XLI CILAMCE
XLI Ibero-Latin American Congress on Computational Methods in Engineering
16-19 November, 2020 FOZ DO IGUAÇU PARANÁ - BRAZIL

Damage Models for Micromorphic Continuum

Pamela D. Nogueira[1], Roque L. S. Pitangueira[1], Leandro L. Silva[1]

[1]*Dept. of Structural Engineering, Federal University of Minas Gerais*
Antônio Carlos Avenue, 6627, Zip-Code 31270-901, Belo Horizonte/Minas Gerais, Brazil
pameladanielanogueira@gmail.com, roque@dees.ufmg.br, leandro@dees.ufmg.br

Abstract. Numerous materials, although they appear macroscopically homogeneous, usually present a heterogeneous microstructure that directly influences the structural behavior. The phenomenological approach followed by the classical continuum mechanics does not individually accounts for this influence, which can be significant, for example, in cases where the structure or the specimen under analysis is small compared to its microstructure or when the material media has a complex microstructure. Within the framework of continuum mechanics, so-called generalized continuum theories are particularly suited to deal with the above issues incorporating the microstructural behavior on the formulation. The micromorphic theory is included in this general class of generalized continua and, more specifically, in the group that incorporates additional degrees of freedom to the material particles. Another aspect of generalized continua is its ability to address the issue of localization in quasi-brittle materials modeled as elastic-degrading media as a result of its non-local character. In order to allow the representation of quasi-brittle media with the micromorphic continuum theory, this work presents a formulation for scalar-isotropic damage models for a micromorphic continuum in the constitutive models framework of the INSANE system, initially conceived for classical media and later expanded for the micropolar continuum. This implementation is based on the tensorial format of a unified constitutive models formulation and homogenization techniques to obtain the micromorphic constitutive relations.

Keywords: Micromorphic continuum, Continuum damage mechanics, Elastic-degrading constitutive models.

1 Introduction

The modeling of damage and fracture in structures is and has always been an important topic in the field of computational mechanics. As a consequence, there is a growing demand for reliable material models that are capable of representing all the phenomena involved in material failure. Concerning *quasi-brittle* materials, a proper characterization of their behavior is of great importance taking into account the large number of materials that falls into this category, e.g., concrete, rocks, coarse-grained ceramics, and most fiber-reinforced materials.

The macroscopic behavior of quasi-brittle materials is mainly due to its characteristics at the *micro* scales and to defects that may exist at such scales, which are closely related to the *heterogeneity* at the microscale. Figure 1 illustrates the heterogeneous microscopic structure of concrete, wherein the features in the microscale can be identified.

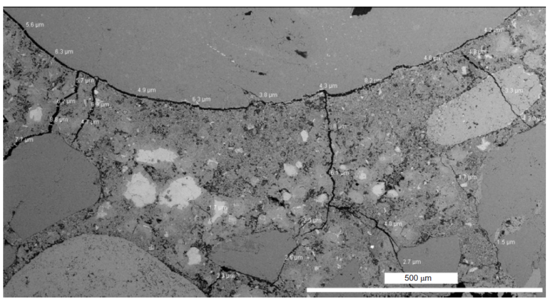

Figure 1. Low-magnification image showing the microcracks on the microstructure of concrete [1]

Therefore, the degradation of a quasi-brittle material is a complex phenomenon strongly correlated to the heterogeneous character of its microstructure. Hence, a detailed modeling of each individual process involved in failure proves to be complicated and often not necessary. For this reason, the discipline of *Continuum Damage Models* (CDM) emerged, accounting for degrading effects in an average sense by incorporating *damage* variables into a standard continuum mechanics description.

In spite of the advantages, finite element computations based on continuum damage models may suffer from a number of issues, e.g., strong mesh dependence. These problems emerge from the *softening* behavior of such models, which is a characteristic of quasi-brittle materials. This softening phase is characterized by a reduction in the load-carrying capacity of the material when a certain deformation threshold is reached, leading to the concentration of the degrading phenomena in a certain part of the body, process that called *strain localization*.

To overcome these shortcomings several methodologies were developed. Among the most efficient approaches are those that introduce at the formulation level an *internal material length* based on the *non-local* character of plasticity and damage. A valid alternative is represented by the *micromorphic continuum* [2–4], a generalized continuum theory in which the internal length is related to an additional field that enriches the continuum kinematics with effects connected to the microstructure of the material.

This paper presents an attempt of extension to the micromorphic continuum of a general elastic-degrading unified formulation focusing on isotropic scalar damage models. The implementations were held in the software *INSANE* (*INteractive Structural ANalysis Environment*) and are based on the tensorial format of a unified constitutive models formulation.

2 Generalized Continuum Mechanics

In the study of material behavior based on the classical continuum mechanics every point in the material is occupied by a small element of the solid called *material particle*. These particles can be idealized as mathematical points as its dimensions are small compared to all characteristics lengths, but are nevertheless large compared to atomic dimensions [5]. In this context, the medium kinematics is described by the translational degrees of freedom of the material particles and by the consequent measures of deformation.

When dealing with composite inhomogeneous materials in the classical continuum, the constitutive equations are developed using the concept of material particle associated to the idea of a *representative volume element* (RVE) [6–8]. As define by Nemat-Nasser and Hori [9], the RVE for a material point of a continuum mass is a material volume which is statistically representative of the infinitesimal material neighborhood of that material point.

In the analysis of usual structures in the engineering field the hypotheses of the classical continuum are sufficient. However, in situations wherein the RVE concept does not represent satisfactorily all the phenomena related to the influence of the substructure or the structural dimensions are small comparatively to the microstructure, theories that incorporate information on the material substructure are necessary.

In this context, *generalized continuum mechanics* were developed. The generalization of standard continuum mechanics of Cauchy begins in Voigt [10] and Cosserat and Cosserat [11] through the expansion of its basic working hypotheses. These generalizations involve higher order gradients of the displacement fields (*higher grade continua*) or/and additional degrees of freedom (*higher order continua*) [12–15]. Higher grade continua are related to the order of the terms of the Taylor series expansion that are considered in the non-linear deformation mapping. On the order hand, higher order continua are characterized by additional degrees of freedom per material point. Embedded in each material point is assumed to be a *microcontinuum*, whose kinematics defines these additional degrees of freedom [2, 13, 15]. The micromorphic theory falls into the last category.

Nevertheless, the micromorphic continuum presents two main drawbacks: the definition of the additional constitutive equations and the determination of the high number of constitutive parameters. For the case of a linear isotropic micromorphic material there are eighteen elastic parameters in contrast to the two Lamé parameters of a classical isotropic continuum [1]. To overcome these limitations, this work is based in the multiscale formulation proposed in Silva [16] in order to obtain the macroscopic micromorphic constitutive relations using parameters defined for the classical theory.

2.1 Micromorphic Continuum

As aforementioned, in the micromorphic continuum, additional degrees of freedom are considered at each material point or, as defined in Eringen [17], "a micromorphic continuum is a continuous collection of deformable

[1]The number of elastic parameters remains the same even for simplified models (e.g., plane or reticulated models) as, to the authors' knowledge, the micromorphic theory was not formulated for these particular cases. The constitutive relations adopted here were obtained applying the simplifying hypotheses of each model to the three-dimensional relations.

point particles". For brevity, the formulation of such theory will not be detailed in this paper and the reader may refer to Eringen [17] and Silva [16]. As this paper is related to constitutive modeling and elastic degrading models, the constitutive relations for micromorphic media will be briefly addressed in the following paragraphs.

Based on the set of strain tensors proposed in Eringen and Şuhubi [3] and Şuhubi and Eringen [4], in a linear approximation and disregarding temperature variations, the internal energy ψ^0 is approximated by

$$\begin{aligned}\psi^0 =& \frac{1}{2}A_{klmn}\epsilon_{kl}\epsilon_{mn} + \frac{1}{2}B_{klmn}e_{kl}e_{mn} + \frac{1}{2}C_{klmnpq}\gamma_{klm}\gamma_{npq} + \\ &+ E_{klmn}\epsilon_{kl}e_{mn} + F_{klmnp}\epsilon_{kl}\gamma_{mnp} + G_{klmnp}e_{kl}\gamma_{mnp}\end{aligned} \quad (1)$$

where A_{klmn}, B_{klmn}, C_{klmnpq}, E_{klmn}, F_{klmnp} and G_{klmnp} are the constitutive moduli; and ϵ_{kl}, e_{kl}, and γ_{klm} are the linear strain tensors.

From eq. (1) and applying symmetry regulations, the linear constitutive equations of an isotropic micromorphic solid can be obtained

$$t_{kl} = A_{klmn}\epsilon_{mn} + E_{klmn}e_{mn}, \quad s_{kl} = E_{mnkl}\epsilon_{mn} + B_{klmn}e_{mn}, \quad \text{and} \quad m_{klm} = C_{lmknpq}\gamma_{npq} \qquad \text{(2a,b,c)}$$

where t_{kl} is the *stress tensor*, s_{kl} is a symmetric stress tensor named *micro-stress average* [3], and m_{klm} is the *stress moments tensor* or, as defined in Eringen and Şuhubi [3], the *first stress moments*.

2.2 Homogenization of a Classical Continuum towards a Micromorphic Continuum

The analytical and discrete formulations of the micromorphic theory are well established in the literature, however, the identification of the corresponding constitutive laws and the determination of the high number of constitutive parameters limit its practical application. As an alternative to circumvent these limitations, the micromorphic homogenization strategy proposed by Silva [16] is used here, which consists in a multiscale formulation for the construction of macroscopic micromorphic constitutive relations in terms of homogenized microscopic quantities obtained from the solution of boundary value problems at the microscale according to the classical continuum theory. This strategy begins with models of the classical continuum on the microscale, without making any constitutive assumptions on the macroscale. Consequently, the necessary material parameters are those of the classical theory. This strategy is based on the micromorphic homogenization of [18] and is illustrated in Fig. 2.

Through this strategy, the initial macroscopic micromorphic constitutive relations are obtained only for the first step of the first iteration by subjecting the material particles to Cauchy stress states resulting from elementary states of strain, which consist of the successive application of component by component of macroscopic micromorphic strain with unit value, while the others components are kept as zero. From the Cauchy stress states, the components of macroscopic micromorphic stress are determined, which, as a result of elementary states of strain, consist of the terms of macroscopic micromorphic constitutive relations. For the subsequent iterations and steps, the initial constitutive relations are degraded through the investigation of the degraded state of the material based on the damage models implemented.

3 Elastic Degradation for Micromorphic Media

The implementation of the isotropic damage models for a micromorphic continuum here presented was based on the unified framework for constitutive modeling presented in Penna [19]. A particularity of this system is the use of a *tensorial* format instead of a *vectorial-matricial* one, which allows the format being independent from the analysis model and from the numerical method. Hence, the generality and the possibility of expansion of the code is greatly increased.

In a geometrically linear context, an elastic-degrading medium is characterized by *total* stress-strain relations

$$\sigma_{ij} = D_{ijkl}\varepsilon_{kl}, \qquad \text{and} \qquad \varepsilon_{ij} = D^{-1}_{ijkl}\sigma_{kl} \qquad \text{(3a,b)}$$

where D_{ijkl} and D^{-1}_{ijkl} are the components of the fourth-order stiffness and compliance tensors. The equations presented correspond to the assumption of an *unloading-reloading* process where the stiffness remains equal to

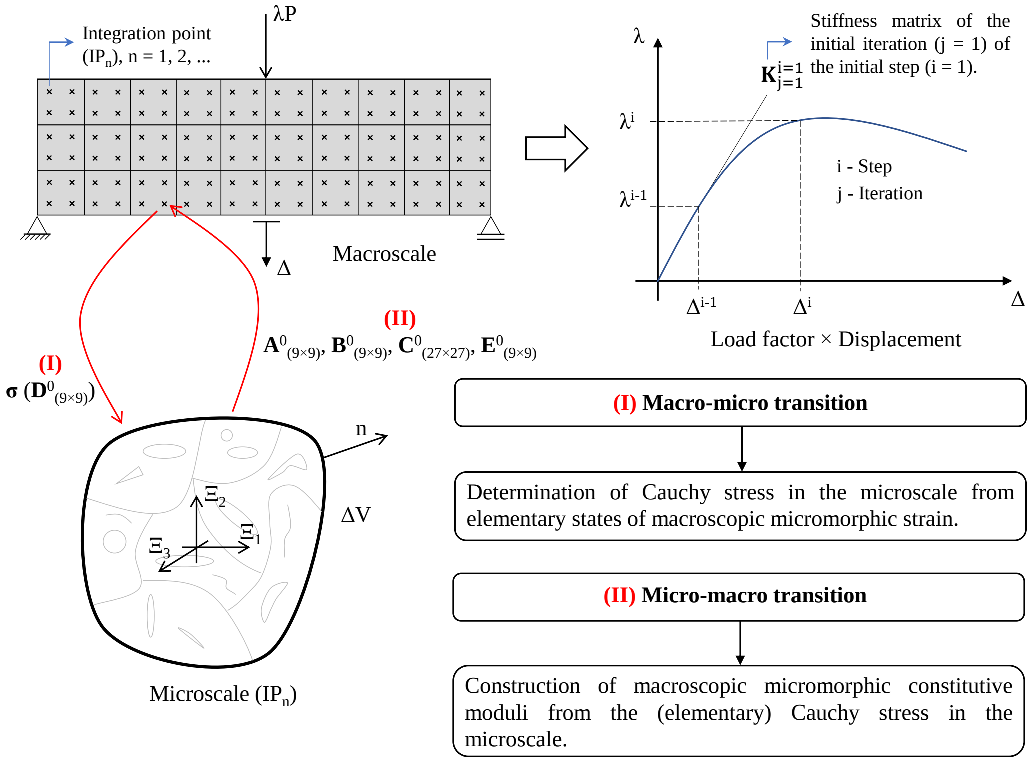

Figure 2. Micromorphic homogenization strategy

the current secant one, i.e., a full unload leads to no permanent strains. This formulation, as initially implemented in the INSANE system, refers to the classical continuum theory, hence, the tensors σ_{ij} and ε_{ij} are the stress and strain tensors of a classical continuum. The unified formulation can be presented in two different representations: a *stress-based* one and a *strain-based* one. [2]

3.1 Isotropic Damage Models in the Unified Framework

This unified theory here employed can encompass many of the continuum damage models proposed in the recent literature. In this section, standard isotropic damage model (as defined by de Borst and Gutiérrez [22]) will be shortly presented in the context of the unified framework for the classical continuum theory.

For the case of isotropic damage evolution, the total stress-strain relation is defined as

$$\sigma_{ij} = (1 - D) D^0_{ijkl} \varepsilon_{kl} \tag{4}$$

where D is the scalar damage variable, D^0_{ijkl} is the initial stiffness and the tensors σ_{ij} and ε_{kl} are the stress and strain tensors of a classical continuum. The loading function can be written as

$$F = f(\mathscr{A}) - h(K) \tag{5}$$

in which $f(\mathscr{A})$ is a function specified for each model that can be written in terms of stress/strain measures, thermodynamic forces or damage. The function $h(K)$ is a *history variable* which has the same nature as $f(\mathscr{A})$. Adopting the strain-based formulation, $f(\mathscr{A})$ can be defined as an *equivalent strain* ε_{eq} (i.e., $f(\mathscr{A}) = \varepsilon_{eq}$) and $h(K) = K(D)$, where $K(D)$ is the history variable related to the equivalent strain written as a function of the damage.

[2] For the theoretical basis for a unified formulation for constitutive models the reader may refer to a number of authors (see, e.g., de Borst [20], Carol et al. [21], de Borst and Gutiérrez [22], Armero and Oller [23, 24], Carol et al. [25, 26]). For the implementation in the INSANE system, see Penna [19].

The classical damage models of *Mazars-Lemaitre* [27], *Simo-Ju* [28], *Marigo* [29, 30], and *Mazars* [22, 31] are defined by the following equivalent strain measures:

$$\varepsilon_{eq} = \begin{cases} \sqrt{\varepsilon_{ij}\varepsilon_{ij}} & \text{(Mazars-Lemaitre)} \\ \sqrt{2\psi^0} & \text{(Simo-Ju)} \\ \sqrt{2\psi^0/E} & \text{(Marigo)} \\ \sqrt{\left[\sum_{k=1}^{3}(<\varepsilon_{(i)}>_+)^2\right]} & \text{(Mazars)} \end{cases} \tag{6}$$

in which $2\psi^0 = D^0_{ijkl}\varepsilon_{ij}\varepsilon_{kl}$ is the internal energy, E is the initial Young's modulus, $\varepsilon_{(i)}$ is the i-th eigenvalue of the strain tensor and $<\varepsilon_{(i)}>_+ = (\varepsilon_{(i)} + |\varepsilon_{(i)}|)/2$ its positive part.

When dealing with the micromorphic theory, there are four constitutive moduli, which causes a problem of consistency (or compatibility) with a framework firstly implemented for a classical theory. In order to address this problem between the formulation for the micromorphic continuum and the existing analogous formulation for classical media previously discussed, a tensorial compact formulation was proposed based on Gori et al. [32].

Until now, the following damage models were elaborated in the context of micromorphic theory and implemented in the INSANE system similarly to Gori [33]:

$$\Gamma_{eq} = \begin{cases} \sqrt{\epsilon_{ij}\epsilon_{ij} + e_{ij}e_{ij} + \gamma_{ijk}\gamma_{ijk}} & \text{(Mazars-Lemaitre)} \\ \sqrt{2\psi^0} & \text{(Simo-Ju)} \end{cases} \tag{7}$$

where Γ_{eq} is the generalized equivalent strain and the internal energy ψ^0 of an isotropic media defined as

$$\psi^0 = \frac{1}{2}A^0_{klmn}\epsilon_{kl}\epsilon_{mn} + \frac{1}{2}B^0_{klmn}e_{kl}e_{mn} + \frac{1}{2}C^0_{klmnpq}\gamma_{klm}\gamma_{npq} + E^0_{klmn}\epsilon_{kl}e_{mn}. \tag{8}$$

4 Numerical simulation

In order to illustrate the utilization of the constitutive models presented in the previous section, an example of a uniaxial stress state is here presented. As can be seen in Fig. 3, a square panel in plane-stress state with unitary thickness and constituted by one finite element is loaded in the x direction. This model is able to demonstrate both models implemented as it yields similar results due to its simplicity.

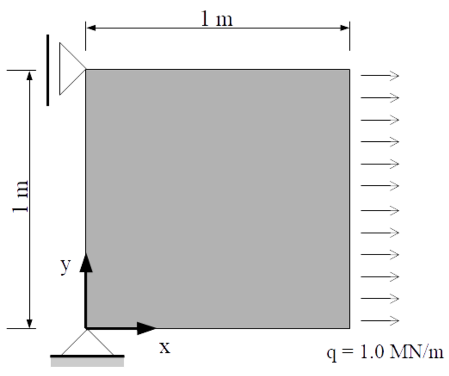

Figure 3. Uniaxial stress state

In order to obtain the initial elastic tensor necessary for isotropic damage models, the homogenization technique (Subsection 2.2) was applied with a square microcontinuum of dimension $0.05\ m$. The equivalent isotropic homogeneous material is characterized by an elastic modulus $E = 20000\ MPa$ and a Poisson's modulus $\nu = 0.2$. The parameters for the constitutive models are: **Mazars-Lemaitre isotropic damage model** [27]: $\alpha = 0.999$, $\beta = 15$, $\kappa_0 = 0.0145$, and exponential damage law [19]; **Simo-Ju isotropic damage model** [28]: $\alpha = 0.999$, $\beta = 2000$, $\kappa_0 = 0.000104$, and exponential damage law [19].

The loading process is driven by the *displacement control method* assuming an increment of $0.000005\ m$ for the horizontal displacement of the loaded face in order to better describe the peak load behavior and a tolerance for the convergence of 10^{-4} in displacement. The results for the analysis are presented in Fig. 4 wherein the relation between the control displacement and the load factor is given.

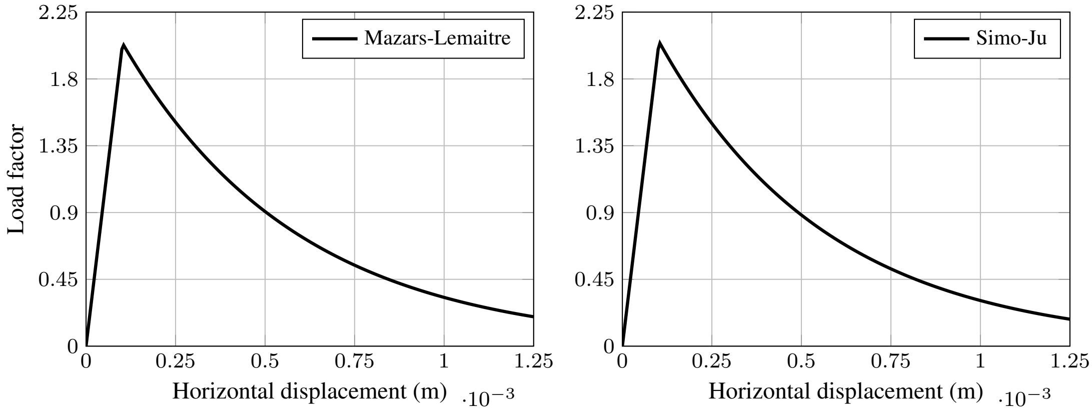

Figure 4. Load factor $\times$ horizontal displacement

5 Conclusions

This paper presented a first attempt to model quasi-brittle media by means of continuum damage models, more specifically scalar-isotropic models, with use of the micromorphic continuum theory. Observing the preliminary results here presented, it is possible to assume that this formulation is viable as it yields consistent results. Applying the homogenization technique to obtain the initial elastic tensor made possible the reduction of the number of elastic parameters necessary for the analysis (E and ν), solving one of the greatest disadvantages of this generalized continuum theory, which requires 18 elastic parameters for a isotropic medium. Furthermore, the proposed tensorial formulation made possible the implementation of models for generalized continua, more specifically the micromorphic continuum, in a unified constitutive framework first conceived for a classical medium.

Acknowledgements. The authors gratefully acknowledge the support from the Brazilian research agencies CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior), FAPEMIG (Fundação de Amparo à Pesquisa do Estado de Minas Gerais; grant PPM-00747-18), and CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico; grant 309515/2017-3).

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] Wong, H. S., Zobel, M., Buenfeld, N. R., & Zimmerman, R. W., 2009. Influence of the interfacial transition zone and microcracking on the diffusivity, permeability and sorptivity of cement-based materials after drying. *Magazine of Concrete Research*, vol. 61, n. 8, pp. 571–589.

[2] Mindlin, R. D., 1964. Micro-structure in linear elasticity. *Archive for Rational Mechanics and Analysis*, vol. 16, pp. 51–78. Communicated by R. A. Toupin.

[3] Eringen, A. C. & Şuhubi, E. S., 1964. Nonlinear theory of simple micro-elastic solids – i. *International Journal of Engineering Science*, vol. 2, n. 2, pp. 189–203.

[4] Şuhubi, E. S. & Eringen, A. C., 1964. Nonlinear theory of simple micro-elastic solids – ii. *International Journal of Engineering Science*, vol. 2, n. 4, pp. 389–404.

[5] Mal, A. K. & Singh, S. J., 1991. *Deformation of elastic solids*. Prentice Hall, Inc., Englewood Cliffs, NJ, USA.

[6] Hashin, Z., 1983. Analysis of composite materials - a survey. *Journal of Applied Mechanics*, vol. 50, n. 3, pp. 481–505.

[7] Kröner, E., 1977. Bounds for effective elastic moduli of disordered materials. *Journal of the Mechanics and Physics of Solids*, vol. 25, n. 2, pp. 137–155.

[8] Willis, J. R., 1981. Variational and related methods for the overall properties of composites. *Advances in Applied Mechanics*, vol. 21, pp. 1–78.

[9] Nemat-Nasser, S. & Hori, M., 1993. *Micromechanics: Overall Properties of Heterogeneous Materials*. Elsevier Science Publishers.

[10] Voigt, W., 1887. Theoretische studien über die elasticitätsverhältnisse der krystalle. *Abhandlungen der Königlichen Gesellschaft der Wissenschaften*, vol. 34, pp. 3–52.

[11] Cosserat, E. & Cosserat, F., 1909. *Théorie des corps déformables*. Librairie Scientifique A. Hermann et Fils, Paris.

[12] Germain, P., 1973a. La méthode des puissances virtuelles en mécanique des millieux continus premiere partie: théorie du second gradient. *Journal de Mécanique*, vol. 12, n. 2, pp. 235–274.

[13] Germain, P., 1973b. The method of virtual power in continuum mechanics. part 2: Microstructure. *SIAM Journal of Applied Mathematics*, vol. 25, n. 3, pp. 556–575.

[14] Forest, S., 1998. Mechanics of generalized continua: construction by homogenization. *Journal de Physique IV*, vol. 8, pp. Pr4–39, Pr4–48.

[15] Hirschberger, C. B., 2008. *A Treatise on Micromorphic Continua. Theory, Homogenization, Computation*. PhD thesis, Technische Universität Kaiserslautern, Kaiserslautern, Germany.

[16] Silva, L. L., 2019. *Estratégia Multiescala para Descrição Micromórfica do Contínuo a partir do Contínuo Clássico*. PhD thesis, Universidade Federal de Minas Gerais, Belo Horizonte, MG, Brasil.

[17] Eringen, A. C., 1999. *Microcontinuum Field Theories: I. Foundations and solids*. Springer, New York.

[18] Hütter, G., 2017. Homogenization of a Cauchy continuum towards a micromorphic continuum. *Journal of the Mechanics and Physics of Solids*, vol. 99, pp. 394–408.

[19] Penna, S. S., 2011. *Formulação Multipotencial para Modelos de Degradação Elástica - Unificação Teórica, Proposta de Novo Modelo, Implementação Computacional e Modelagem de Estruturas de Concreto*. PhD thesis, Universidade Federal de Minas Gerais, Belo Horizonte, MG, Brasil.

[20] de Borst, R., 1987. Smeared cracking, plasticity, creep, and thermal loading–a unified approach. *Computer Methods in Applied Mechanics and Engineering*, vol. 62, pp. 89–110.

[21] Carol, I., Rizzi, E., & Willam, K., 1994. A unified theory of elastic degradation and damage based on a loading surface. *International Journal of Solids and Structures*, vol. 31, n. 20, pp. 2835–2865.

[22] de Borst, R. & Gutiérrez, M. A., 1999. A unified framework for concrete damage and fracture models including size effects. *International Journal of Fracture*, vol. 95, pp. 261–277.

[23] Armero, F. & Oller, S., 2000a. A general framework for continuum damage models. I. infinitesimal plastic damage models in stress space. *International Journal of Solids and Structures*, vol. 37, n. 48-50, pp. 7409–7436.

[24] Armero, F. & Oller, S., 2000b. A general framework for continuum damage models. II. integration algorithms, with applications to numerical simulation of porous metals. *International Journal of Solids and Structures*, vol. 37, n. 48-50, pp. 7437–7464.

[25] Carol, I., Rizzi, E., & Willam, K., 2001a. On the formulation of anisotropic elastic degradation. I. theory based on a pseudo-logarithmic damage tensor rate. *International Journal of Solids and Structures*, vol. 38, n. 4, pp. 491–518.

[26] Carol, I., Rizzi, E., & Willam, K., 2001b. On the formulation of anisotropic elastic degradation. II. generalized pseudo-rankine model for tensile damage. *International Journal of Solids and Structures*, vol. 38, n. 4, pp. 519–546.

[27] Mazars, J. & Lemaitre, J., 1985. *Application of Continuous Damage Mechanics to Strain and Fracture Behavior of Concrete*, pp. 507–520. Springer Netherlands, Dordrecht.

[28] Simo, J. & Ju, J., 1987. Strain- and stress-based continuum damage models–i. formulation. *International Journal of Solids and Structures*, vol. 23, n. 7, pp. 821 – 840.

[29] Marigo, J., 1985. Modelling of brittle and fatigue damage for elastic material by growth of microvoids. *Engineering Fracture Mechanics*, vol. 21, n. 4, pp. 861 – 874.

[30] Lemaitre, J. & Desmorat, R., 2005. *Engineering Damage Mechanics: Ductile, Creep, Fatigue and Brittle Failures*. Springer.

[31] Mazars, J., 1984. *Application de la mécanique de l'endommagement au comportement non lineaire et á la rupture du béton de structure.* PhD thesis, Université Pierre et Marie Curie, Paris, France.

[32] Gori, L., Penna, S. S., & Pitangueira, R. L., 2017. An enhanced tensorial formulation for elastic degradation in micropolar continua. *Applied Mathematical Modelling*, vol. 41, pp. 299–315.

[33] Gori, L., 2018. *Failure analysis of quasi-brittle media using the micropolar continuum theory, elastic-degrading constitutive models, and smoothed point interpolation methods*. PhD thesis, Universidade Federal de Minas Gerais, Belo Horizonte, MG, Brasil.