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Structural optimization of a continuous reinforced concrete beam considering risks and progressive collapse

Lucas da Rosa Ribeiro[1], André T. Beck[1]

[1]*Dept. of Structural Engineering, University of São Paulo*
Av Trabalhador São-carlense, 400, 13566-590, São Carlos, SP, Brazil
lucasribeiro@usp.br , atbeck@sc.usp.br

Abstract. Current normatives that guides the structural design against progressive collapse adopts a damage-tolerant approach, where the system is design to withstand the loss of individual vertical elements due to abnormal load conditions. However, the actual guidelines consider this individual element loss in a deterministic manner, which can overestimate the damage occurrence and substantially increase the total expected costs. Aiming to analyze how the optimal design of usual structural systems is affected by a column removal scenario, a Risk Optimization is performed on a continuous beam of reinforced concrete subjected to the loss of the internal support, which is considered by means of a latent probability of failure. In order to increase the efficiency of the optimization process, an adapted system single loop approach to the risk optimization is employed herein, allowing a very fast convergence to the optimal design. Considering the steel rebar areas as the optimal parameters, it is found that the latent probability of failure substantially increases the steel rebar area directly affect by the internal column loss when compared to the semiprobabilist design presented by the current normative. When the smallest latent probability is considered, the optimal steel rebar area is identical to when this probability is null, however, when the target reliability is over the reliability of the reference design, this area increases very fast. It is also identified evidences of a threshold column loss probability, but only for the rebar area not affected by the column loss removal, meaning that its design is indifferent to the objective consideration of the column loss.

Keywords: column removal; continuous beam; progressive collapse; reinforced concrete; risk optimization.

1 Introduction

Extraordinary events are known to generate loading conditions able to generate structural collapses. Some examples are the gas explosion at the Ronan Point Tower (UK, 1968), terrorist attacks like the World Trade Center (NY, 9/11, 2001), and earthquakes like the one at Wenchuan (China, 2008). Since these abnormal loads have very low probabilities of occurrence, structural elements are not usually designed to withstand them. Instead, the structural system is designed to bridge over the loss of individual elements [1-6]. When under multiple hazards, the probability of structural collapse p_c is given as:

$$p_c = P[C] = \sum_{H} \sum_{LD} P[C|LD, H]\, P[LD|H]\, P[H] \tag{1}$$

where $P[H]$ is the probability of hazard occurrence; $P[LD|H]$ is the conditional probability of local damage for a given hazard H; and $P[C|LD, H]$ is the conditional probability of collapse for a given LD and H. According to the damage-tolerant approach currently adopted, the structural system is designed by limiting $P[C|LD, H]$. Since this term only involves structural analysis, it can be reduced by incorporating redundancy, alternate load paths, compressive arch or catenary actions, structural fuses, segmentation, and others [7,8]. However, the actual guidelines consider this individual element loss in a determinist manner, which can overestimate the damage occurrence and substantially increase the total expected costs.

In addition, continuous reinforced concrete (RC) beams and, more generally, RC moment frames, are particular structures worldwide employed that can strongly exemplify how the controlling of $P[C|LD, H]$ may influence over the structural reliability against progressive collapse.

In view of that, such structures, alongside the probabilistic consideration of column removal scenarios, are the study objects of this manuscript. Practical and specific design measures against progressive collapse are not addressed herein.

2 Formulation and implementation

This work addresses optimal design of a continuous beam subject to usual gravity loads and to a column loss scenario. The normal loading condition (NLC) is considered as one particular hazard in Eq. (1) associated to $P[NLC] = 1$. Since the normal loading condition does not lead to immediate local damage, the probability of collapse is given directly by $p_C = P[C|NLC]$.

The local damage (LD) condition herein considered is the Internal Column Loss (ICL). Since the evaluation of $P[CL|H]$ and $P[H]$ involves a risk analysis addressing the structural purpose and its environment, which is out of the scope of this manuscript, the column loss probability is given herein as $p_{CL} = \sum_H P[CL|H]P[H]$. This allows it to be considered as an independent parameter, making the formulation threat-independent.

Column loss analysis is not required when the threat probability is smaller than the $h = 10^{-7}$ per year [10]. Thus, considering a design life of $t = 50$ years, this is equivalent to $p_{CL}^{min} = h \times t = 5 \times 10^{-6}$. Therefore, p_{CL} is considered as an independent parameter ranging from p_{CL}^{min} up to 1.

2.1. Reference design

The continuous RC beam employed is initially designed according to the current national normative [11], allowing the obtaining of a reference cost C_{Ref} that is used to turn the total expected cost dimensionless. Details of the geometry and loading are shown in Fig. 1.

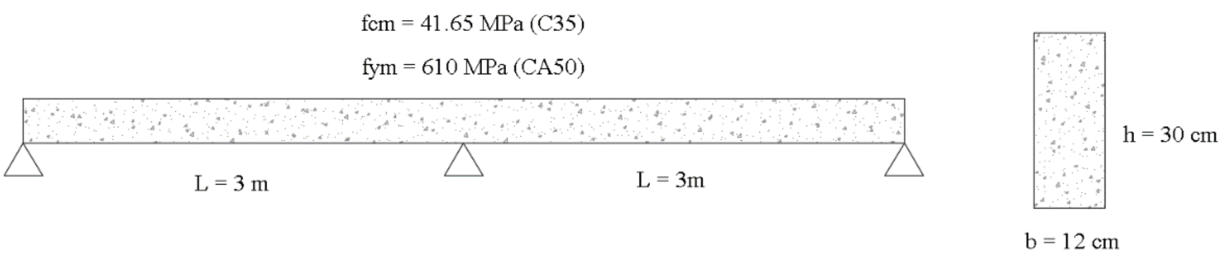

Figure 1. Continuous beam herein analyzed.

The beam is subjected to a uniform load composed by the sum of a dead load and live load of 10 kN/m each, totalizing 28 kN/m due to the load factor of 1.4. Regarding the material parameters, it is noticed that the mean values of concrete resistance and steel yielding are used, which are obtained by the Bias factors found by Santiago [12]. Details of the usual designing according to the current national normative are shown in Tab. 1.

Table 1. Usual design according to the current Brazilian normative [11]

Bending moment (kNcm)	d = h – 4cm (cm)	kc (cm²/kN)	ks (cm²/kN)	$\beta_X = \frac{x}{d}$	Domain	Steel rebar area (cm²)
-3150.000	26	2.575	0.026	0.22	2	3.15
1771.875	26	4.578	0.024	0.42	3	1.64

The SINAPI database (04/2020 unburdened) [13] is used to evaluate C_{Ref}, where the individual costs of wooden formwork montage, industrial concrete obtaining, concrete pouring, steel rebar acquisition, and placing of the reinforcement are considered, leading to the value given by $C_{Ref} = R\$\ 678.32$.

2.2. Total expected cost

The initial construction cost is also obtained using the SINAPI database (04/2020 unburdened) [13], and is made non-dimensional by dividing by the reference construction cost C_{Ref}.

Consequences of structural collapse involve the cost of shut-down, costs for removing debris and rebuilding, damage to building contents and surroundings, injury, death, and environmental damage. Since only the cost of reconstruction depends on design safety margins, consequences are considered herein by an independent cost parameter k times the reference cost C_{Ref}. This cost term is also made non-dimensional by dividing by C_{Ref}:

$$C_{collapse}(\boldsymbol{d}) = k\, C_{Ref} \frac{1}{C_{Ref}} = k \tag{2}$$

Following the Joint Committee on Structural Safety (JCSS) [14], a full cost-benefit analysis is recommended for $k \geq 10$. Therefore, the values $k = 10$ and $k = 20$ are considered. The expected cost of collapse is given by the product of collapse cost and collapse probability. Therefore, the total expected cost C_{TE} is obtained by (3):

$$C_{TE}(\boldsymbol{d}) = \frac{C_{construction}(\boldsymbol{d})}{C_{Ref}} + k_{NLC}\, \Phi[-\beta_{\text{NLC}}(\boldsymbol{d}, \boldsymbol{X})] + k_{ICL} p_{ICL}\, \Phi[-\beta_{\text{ICL}}(\boldsymbol{d}, \boldsymbol{X})] \tag{3}$$

where $\Phi[\ \ ]$ is the standard Gaussian cumulative distribution function, β is the reliability index, and p_{ICL} is the internal column loss probability. In view of that, the risk optimization employed herein is given by:

$$\begin{gathered} \text{Find } \boldsymbol{d} \\ \text{which minimizes } C_{TE}(\boldsymbol{d}) \\ \text{subjected to } \boldsymbol{d} \in \mathcal{D} \end{gathered} \tag{4}$$

2.3. Statistics

The uncertainties herein considered are the dead and live loads, as show in Table 1. Despite the live load having a Gumbel distribution, in this manuscript it is considered with normal distribution in order to keep the limit state equations linear and to quickly estimate some failure probabilities inside the SLA loop.

Table 1. Statistics used

Variable	Mean (μ)	C.O.V. (σ/μ)	Reference
Dead load (D)	$1.06\ L_n$	0.12	[12]
Live load, 50 year (L_{50})	$1.00\ L_n$	0.40	[12]

2.4. System single loop approach adapted to risk optimization

In order to increase the efficiency of the optimization process, an adapted system single loop approach (SLA) to the risk optimization is employed herein, allowing a very fast convergence to the optimal design. This technique was first used to solve reliability based design optimization problems of series systems by Liang et al [15], and was later expanded to generic systems by Nguyen et al [16]. Since the method uses the target reliability indexes of the individual components as design variables, the problem presented in (4) is given as:

$$\begin{gathered} find\ \{\boldsymbol{d}, \beta_{Ti}\} \\ which\ minimizes\ C_{ET}(\boldsymbol{d}) \\ while\ subjected\ to\ \{P_{fSYS}(\boldsymbol{d}) \leq P_{fSYS,T}\}\ and\ \{g_{i(NLC)}(\boldsymbol{d}, X) \geq 0, i = 1, \ldots, n_{LS}\}\ and\ \{\boldsymbol{d} \in \mathcal{D}\} \end{gathered} \tag{5}$$

Even though all the limit state equations and its gradients are used in the search of the minimal performance point, only the limit state equations related to the normal loading condition are employed in the constraint. Also, the system failure probability P_{fSYS} located in the constraint is given as:

$$P_{fSYS} = (1 - P_{ICL})P_{f|NLC} + P_{ICL}P_{f|ICL} \tag{5}$$

where $P_{f|NLC}$ is evaluated by the superior unimodal limit obtained by the new design variables β_{Ti}, and $P_{f|ICL}$ is evaluated via First Order Second Moment method (FOSM).

2.5. First order second moment method (FOSM)

Without any consideration to non-Gaussian distributions or possible correlations between the random variables, and considering linear limit state equations, the FOSM leads to the Cornell reliability index given as:

$$\beta(d, X) = \frac{E[g(d, X)]}{\sqrt{Var[g(d, X)]}} \tag{6}$$

The limit state equations used to obtain $\beta(d, X)$, and also used as constraints (except $g_{ICL}(d, X)$) are:

$$\begin{gathered} g_{NLC,POS}(d, X) = M_{R,POS} - M_{NLC,POS} \\ g_{NLC,NEG}(d, X) = M_{R,NEG} - M_{NLC,NEG} \\ g_{ICL}(d, X) = M_{R,POS} - M_{ICL} \end{gathered} \tag{7}$$

where:

$$\begin{gathered} M_{R,POS} = 0.68\, f_{cm}\, b\, x_{inf} \left(d - 0.4x_{inf}\right) \quad M_{R,NEG} = 0.68\, f_{cm}\, b\, x_{sup}\, (d - 0.4x_{sup}) \\ M_{NLC,POS} = \frac{9qL^2}{128} \qquad M_{NLC,NEG} = \frac{qL^2}{8} \qquad M_{ICL} = \frac{(2q)(2L)^2}{8} \\ x_{inf} = \frac{A_{s,inf}\, f_{ym}}{0.68\, b\, f_{cm}} \qquad x_{sup} = \frac{A_{s,sup}\, f_{ym}}{0.68\, b\, f_{cm}} \end{gathered} \tag{8}$$

It must be noticed that for the internal column loss scenario the loading is increased by a factor of 2.0 in order to represent, in the static analysis, the dynamic effects of the sudden support removal, as given by the GSA [6].

3 Optimal design of a continuous reinforced concrete beam

The evolution of the superior and inferior steel rebar areas, and also of the objective function for different values of P_{ICL} are shown in Figs. 2, 3 and 4, respectively.

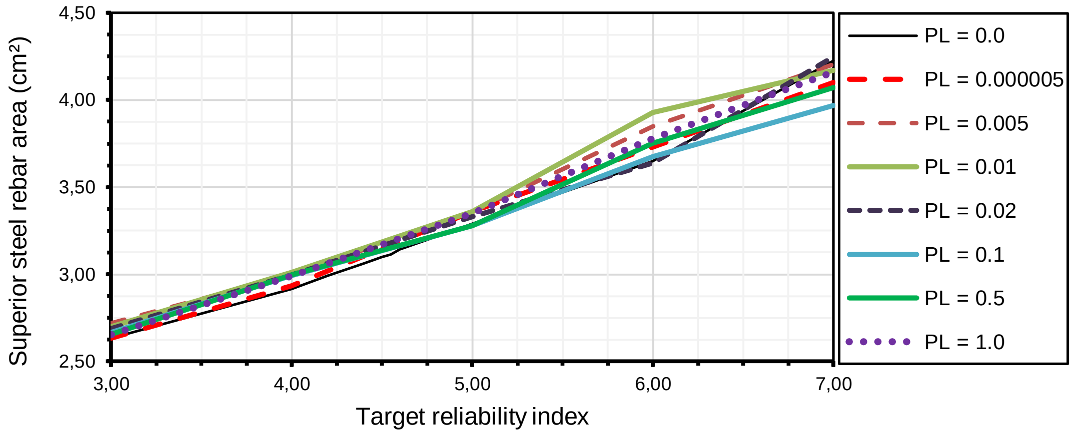

Figure 2. Evolution of $A_{s,sup}$ with $\beta_{T,SYS}$ for different values of P_{ICL}.

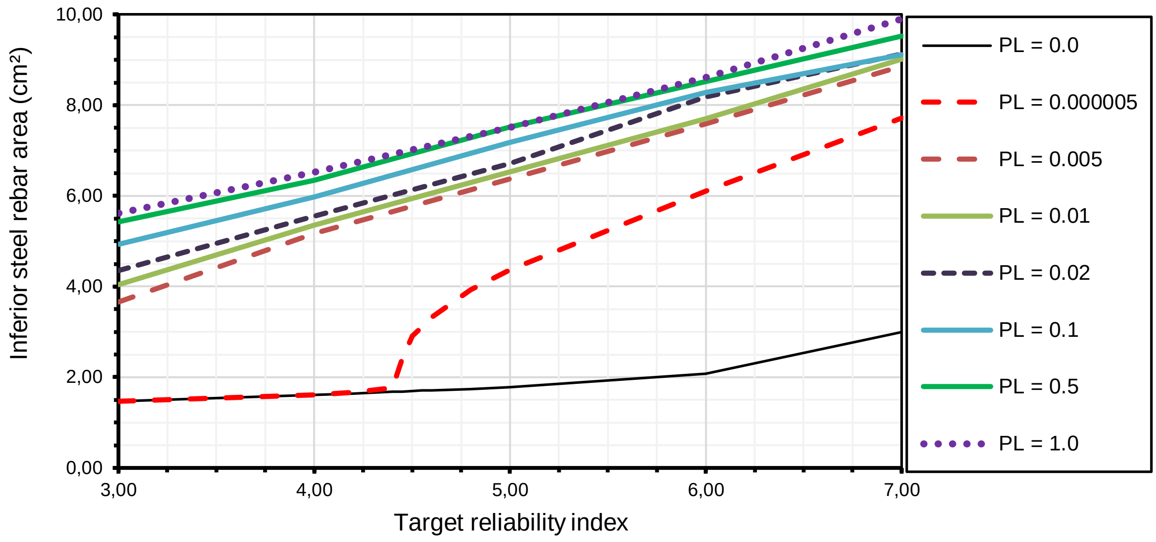

Figure 3. Evolution of $A_{s,inf}$ with $\beta_{T,SYS}$ for different values of P_{ICL}.

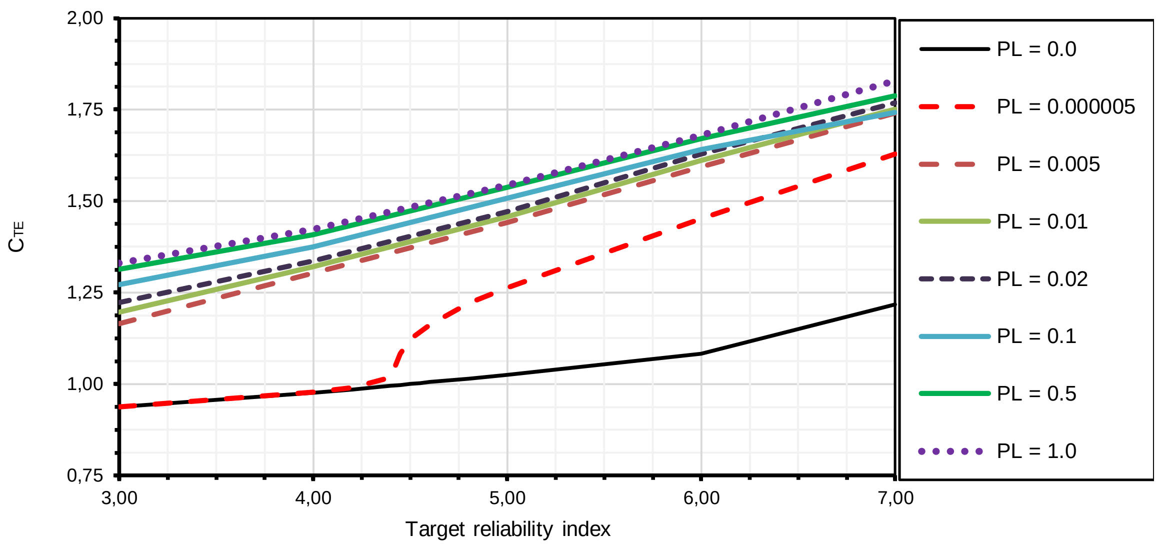

Figure 4. Evolution of C_{TE} with $\beta_{T,SYS}$ for different values of P_{ICL}.

Considering the steel rebar areas as the optimal parameters, it is found that the latent probability of failure substantially increases the inferior steel rebar area (directly affect by the internal column loss) when compared to the design with $P_{ICL} = 0.0$. The greater the value of P_{ICL}, the greater is the optimal inferior steel rebar area for a given target system reliability index.

However, when the smallest latent probability is considered, this optimal steel rebar area is identical to when this P_{ICL} is null, but when the target reliability is over the reliability of the reference design (and also $C_{TE} > 1$), this area increases very fast.

However, as shown in Fig. 3, this behavior is not verified for the superior steel rebar area. Since this rebar is only affected in the no column loss scenario, there would be no reason for this rebar to significantly differ from the original condition ($P_{ICL} = 0.0$) when P_{ICL} increases.

In addition, the surfaces of C_{TE} for different values of P_{ICL} are shown in Fig. 5.

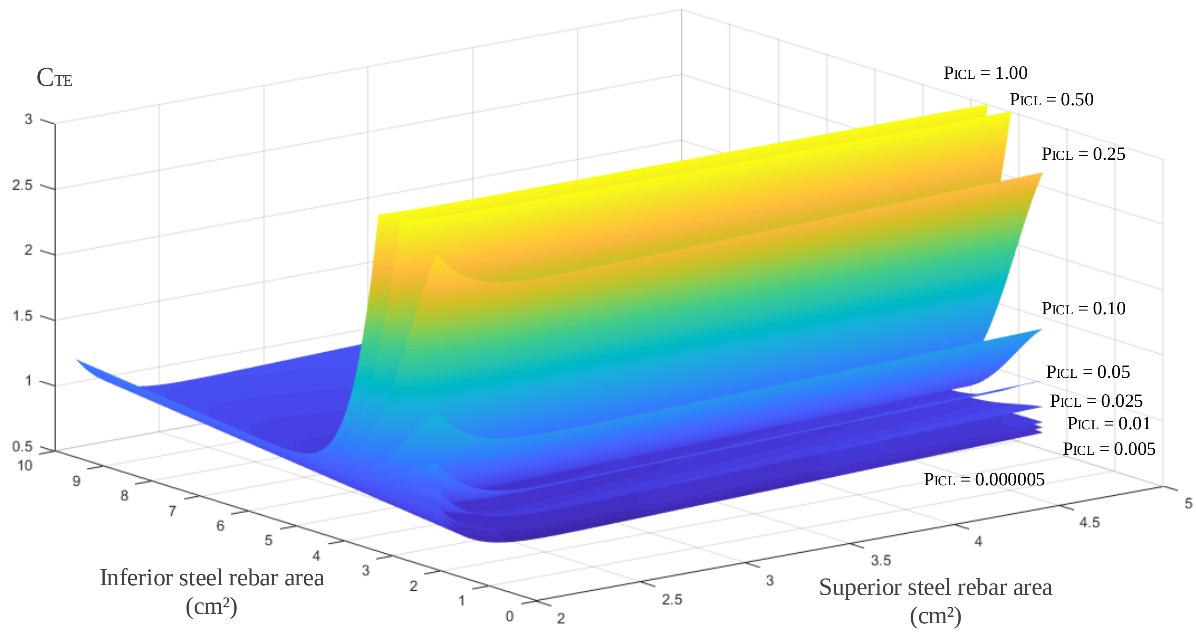

Figure 5. C_{TE} for different values of P_{ICL}.

As shown in Fig. 5 and, in more detail, in Fig. 6, it is identified a plateau in C_{TE}, but only in the direction of the superior rebar area (not affected by the column loss removal) and small values of inferior rebar area. This indicates the presence of a threshold column loss probability for this design variable. This means that, for the superior rebar area, its optimal value is indifferent to the objective consideration of the column loss. The presence of a threshold is in accordance with the evolution of the superior rebar area with the target system reliability shown in Fig. 2, where the total expected costs were almost not affected by the increasing of P_{ICL}.

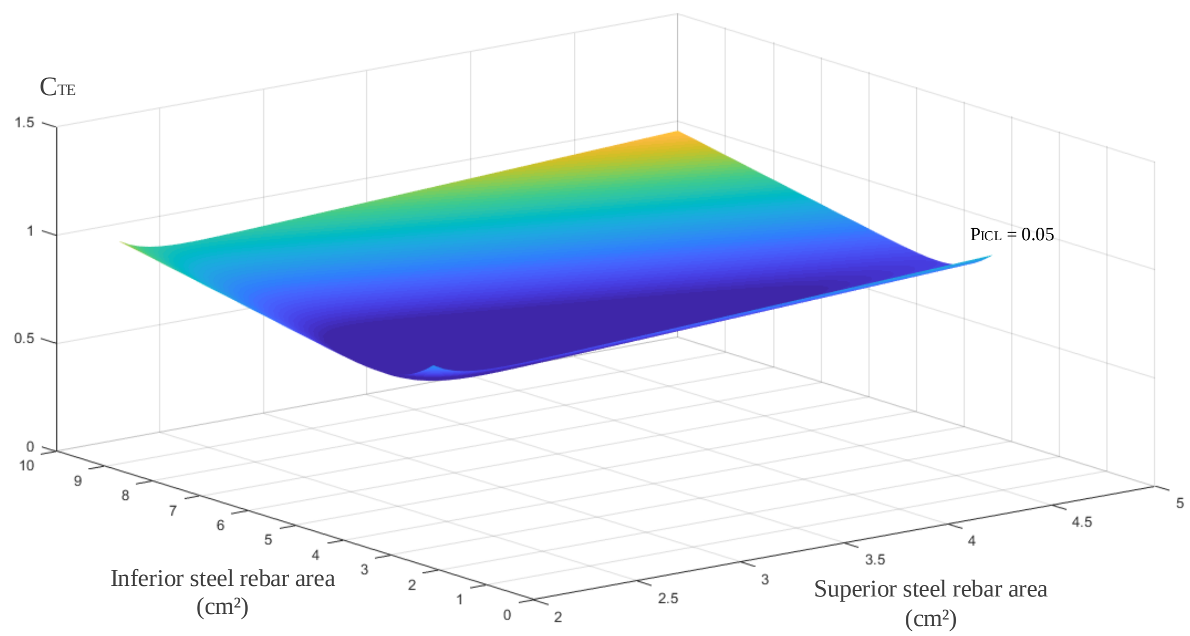

Figure 6. Threshold column loss probability ($P_{ICL} = 0.05$) verified for the superior rebar area .

4 Conclusions

In this manuscript, it is verified the influence of the column loss probability over the optimal design of a continuous RC beam, the global behavior of the total expected costs for the space of project, and a possible threshold column loss probability for the superior rebar area. Also, it is verified the efficiency of the adapted system SLA method for the risk optimization, allowing a very fast convergence of the results. Regarding the steel rebar areas, only the inferior rebar is significantly affected by the column loss scenarios, resulting in optimal values almost 5 times greater for the higher latent probabilities and target reliability indexes. In addition, when the smallest latent probability is considered, the optimal inferior steel rebar area is identical to the case when this probability is null, but only until the total expected cost is equal to the unity, growing very fast after then.

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