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An approach to solve structural reliability problems combining the weighted average simulation method and Kriging

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Abstract. Reliability analyses of structural systems remain a challenge due to the number of performance function calls, associated with the considerable computational efforts necessary for the evaluation of some mechanical system models. Recently, Kriging surrogate models have been employed to provide predictions of the limit state function, in order to reduce the number of required evaluations. However, the accuracy of the results depends on the sample points used to build the surrogate model. Over the last few years, several developments based on learning functions have been done to choose the appropriate sample points. The aim of this paper is to combine Kriging and the weighted average simulation method (WASM) and analyze the performance of three learning functions from the literature, i.e, the U, EFF and UWS functions. The methodology is applied in several examples and the results are compared taking the evaluation of failure probabilities by WASM as a reference. Results show that all active learning functions lead to accurate solutions in terms of failure probabilities. In addition, it was observed that the UWS-function requires a fewer number of sample points to achieve the convergence.

Keywords: Structural reliability, weighted average simulation method, Kriging surrogate model, active learning.

1 Introduction

Reliability analyses have an important role due to the capability of handling with uncertainties, which are present in most of the engineering systems (Xiao et al. [1], Lelièvre et al. [2]). In the past few decades, many reliability methods were developed. The approximation methods such as First Order Reliability Method (FORM) and Second Order Reliability Method (SORM), in which the probability of failure is computed based on the most probable point of failure (Ditlevsen et al. [3], Melchers et al. [4]), and the simulation methods such as Monte Carlo Simulation (MCS) (Fishman [5]). The simulation methods are robust to estimate the probability of failure for highly non-linear problems. However, those methods usually require a large number of samples when dealing with small failure probabilities, making the computational cost one challenge to obtain accurate results.

Over the last years, developments such as Subset Simulation (SS) (Au et al. [6]) and the Weighted Average Simulation Method (WASM) (Rashki et al. 7, Okasha [8]) were developed to improve the computational efficiency of the reliability analyses by simulation. However, these methods may still require many performance function evaluations to achieve convergence. In order to deal with this task, many approaches have been developed combining reliability methods and surrogate models, which include Neural Networks (Chojaczyka et al. [9], Gomes [10], Gomes [11]), Support Vector Regression (Roy et al. [12]) and Kriging. The approaches have been showing promising results especially in terms of number of performance function evaluations.

Recently, the combination of reliability methods and Kriging has been gaining attention, mainly because Kriging can provide both the prediction and its mean square error. Based on the prediction and its variance, several studies have been done to develop learning functions in order to identify potential new sample points to update the Kriging model. The objective is basically to obtain high accuracy for the failure probability without a large set of sample points (Echard et al. [13], Huang et al [14], Peijuan et al. [15], Levière et al. [2], Xiao et al. [1] Xiao(a) et al. [16]). To further reduce the number of performance functions evaluations, in this paper a combination of WASM and Kriging surrogate model is proposed. The surrogate model is built considering a small set of samples and then updated in an iterative process, where three learning functions from the literature are tested. By utilizing the Kriging response, the failure probability is evaluated through WASM and the results obtained by employing each one of the learning functions are compared to those obtained without the use of the surrogate model.

2 Reliability analysis combining WASM and Kriging

2.1 Weighted average simulation method, WASM

The weighted average simulation method is a type of importance sampling estimate, developed by Rashki et al. [7] for determining the probability of failure by considering the weight index of the samples. Following a procedure proposed by Rashki et al. [7], the first step of this method consists of generating a number of samples for each random variable, based on its probability density function, by using the Monte Carlo Simulation (MCS). Then, the minimum and maximum values obtained are selected as the bounds of each random variable. Based on these bounds, the second step consists of generating samples for each random variable following a uniform distribution, the use of Halton sequences is suggested by Xiao et al. [1]. In the third step, the weight index of each sample i, $W(i)$, is determined by:

$$W(i) = \prod_{j=1}^{n} f_j(i) \tag{1}$$

where n is the number of random variables and f_j is the probability density function of the jth variable.

Finally, the probability if failure is computed as:

$$P_f = \frac{\sum_{i=1}^{N} I(i) \cdot W(i)}{\sum_{i=1}^{N} W(i)} \tag{2}$$

where N is the sample size and $I(i)$ is equal to 1 for samples located in the failure region an to 0 to the other samples. In comparison with simple Monte Carlo Simulation, this method requires significantly less samples to achieve convergence.

2.2 Kriging surrogate model

Kriging is an interpolation method that aims to approximate a function from a given dataset of input and output parameters. This model is based on the idea that the performance function $G(\mathbf{x})$, a function of the input parameters, $(\mathbf{x})$, may be seen as the realization of a Gaussian process, given by (Echard et al. [13], Xiao et al. [1]):

$$G(\mathbf{x}) = \mathbf{f}^T(\mathbf{x})\boldsymbol{\beta} + z(\mathbf{x}) \tag{3}$$

where $\mathbf{f}^T(\mathbf{x})\boldsymbol{\beta}$ is the mean value of the Gaussian process and $z(\mathbf{x})$ is a Gaussian process with zero mean and the covariance COV. The covariance between two points $\mathbf{x}_i$ and $\mathbf{x}_j$ is given as follow (Forrester et al. [17]):

$$COV(\mathbf{x}_i, \mathbf{x}_j) = \sigma^2 R_\theta(\mathbf{x}_i, \mathbf{x}_j) \tag{4}$$

where σ^2 is the process variance and R_θ is the correlation function which depends on a set of parameters θ.

There are several correlation functions which can be employed. In this paper the Gaussian correlation function is chosen. It can be formulated as:

$$R_\theta(\mathbf{x}_i, \mathbf{x}_j) = \exp\left[-\sum_{k=1}^{n} \theta_k \left| x_k^i - x_k^j \right|^2\right] \tag{5}$$

where n is the number of random variables and θ_k are the unknown correlation parameters.

Based on the design of experiments $\left[\mathbf{x}^{(1)}, ..., \mathbf{x}^{(p)}\right]$ and the corresponding responses $\left[G(\mathbf{x}^{(1)}), ..., G(\mathbf{x}^{(p)})\right]$, on p training points, the unknown parameters can be estimated according to:

$$\hat{\boldsymbol{\beta}} = (\mathbf{F}^T\mathbf{R}_\theta^{-1}\mathbf{F})^{-1}\mathbf{F}^T\mathbf{R}_\theta^{-1}\mathbf{G} \qquad \hat{\sigma}^2 = \tfrac{1}{p}(\mathbf{G} - \mathbf{F}\hat{\boldsymbol{\beta}})^T\mathbf{R}_\theta^{-1}(\mathbf{G} - \mathbf{F}\hat{\boldsymbol{\beta}}) \tag{6}$$

where $\mathbf{R}_\theta$ is the correlation matrix of all the observed data and σ^2 is the process variance.

At a new training point, $\mathbf{x}$, the prediction of $G(\mathbf{x})$ follows a normal distribution. The mean prediction $\mu_{\hat{G}}(\mathbf{x})$ and the Kriging variance $\sigma^2(\mathbf{x})$, defined as the minimum of the mean squared error between $\hat{G}(\mathbf{x})$ and $G(\mathbf{x})$, can be determined as:

$$\begin{aligned} \mu_{\hat{G}}(\mathbf{x}) &= \mathbf{f}^T(\mathbf{x})\hat{\boldsymbol{\beta}} + \mathbf{r}^T(\mathbf{x})\mathbf{R}_\theta^{-1}(\mathbf{G} - \mathbf{F}\hat{\boldsymbol{\beta}}) \\ \sigma^2_{\hat{G}}(\mathbf{x}) &= \hat{\sigma}^2\left(1 + \mathbf{u}^T(\mathbf{x})(\mathbf{F}^T\mathbf{R}_\theta^{-1}\mathbf{F})^{-1}\mathbf{u}(\mathbf{x}) - \mathbf{r}^T(\mathbf{x})\mathbf{R}_\theta^{-1}\mathbf{r}(\mathbf{x})\right) \end{aligned} \tag{7}$$

where $\mathbf{r}^T(\mathbf{x}) = [R_\theta(\mathbf{x}, \mathbf{x}_1), R_\theta(\mathbf{x}, \mathbf{x}_2), ...R_\theta(\mathbf{x}, \mathbf{x}_p)]$, and $\mathbf{u}(\mathbf{x}) = \mathbf{F}\boldsymbol{R}_\theta^{-1}\mathbf{r}(\mathbf{x}) - \beta(\mathbf{x})$. In this paper, the procedure proposed by Forrester et al. [17] is used to build the Kriging surrogate model and to obtain its predictions.

2.3 Active learning combining Kriging surrogate model and weighted average simulation method

The accuracy of the Kriging prediction is directly dependent on the design of experiment (DoE) used to construct the surrogate model. The active learning method consists of increasing the accuracy of the surrogate model by adding training points in the interest region, which means the vicinity of the limit state function (Huang et al [14]) in reliability problems. The methodology used herein consists of an active learning reliability method combining Kriging and WASM, and three learning functions are used to obtain the new training points.

The first learning function, proposed by Echard et al. [13] in the context of simple Monte Carlo Simulation, consists of identifying the points close to the limit state function having a high Kriging variance, it is given by:

$$U(\mathbf{x}) = \left| \frac{\mu_{\hat{G}}(\mathbf{x})}{\sigma_{\hat{G}}(\mathbf{x})} \right| \tag{8}$$

The point that minimizes the U-function is chosen as the new sample point and added to the DoE. The active learning process is continued until the stopping criterion, corresponding to $\min(U) \geq 2$, is achieved. It corresponds to the case that a sample $\mathbf{x}$ is classified, in the failure or safe region, with a probability of 0.977. Due to its good performance and simplicity, this function has been accepted widely.

The second learning function used in this paper is the Expected Feasibility Function (EFF), developed by Bichon et al. [18] in the context of a multimodal adaptive importance sampling, which represents a balance between the search in the vicinity of limit state function and a global search. This function indicates how well the true value of the performance function in a sample point $\mathbf{x}$ satisfies the equality constraint $G(\mathbf{x}) = z$ over a region $z \pm \epsilon$. It is expressed as:

$$\begin{aligned} EFF(\mathbf{x}) = \left(\hat{G}(\mathbf{x}) - z\right) \left[2\Phi\left(\frac{z-\mu_{\hat{G}}(\mathbf{x})}{\sigma_{\hat{G}}(\mathbf{x})}\right) - \Phi\left(\frac{(z-\epsilon)-\mu_{\hat{G}}(\mathbf{x})}{\sigma_{\hat{G}}(\mathbf{x})}\right) - \Phi\left(\frac{(z+\epsilon)-\mu_{\hat{G}}(\mathbf{x})}{\sigma_{\hat{G}}(\mathbf{x})}\right)\right] \\ -\sigma_{\hat{G}}(\mathbf{x}) \left[2\phi\left(\frac{z-\mu_{\hat{G}}(\mathbf{x})}{\sigma_{\hat{G}}(\mathbf{x})}\right) - \phi\left(\frac{(z-\epsilon)-\mu_{\hat{G}}(\mathbf{x})}{\sigma_{\hat{G}}(\mathbf{x})}\right) - \phi\left(\frac{(z+\epsilon)-\mu_{\hat{G}}(\mathbf{x})}{\sigma_{\hat{G}}(\mathbf{x})}\right)\right] \\ +\epsilon \left[\Phi\left(\frac{(z+\epsilon)-\mu_{\hat{G}}(\mathbf{x})}{\sigma_{\hat{G}}(\mathbf{x})}\right) - \Phi\left(\frac{(z-\epsilon)-\mu_{\hat{G}}(\mathbf{x})}{\sigma_{\hat{G}}(\mathbf{x})}\right)\right] \end{aligned} \tag{9}$$

where Φ and ϕ are the standard normal cumulative distribution function and the standard normal density function, respectively. The value of z is taken as zero in reliability problems, and $\epsilon = 2\sigma_{\hat{G}}(\mathbf{x})$. The sample point which maximizes the EEF is chosen and added to the DoE. The stopping criterion is suggested as $\max(EEF) \leq 10^{-3}$.

The last learning function investigated in this paper is proposed by Xiao et al. [1] for multiples failure modes, in the context of WASM. The learning function and the corresponding stopping criterion are directly linked with the probability of failure. The learning function is defined as:

$$UWS = W_i^2 c_i (1 - c_i) \qquad \begin{cases} c_i = 1 - \prod_{j=1}^{m} \left[1 - \Phi\left(\frac{-\mu_{\hat{G}_j}(\mathbf{x}_i)}{\sigma_{\hat{G}_j}(\mathbf{x}_i)}\right)\right] & \text{for series system} \\ c_i = \prod_{j=1}^{m} \left[\Phi\left(\frac{-\mu_{\hat{G}_j}(\mathbf{x}_i)}{\sigma_{\hat{G}_j}(\mathbf{x}_i)}\right)\right] & \text{for parallel system} \end{cases} . \tag{10}$$

where W_i is the weight index of each sample, defined in Section 2.1, c_i is the expectation of the $I(i)$, defined according to the failure mode of the system and m is the number of failure modes.

The new training point is the one that maximizes the UWS function. When considering multiples failure modes, this method defines which mode should be updated with the new training sample. The failure mode is chosen as the $\max(1/2^{m-1} UWS^*)$, where UWS^* is the value of the learning function at the new training point for each failure mode. The stopping criterion is defined by using the $Var(P_f)$ and the $E(P_f)$ as follow:

$$\frac{\sqrt{Var(P_f)}}{E(P_f)} < \left| \frac{\epsilon_s}{\Phi^{-1}(\alpha/2)} \right| \qquad \text{with:} \qquad Var(P_f) = \frac{1}{\left(\sum_{i=1}^{N} W_i\right)^2} \sum_{i=1}^{N} UWS_i \tag{11}$$

where $E(P_f)$ is the expected probability of failure obtained by using the metamodel, $\epsilon_s \in [5 \times 10^{-4}, 2 \times 10^{-2}]$, $\alpha \in [1 \times 10^{-3}, 2 \times 10^{-2}]$ and N is the sample size. According to Xiao et al. [1], the main advantage of this active learning function is the reduction of the variance in each iteration and the guarantee of a small value for the variance of the probability of failure when the leaning process is stopped.

In this paper, the sample size N and the parameters ϵ and α are chosen in each example. On the other hand, the number p of initial training points is selected as 12 in all them. The computational codes were developed in MATLAB. Figure 1 describes each steps of the methodology adopted herein.

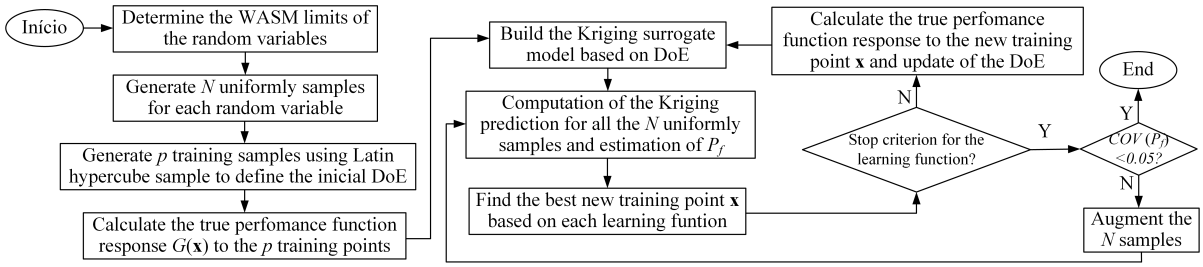

Figure 1. Flowchart of the methodology

3 Numerical Examples

3.1 Series system with four branches

The first example consists of a series system with four branches and two random variables which has been studied in Echard et al. [13], Huang et al. [14] and Liu et al. [19]. The performance function is expressed as:

$$G(X_1, X_2) = \min \begin{cases} 3 + 0.1(X_1 - X_2)^2 - \frac{(X_1 - X_2)}{\sqrt{2}}; 3 + 0.1(X_1 - X_2)^2 + \frac{(X_1 - X_2)}{\sqrt{2}}; \\ (X_1 - X_2) + \frac{7}{\sqrt{2}}; (X_2 - X_1) + \frac{7}{\sqrt{2}} \end{cases} . \tag{12}$$

The random variables X_1 and X_2 are standard normal distributed. The parameters considered in the UWS are set as $\epsilon_s = 1.0 \times 10^{-3}$ and $\alpha = 1.0 \times 10^{-2}$, and the WASM with 10^5 samples is used.

Figure 2 illustrates the added points, starting from the same initial DoE, considering the three different learning functions. It can be observed that for all active learning functions the new samples are added in the regions of interest, which means around the limit state, especially when the UWS is employed.

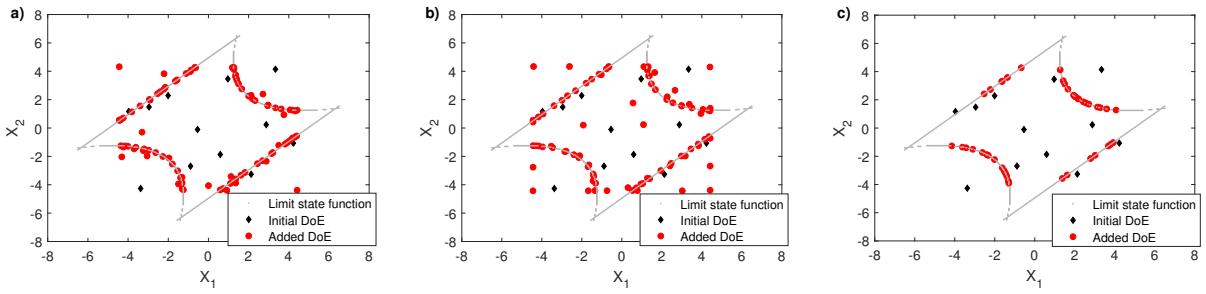

Figure 2. Added points by using the a) U, b) EFF, and c) UWS functions, respectively, for the series system example

The average probability of failure (P_f), obtained over ten runs of the algorithms, the percentage difference (ϵ) and the number of performance functions evaluation (Number of calls) are summarized in Table 1. Figure 3 shows a boxplot of the probability of failure for all runs and the convergence of the failure probability along the adaptive process for one of these runs. As can be seen in Table 1, the combination of WASM and Kriging leads to good results. In particular, WASM-UWS presents a very fast convergence, as can be also seen in Fig. 3.

Table 1. Average results for the series system example

Method	P_f	$\epsilon(\%)$	Number of calls
WASM	2.215×10^{-3}	*	10^5
WASM-U	2.215×10^{-3}	6.0×10^{-3}	12+112.7
WASM-EFF	2.215×10^{-3}	1.3×10^{-2}	12+121
WASM-UWS	2.215×10^{-3}	7.0×10^{-3}	12+30.1

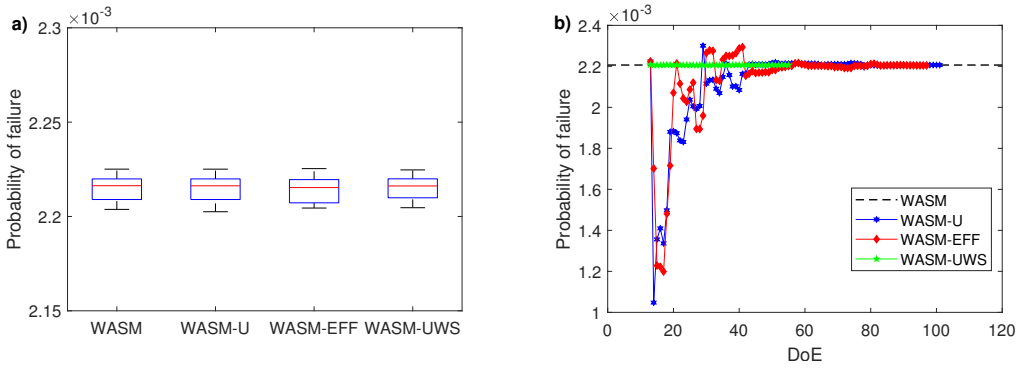

Figure 3. Example 1: a) Boxplot of the probability of failure b) failure probability over the adaptive process

3.2 The modified Rastringin function

The modified Rastringin function is often used in the literature to test the combination of reliability methods and Kriging, as can be seen in Echard et al. [13], Huang et al. [14], Lelièvre et al. [2] and Liu et al. [19]. The performance function is expressed as follows:

$$G(X_1, X_2) = 10 - \sum_{i=1}^{2}(X_i^2 - 5\cos(2\pi X_i)) \tag{13}$$

The random variables X_i are standard normal distributed. The parameters considered in the WASM-UWS are set as $\epsilon_s = 1.0 \times 10^{-3}$, $\alpha = 1.0 \times 10^{-2}$, and the WASM with 10^5 samples is used. Figure 4 illustrates the sampling process for each active learning function. This illustration indicates that WAMS-UWS presents the best results by far.

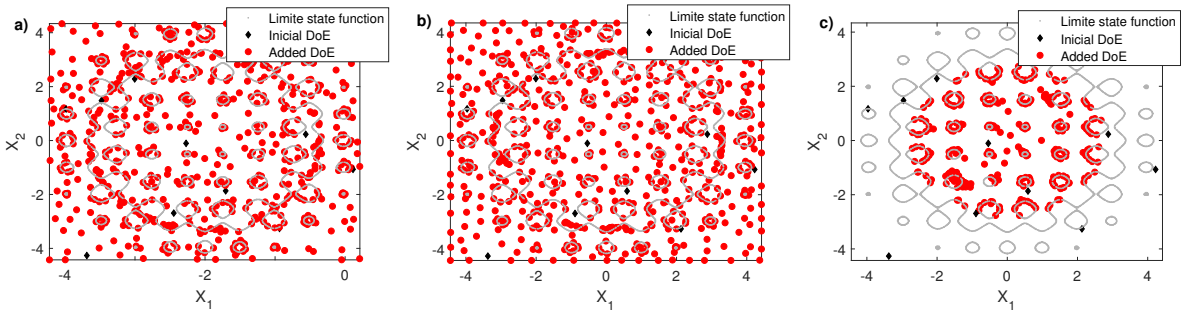

Figure 4. Added points by using the a) U, b) EFF, and c) UWS functions, respectively, for the modified Rastringin example

Table 2 compares the average results obtained for each method. Thought the UWS-function requires fewer samples to achieve convergence, it can be noticed that it also leads to higher differences in comparison with the reference result. However, changes on the parameters of the stopping criteria could lead to more accurate results. Figure 5 shows a boxplot of the probability of failure for all runs and the convergence of the failure probability along the adaptive process for one of these runs. The convergence is slow, but is achieved in all cases, except for the small differences found in the WASM-UWS case.

Table 2. Average results for the modified Rastringin function

Method	P_f	$\epsilon(\%)$	Number of calls
WASM	7.296×10^{-2}	*	10^5
WASM-U	7.296×10^{-2}	1.3×10^{-5}	12+638.1
WASM-EFF	7.296×10^{-2}	6.8×10^{-6}	12+694.4
WASM-UWS	7.307×10^{-2}	1.5×10^{-1}	12+423.7

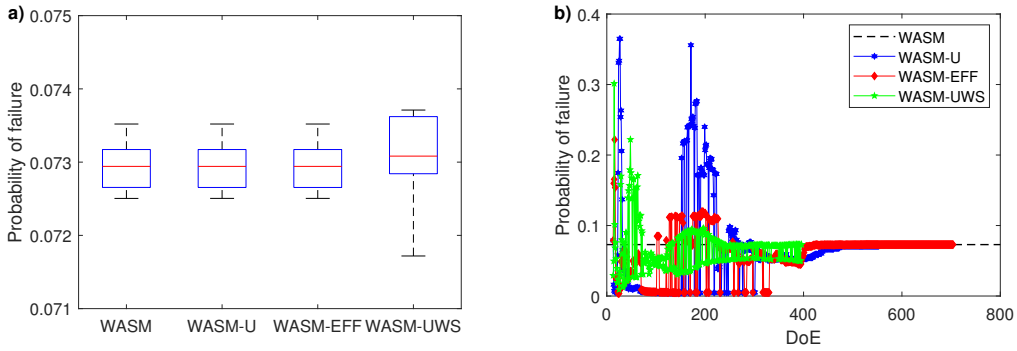

Figure 5. Example 2: a) Boxplot of the probability of failure b) failure probability over the adaptive process

3.3 Dynamic response of a non-linear oscillator

The last example consists of a non-linear undamped single degree of freedom system. It has been studied in Echard et al. [13], Huang et al. [14], Peijuan et al, [15] Lelièvre et al. [2] and Liu et al. [19]. The performance function is given by:

$$G(c_1, c_2, m, r, t_1, F_1) = 3r - \left| \frac{2F_1}{m\omega^2} \sin \frac{\omega t_1}{2} \right|. \tag{14}$$

where $w = \sqrt{(C_1 + C_2)/m}$, The six random variables are normally distributed: $m \sim N(1, 0.05)$, $c_1 \sim N(1, 0.1)$, $c_2 \sim N(0.1, 0.01)$, $r \sim N(0.5, 0.05)$, $F_1 \sim N(1, 0.2)$ and $t_1 \sim N(1, 0.2)$. The stopping criterion considered in the UWS is defined by $\epsilon_s = 5.0 \times 10^{-3}$ and $\alpha = 1.0 \times 10^{-2}$, and the WASM with 10^5 samples is used.

Table 3 compares the average results obtained for each method. Figure 5 shows a boxplot of the probability of failure for all runs and the convergence of the failure probability along the adaptive process for one of these runs. It can be seen that the UWS-function requires fewer samples to achieve convergence, but all cases leads to good results.

Table 3. Average results for the non-linear oscillator

Method	P_f	$\epsilon(\%)$	Number of calls
WASM	2.820×10^{-2}	*	10^5
WASM-U	2.820×10^{-2}	1.0×10^{-4}	12+104.2
WASM-EFF	2.820×10^{-2}	9.0×10^{-3}	12+69.8
WASM-UWS	2.820×10^{-2}	1.0×10^{-2}	12+16.4

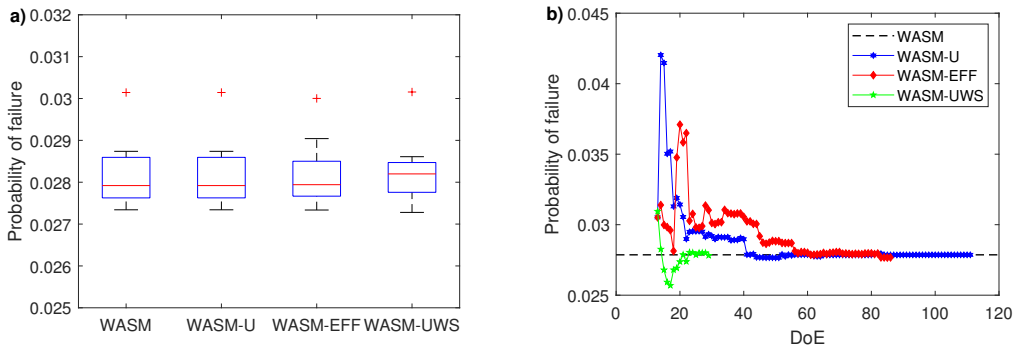

Figure 6. Example 3: a) Boxplot of the probability of failure b) failure probability over the adaptive process

4 Conclusions

This paper combines Kringing and WASM and compares the effectiveness of three learning functions from the literature in this context. Application to three benchmark problems showed that the methodology lead to promising results, in all cases, with points being added in the region of interest and accurate failure probability estimates being obtained. Among the learning functions considered, the UWS led to faster convergence, although a better tuning of the parameters of its stopping criterion is required to further improve its accuracy.

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