1.93

Figure 2. Confidence intervals of the first five vibration modes of the concrete block.

Then, in the case of non-stationary, when resampling the time series, some blocks may have different variances and therefore the correlation functions computed may have a particular bias due to these particular blocks. Randomness in block length and number of blocks in the stationary method (SB) causes greater changes in the bootstrapped correlation functions resulting in increased variability in the identified damping ratios. This can be seen in obtained results. If we calculate the error between the parameters identified with the original time series and the parameters obtained by bootstrapping, the stationary boostrap (SB) presents the highest error being more evident for the damping ratios. Finally, for the moving blocks (MV) and block of blocks (BB) methods, the errors with respect to the parameters obtained are fairly even and the mean bootstrap is close to the mean of the original series, however, this can be improved by increasing the bootstrap samples and by changing the block length.

6 Conclusions

The SSI-COV method was applied to identify the modal parameters of a concrete block, then, we explore the uncertainties in these modal parameters through the estimation of the confidence intervals by appling three different block bootstrap methods. The results based on 300 bootstrap samples show that the parameters obtained by bootstrapping are consistent with the modal parameters obtained with the original acceleration time series. Damping ratios exhibited larger variability due to changes in correlation functions. The stationary bootstrap, due to its randomness in selecting the block length and number of blocks induces more changes in the correlation functions and therefore, of the three methods, it showed the greatest error between the bootstrap parameters and the original modal parameters.

Acknowledgements. The authors acknowledge UNILA (University of Latin American Integration) for financial support and CEASB for laboratories to perform simulations.

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