Assessment of failure probability of planar steel frames to plastic collapse by advanced structural analysis

Danilo Luiz Santana Mapa, Endersom Lisboa Nadir, Marcílio Sousa da Rocha Freitas, Ricardo Azoubel da Mota Silveira, Andréa Regina Dias Silva

Programa de Pós-Graduação em Engenharia Civil, Universidade Federal de Ouro Preto
Departamento de Engenharia Civil, Escola de Minas, Campus Universitário s/n, Morro do Cruzeiro, 35400-000, Ouro Preto, Minas Gerais, Brasil
danilo.mapa@aluno.ufop.edu.br, endersom.nadir@aluno.ufop.edu.br, marcilio@ufop.edu.br, ricardo@ufop.edu.br, andreardsilva@ufop.edu.br

Abstract. The current advanced analysis techniques for steel frames generally use structural analyses with geometric and material non-linearities to capture the collapse strength of the steel frame. Advanced analysis has the potential to result in more efficient designs, due to more accurate predictions of true strength of the structural system. Unfortunately, even with the advanced nonlinear structural analysis method, the true strength of a steel frame cannot be predicted with certainty because of ever-present uncertainties of the most significant design variables, which are the properties of the material, the applied external loads and the geometric properties of the cross sections of the steel profiles. Reliability methods allow the evaluation of the safety level of structures in probabilistic terms, by the direct evaluation of the failure probability of the structural system. In the present work, the First Order Reliability Method (FORM) was used to calculate the probability of failure of planar steel frames to plastic collapse. The advanced analyses were performed using the program MASTAN2 and considered the geometric nonlinearities and the inelasticity of the steel. The failure probabilities of numerical examples of planar steel frames were evaluated and compared to other authors. The results of the numerical examples showed that it is essential to obtain the probability of failure as part of the structural design to account for uncertainties inherent to design variables in order to obtain safer structures.

Keywords: failure probability, steel frames, plastic collapse, FORM, MASTAN2.

1 Introduction

The current advanced analysis techniques for steel frames generally use structural analyses with geometric and material non-linearities to capture the collapse strength of the steel frame. Advanced analysis has the potential to result in more efficient designs, due to more accurate predictions of true strength of the structural system. It makes possible to simultaneously evaluate the strength and stability of the structure, without the need to individually check the capacity of the members.

Advanced analysis generally falls into two main categories: refined plastic hinge method and plastic zone method. In the plastic zone method, the cross section of each finite element is discretized into fibers. Second order effects and residual stresses can be considered directly in the analysis. However, the plastic zone method requires an intense computational effort [1]. In the refined elastic-plastic hinge method, a tangent modulus is used to capture the plasticity spread along the member under large axial, bending forces and residual stresses [2].

Unfortunately, even with the advanced nonlinear structural analysis method, the true strength of a steel frame cannot be predicted with certainty because of ever-present uncertainties of the most significant design variables, which are the properties of the material, the applied external loads and the geometric properties of the cross sections of the steel profiles. Current codes have a deterministic format; however the effect of uncertainties is considered through the application of safety factors [3].

In order to supply such uncertainties and guarantee a target level of structural reliability, a resistance factor at the system level is applied by standards. However, this semi-probabilistic method does not allow real knowledge of the safety levels of the structure in service. Reliability methods allow the evaluation of the safety level of structures in probabilistic terms, by the direct evaluation of the failure probability of the structural system.

In the present work, the First Order Reliability Method (FORM) was used to calculate the probability of failure of planar steel frames to plastic collapse. The FORM method uses the probability density function of each uncertain variables to determine the probability of failure, being considered sufficient to this kind of analyzes. The FORM method is also efficient in computational cost during reliability analyzes.

This paper examines the structural reliability of steel frames designed by LRFD provisions. A version of the computer reliability program, in MATLAB® language, developed by Mapa [4] was used, where the reliability analysis is performed based on the First Order Reliability Method (FORM), which acts interactively with the program MASTAN2 [5], which performs second-order inelastic structural analyzes. The safety of the structures was discussed by the probability of failure for ultimate limit states of resistance related to the plastic collapse. The results were compared with those obtained by other authors and the implications of reliability of this methodology and the adequacy of the resistance factor applied to the system are discussed.

2 Structural Reliability

In the structural reliability analysis, the maximum demand (load effect), S, and the available resistance (capacity), R, are modeled by random variables. The objective of the reliability analysis is to ensure the event $R > S$ throughout the useful life of the structure in terms of probability. The failure occurs if R is less than S and this event can be represented in terms of probability as $P(R < S)$. If both the R and S random variables have normal distribution and are statistically independent, then the Z random variable can be entered as $Z = R - S$. The failure probability can be defined as:

$$P_f = P(Z < 0) = \int_{-\infty}^{0} f_Z(z).dz = \Phi\left(\frac{0-\mu_Z}{\sigma_Z}\right) = \Phi(-\beta) \tag{1}$$

where $\mu_Z = \mu_R - \mu_S$, $\sigma_Z = \sqrt{\sigma_R{}^2 + \sigma_S{}^2}$, Φ is the CDF of the standard normal distribution and β is the Cornell [6] reliability index, defined below:

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}}. \tag{2}$$

Initially, the reliability index was evaluated simply as a function of the means and standard deviations of the available resistance, R, and the maximum demand, S, as indicated in eq. (2). Subsequently, the reliability index started to be obtained by analytical methods based on approximations in first-order Taylor series (FORM method).

2.1 First-Order Reliability Method (FORM)

In the FORM method, the random variables U, whose distributions are any and may be dependent on each other or not, are transformed into standard normal V variables that are statistically independent, with the failure function $G(U)$ written in the space of the reduced variables (space V) as $g(V)$. After, the failure surface defined by $g(V) = 0$ is approximated by a linear surface (or hyperplane) at the point with the shortest distance to the origin, identified as V^* (design point in the space of the reduced variables). One of the steps of the FORM method is the transformation of U variables with any distributions into statistically independent standard V variables, using the Nataf [7] transformation:

$$V = \Gamma \sigma^{-1}(U - m) \tag{3}$$

where m is the vector with the means of the variables U, σ is the diagonal matrix containing the standard deviations of the variables U and $\Gamma = L^{-1}$, where L is the lower triangular matrix obtained from the Choleski decomposition of the matrix of the correlation coefficients of U. Another important step of the FORM method is the search for the point on the failure surface closest to the origin of the reduced system, also called design point V^*. To find the design point, an optimization problem P is formulated:

$$\begin{aligned} &P: \text{ minimize } |V| \\ &\text{Subject to: } g(V) = 0. \end{aligned} \tag{4}$$

The algorithm called HLRF, developed by Hasofer and Lind [8] and improved by Rackwitz and Fiessler [9], is commonly used to solve the optimization problem presented by eq. (4). The iterative process generated by the HLRF algorithm goes in search of the design point by solving the following equation:

$$V^{i+1} = \frac{1}{\left|\nabla g\left(V^i\right)\right|^2}.\left[\nabla g\left(V^i\right)^T V^i - g\left(V^i\right)\right].\nabla g\left(V^i\right). \tag{5}$$

During the iterative process, the reliability index β is determined by calculating $|V^{i+1}|$ and the process is interrupted when the relative variation of the β value is less than a tolerance. The failure probability can then be obtained using eq. (1).

3 Advanced structural analysis

Advanced analysis of structures is characterized when nonlinear effects are considered in the formulation of the structural element. The current advanced analysis techniques for steel frames generally use structural analyses with geometric and material non-linearities to capture the collapse strength of the frame.

Nonlinear behavior can be produced by changes in the frame geometry, which is commonly referred to in the literature as geometric non-linearity or second-order effects. The main geometric effects related to geometric non-linearity are the global effect P-Δ as a result of lateral displacements in the structure, and the local effect P-δ in the elements, associated with local deformations.

Another source of non-linearity that must be considered in an advanced structural analysis concerns the material's inelasticity. The inelasticity of the steel allows the load redistribution after the formation of plastic hinges. In the advanced analyzes performed, steel was idealized as an elastic-perfectly-plastic material with no strain hardening. Figure 1 on the left represents a steel frame with elastic loading. With the increase in load, the sequence of formation of plastic hinges begins, indicated in Fig. 1 by the numbering from 1 to 4. When the ultimate limit load F_u is reached, the fourth plastic hinge develops and the plastic collapse mechanism occurs, with the structure deforming indefinitely without any increase in load.

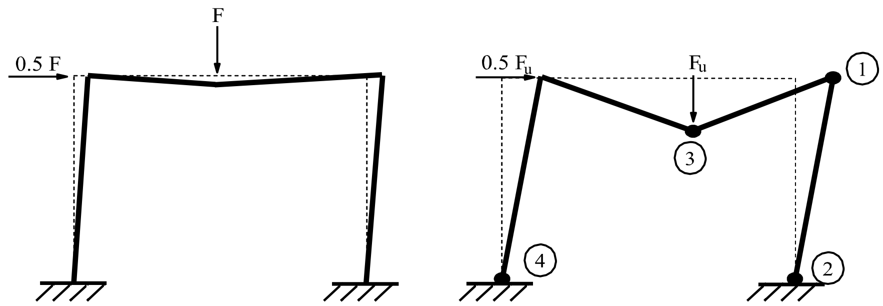

Figure 1. Plastic collapse mechanism of a steel frame

3.1 Advanced analysis in MASTAN2

In this paper, two-dimensional advanced analyses were performed using the software MASTAN2 [5]. Version 3.5.5 of MASTAN2 was used for second-order inelastic analyzes and to obtain the ultimate load factor, necessary to assess the performance function in structural reliability analyzes. In the nonlinear structural analyzes of the frames were used: the plastic hinge formulation present in MASTAN2, strategy of constant increase of the load parameter, predictor-corrector solution, modified tangent elastic modulus (E_{tm}), incremental load factor fixed at 1% of the total load. The discretization of each beam and column was 4 finite elements. The yield surface used in MASTAN2 is a function of a member's axial force and bending moment. The yield surface, which was developed by McGuire, Gallagher and Ziemian [5], is expressed by the polynomial equation:

$$p^2 + m_x^2 + 3.5p^2 m_x^2 = 1 \tag{6}$$

where $p = P/P_y$ is the ratio of the axial force to the squash load, and $m_x = M_x/M_{px}$ is the ratio of the strong axis bending moment to the corresponding plastic moment. Squash load, P_y, and the plastic moment, M_{px}, are, respectively, the section's area and plastic section modulus times σ_y.

The performance function (limit state equation) are usually implicit functions of random variables in the analysis of the reliability of complex structures. The reliability analyzes performed were a combination of the FORM method and the deterministic finite element method implemented in MASTAN2. The reliability analysis of the steel frames was performed by the computational tool developed by Mapa [4], which works coupled to MASTAN2 for evaluating the performance function at each iteration of the FORM method. The performance function was formulated in function of the available resistance (R) of the structural system and in function of the maximum load (S) in the structural system. The performance function was formulated according to the equation:

$$G(U) = 1 - \frac{S}{R} = 1 - \frac{1}{\lambda}. \tag{7}$$

In eq. (7) the overall resistance of the structure was expressed as a function of a load factor $\lambda = R/S$, which provides how many times the resistance to the plastic collapse of the structure is greater than the acting load, based on advanced analysis of the structure. This load factor can also express the necessary load for the formation of the first plastic hinge in the structure.

4 Numerical examples

The results of the structural reliability analysis of steel structures will be presented in this section. Reliability analyzes were performed using the computational tool developed by Mapa [4], which made it possible to assess the safety levels of the ultimate limit states of the structures, in order to know the safety level of the structures when designed by the ANSI 360 [10]. By analyzing the obtained results and comparing to the ones found by other authors, it was possible to validate the computational implementation, attesting its accuracy and efficiency in the structural reliability analysis of steel frames.

In the first example, a continuous beam will be presented and the safety levels with respect to the plastic collapse were investigated. In the second example, the probability of failure of an asymmetric steel frame with two-bay and two-story was obtained. The two structures have significant load redistribution capacity following initial yielding. All beams and columns were compact and laterally braced so that the plastic capacity of each section can be achieved without local buckling. Connections were assumed to be fully rigid. The steel material property is modeled as elastic-perfectly-plastic.

4.1 Three-span continuous beam

A continuous beam subjected to a vertical concentrated load in the middle span is considered in this example. The geometric dimensions, cross-sections, load and support conditions of the structure are shown in the Fig. 2. The following load combination suggested in the ASCE 7-10 [11] are used to select the size of the members: $1.2D_n + 1.6L_n$, where D_n and L_n are nominal dead load and nominal live load, respectively. All members are made of the same grade of steel: the nominal yield stress (F_{yn}) is 345 *MPa* with a nominal Young's modulus (E_n) of 200 *GPa*.

Performing the inelastic analysis of the continuous beam and reduce nominal values of yield stress ($0.9F_{yn}$) and modulus of elasticity ($0.9E_n$) for all members according to ANSI 360 [10], it was found that the first plastic hinge is formed in section B, with a load factor $\lambda_1 = 0.981$, the second plastic hinge is formed in section C with a load factor $\lambda_2 = 1.20$ and the third plastic hinge is formed in section D with a load factor $\lambda_u = 1.29$. Zhang *et al.* [12] also performed the advanced analysis of this beam and they came to the same conclusion that the continuous beam supports approximately 129% ($\lambda_u = 1.29$) of the total load P_0 applied and the first hinge is formed with a load factor $\lambda_1 = 1.0$. The beam had a significant capability for redistributing forces after first yield.

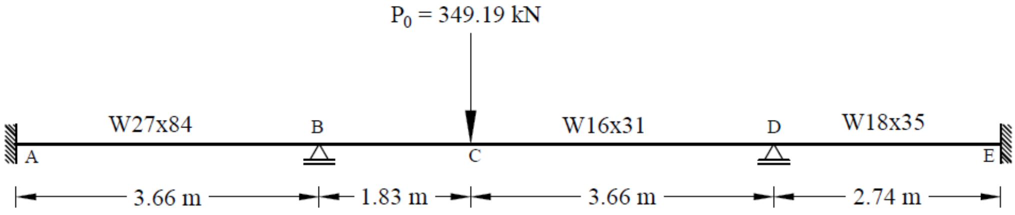

Figure 2. Three-span continuous beam

In order to investigate the safety levels of the continuous beam, reliability analyzes were performed considering the basic random variables: live load (L), dead load (D), cross-sectional area (A), moment of inertia (I), yield strength (F_y) and Young's modulus (E). Table 1 summarizes the statistical information for these basic random variables. The structural load shown in Fig. 2 represent the gravity load combination $P_0 = 1.2D_n + 1.6L_n$, with the nominal live-to-dead load ratio assumed to be $L_n = 1.5D_n$. Table 2 summarizes the reliability indexes obtained for the collapse limit state of the beam for two load levels: load to form the first plastic hinge ($D_n = 97\ kN$ and $L_n = 145{,}5\ kN$) and load to the plastic collapse ($D_n = 125{,}13\ kN$ and $L_n = 187{,}7\ kN$).

Based on the results of the reliability analysis in Tab. 2, some observations can be made. The reliability index $\beta = 4.05$ obtained when we design the beam to a load level for the formation of the first plastic hinge results in a probability of failure of the structural system in the order of 0.00256%. The reliability index $\beta = 2.95$ obtained when we design the beam to a load level of the plastic collapse, results in a probability of failure of the structural system in the order of 0.15889%. The probability of failure of the structural system for design based on plastic collapse is about 62 times greater than the probability of failure of the structural system for design based on the formation of the first plastic hinge.

Table 1. Description of basic random variables

Variable	Mean	Coefficient of Variation (COV)	Distribution	Reference
D (kN)	$1{,}05.D_n$	0.10	Normal	Ellingwood *et al.* [13]
L (kN)	L_n	0.25	Type I Largest	Ellingwood *et al.* [13]
F_y (MPa)	$1{,}10.F_{yn}$	0.06	Lognormal	Bartlett *et al.* [14]
E (MPa)	E_n	0.04	Lognormal	Bartlett *et al.* [14]
A (cm^2)	A_n	0.05	Normal	Ellingwood *et al.* [13]
I (cm^4)	I_n	0.05	Normal	Ellingwood *et al.* [13]

Table 2. Reliability indexes obtained for the continuous beam

Load Level	*Reliability index (present paper)*	*Reliability index* Zhang *et al.* [12]
First plastic hinge	$\beta = 4.05$	$\beta = 3.90$
Plastic collapse	$\beta = 2.95$	$\beta = 2.76$

Comparing the reliability indices obtained in the present study with those obtained by other authors, it is observed in Tab. 2 that the reliability indices obtained by Zhang *et al.* [12] are close to those obtained in the present study and the probabilities of failure are close. Table 2 also shows that the reliability indexes obtained by Zhang *et al.* [12] are slightly lower than the rates obtained in the present study. This small difference between the results can be justified because Zhang *et al.* [12] used the Monte Carlo direct simulation method to assess the probability of failure, used the plastic zone method (discretization of the cross-section in fibers) with residual stresses in the inelastic analysis and incorporated the strain hardening effect in the steel stress-strain curve.

4.2 Two-story unsymmetrical frame

An unsymmetrical two-story, two-bay rectangular steel frame as shown in Fig. 3 is considered next. The geometric dimensions, support conditions and loads are shown in the figure. All members are made of the same grade of steel: the nominal yield stress (F_{yn}) is 248 *MPa* with a nominal Young's modulus (E_n) of 200 *GPa*. The cross-sections of the frame are laminated steel profiles: W12×19 (W310×28.3) assigned to column C1; W14×159 (W360×237) assigned to column C2; W14×145 (W360×216) assigned to columns C3, C5 and C6; W6×9 (W150×13.5) assigned to column C4; W30×116 (W760×173) assigned to beam B1 and B4; W36×182 (W920×271) assigned to beam B2 and W24×55 (W610×82) assigned to beam B3. The reference load P_0 is $146.95\ kN/m$.

Performing the inelastic analysis of the steel frame and reduce nominal values of yield stress ($0.9F_{yn}$) and modulus of elasticity ($0.9E_n$) for all members, it was found that the first plastic hinge is formed with a load factor $\lambda_1 = 1.033$ and the collapse plastic is reached when a load ratio $\lambda_u = 1.168$ is applied. Zhang *et al.* [12] also performed the advanced analysis of this structure and they came to the conclusion that the steel frame supports approximately 119% ($\lambda_u = 1.19$) of the total load P_0 applied in Fig. 3, and the first hinge is formed with a load factor $\lambda_1 = 1.0$. This is due to the significant load redistributing ability of the frame.

In order to investigate the safety levels of the steel frame, reliability analyzes were performed considering the same basic random variables summarized in Tab. 1. The structural load shown in Fig. 3 represent the gravity load combination $P_0 = 1.2D_n + 1.6L_n$, with the nominal live-to-dead load ratio assumed to be $L_n = 1.5D_n$. Table 3 summarizes the reliability indexes obtained for the collapse limit state of the frame for two load levels: load to form the first plastic hinge ($D_n = 40.82\ kN/m$ and $L_n = 61.23\ kN/m$) and load to the plastic collapse ($D_n = 47.68\ kN/m$ and $L_n = 71.52\ kN/m$).

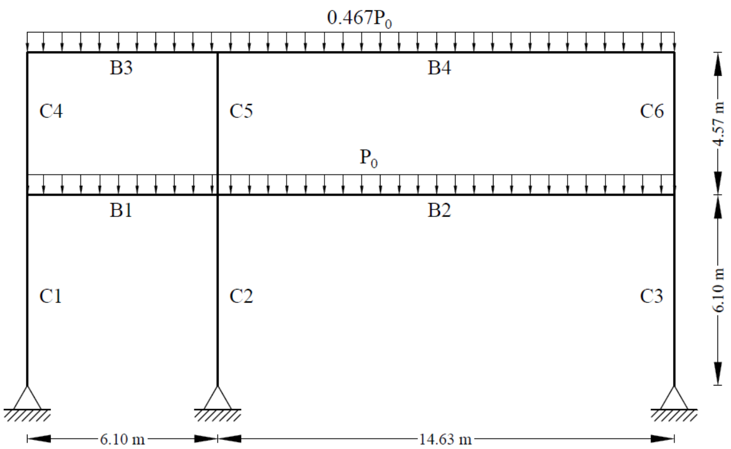

Figure 3. Two-story unsymmetrical frame

Table 3. Reliability indexes obtained for the two-story unsymmetrical frame

Load Level	*Reliability index (present paper)*	*Reliability index* Zhang *et al.* [12]
First plastic hinge	$\beta = 3.61$	$\beta = 3.62$
Plastic collapse	$\beta = 2.94$	$\beta = 2.89$

Based on the results of the reliability analysis in Tab. 3, some observations can be made. The reliability index $\beta = 3.61$ obtained when we design the beam to a load level for the formation of the first plastic hinge results in a probability of failure of the structural system in the order of 0.01531%. The reliability index $\beta = 2.94$ obtained when we design the beam to a load level of the plastic collapse, results in a probability of failure of the structural system in the order of 0.16411%. The probability of failure of the structural system for design based on plastic collapse is about 11 times greater than the probability of failure of the structural system for design based on the formation of the first plastic hinge.

Comparing the reliability indices obtained in the present study with those obtained by other authors, it is observed in Tab. 3 that the reliability indices obtained by Zhang *et al.* [12] are close to those obtained in the present study and the probabilities of failure are close. This small difference between results was justified in the previous example.

5 Conclusions

In the present work, reliability analyzes of two steel structures were carried out through advanced structural analysis considering the effects of geometric nonlinearity and steel inelasticity. The FORM method was used to assess the probability of failure of the system in relation to the ultimate limit state of the plastic collapse. The analyzed structures have significant capacity for redistribution of inelastic load, and through the reliability analysis it was possible to know the safety level for the two design load levels: formation of the first plastic hinge and the plastic collapse. The results of the numerical examples showed that it is essential to obtain the probability of failure as part of the structural design to account for uncertainties inherent to design variables in order to obtain safer structures.

Acknowledgements. The authors would like to thank CAPES and CNPq (Federal Research Agencies), FAPEMIG (Minas Gerais Research Agency), Gorceix Foundation and UFOP/PROPP for their support during the preparation of this research work.

Authorship statement. The authors hereby confirm that they are the sole liable persons responsible for the authorship of this work, and that all material that has been herein included as part of the present paper is either the property (and authorship) of the authors, or has the permission of the owners to be included here.

References

[1] G. A. Gonçalves. *Modelagem do comportamento inelástico de estruturas de aço: membros sob flexão em torno do eixo de menor inércia*. Dissertação (Mestrado em Engenharia Civil), Universidade Federal de Ouro Preto, Ouro Preto, 2013.
[2] H. Liu. *System reliability calibrations for the Direct Design Method of planar steel frames with partially restrained connection.* PhD thesis, The University of Sydney, 2019.
[3] S. Shayan. *System reliability-based design of 2d steel frames by advanced analysis.* PhD thesis, The University of Sydney, 2013.
[4] D. L. S. Mapa. *Confiabilidade estrutural de pórticos metálicos planos*. Dissertação (Mestrado em Engenharia Civil), Universidade Federal de Ouro Preto, Ouro Preto, 2016.
[5] W. McGuire, R. H. Gallagher and R. D. Ziemian. *Matrix Structural Analysis*. Faculty Books, 2000.
[6] C. A. Cornell, "A Probability-Based Structural Code". *Journal of the American Concrete Institute*, Vol. 66, n. 12, pp. 974-985, 1969.
[7] R. E. Melchers. *Structural reliability analysis and prediction*. John Wiley & Sons, 2. ed., 431 p, 1999.
[8] A. M. Hasofer and N.C. Lind, "An exact and invariant first-order reliability format". *Journal of Engineering Mechanics*, v. 100, p. 111-121, 1974.
[9] R. Rackwitz and B. Fiessler, "Structural reliability under combined random load sequences". *Computer and Structures*, v. 9, pp. 489-494,1978.
[10] AISC. *Specification for structural steel buildings*. AISC 360-10, Chicago, 2010.
[11] ASCE. "Minimum design loads for buildings and other structures". *ASCE 7-10*, Reston, VA, 2010.
[12] H. Zhang, H. Liu, B. R. Ellingwood, and K. J. R. Rasmussen, "System Reliabilities of Planar Gravity Steel Frames Designed by the Inelastic Method in AISC 360-10". *J. Struct. Eng*., 144(3): 04018011, 2018.
[13] B. R. Ellingwood, J. G. MacGregor, T. V. Galambos and C. A. Cornell, "Probability based load criteria: Load factors and load combinations." *J. Struct. Div*., 108(5), 978–997, 1982.
[14] F. M. Bartlett, R. J. Dexter, M. D. Graeser, J. J. Jelinek, B. J. Schmidt and T. V. Galambos, "Updating standard shape material properties database for design and reliability." *Eng. J. AISC*, 40(1), 2–14, 2003.