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Optimal design of regular frames considering column removal

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Abstract. Designing a frame to withstand the loss of columns has a great impact over the construction costs, and it may not be viable since not all buildings are likely to be subjected to extraordinary events that might remove these supporting elements. In view of that, this paper employs a formulation for the optimal design of framed structures in the design of a continuous beam. This formulation allows independent column loss probability, and combines the intact structural condition with all the column loss conditions in one objective function. It is shown a threshold column loss probability for with the optimal design becomes indifferent to the objective consideration of column loss. Such threshold varies for different structures, column loss scenarios, and the cost multipliers employed. It is also found that designing by a discretionary column removal is only beneficial if the column loss probability is higher than this threshold.

Keywords: column removal; framed structures; progressive collapse; risk optimization; structural reliability.

1 Introduction

A series of extraordinary events leading to partial or full structural collapses are recorded, such as the gas explosion at Ronan Point Tower (UK, 1968), the construction accident at Skyline Plaza (US, 1973), terrorist attacks like those at Oklahoma City (1995) and World Trade Center (NY, 9/11, 2001), and earthquakes like the one at Wenchuan (China, 2008). Even though the abnormal loads generated by these events have very large impact, they have very low probabilities of occurrence, so structural elements are not usually designed to withstand them. Instead, the structural system is designed to withstand the loss of individual elements due to such actions [1]. The probability of collapse of a structure under multiple hazards is evaluated as:

$$p_c = P[C] = \sum_H \sum_{LD} P[C|LD,H]\, P[LD|H]\, P[H] \tag{1}$$

where $P[H]$ is the probability of hazard occurrence; $P[LD|H]$ is the conditional probability of local damage for a given hazard H; $P[C|LD,H]$ is the conditional probability of collapse for a given local damage LD and hazard H. In Eq. (1), the sum over H indicates the multiple hazards the structure is exposed to, such as loads due to vehicular collisions, explosion, fire and terrorist attacks, while the sum over LD represents the different initial damage states the structure can experience, such as local damage, internal/external column loss, penultimate column loss, etc.

The three basic approaches used in order to mitigate the risk against progressive are: controlling the hazard or its rate of occurrence ($P[H]$); controlling or limiting the local damage ($P[LD|H]$); or controlling or limiting the damage propagation, which is related to the total or partial structural collapse ($P[C|LD,H]$).

Reducing the rate of occurrence of extreme loading events ($P[H]$) generally requires non-structural interventions, such as controlling the personal inside the building, construction of physical barriers to deter vehicle impact, education and training, and others. Controlling $P[LD|H]$ involves measures to strengthen the structural element directly affected by the extreme event. However, there is a very large uncertainty surrounding abnormal loads, so designing an element to withstand them is usually non-economical. Since these measures are addressed by risk analysis, both of them are out of the scope of this manuscript.

Modern design codes [2-6] have adopted a damage-tolerant approach, where localized damages due to extreme actions are acceptable if the structural system has enough robustness. Therefore, the limiting $P[C|LD,H]$ approach is the most currently adopted. Since $P[C|LD,H]$ only involves structural analysis, this term can be controlled via redundancy, alternate load paths, compressive arch or catenary actions, structural fuses, segmentation, and others [7,8]. This term is also addressed by specific code requirements by load combinations for extraordinary loading events [4]:

$$\phi\, R_m \geq 1.2\, D_n + 0.5\, L_n \tag{2}$$

where R_m is the mean value of resistance, D_n is the nominal dead load, L_n is the nominal value for live load, and the load factors (1.2 and 0.5) were adjusted over the years from values derived by Ellingwood & Leyendecker [9]. The strength factor ϕ is considered herein as $\phi = 1$, making the interpretation of optimal designs more straightforward. This design equation (eq. 2), alongside $P[C|LD,H]$, are the study objects of this manuscript.

2 Formulation

This work addresses optimal design of a continuous beam subject to usual gravity loads and to column loss scenarios. Normal loading condition can be considered as one particular hazard in Eq. (1), with $H = NLC$ for "Normal Loading Condition" and $P[NLC] = 1$. Since the normal loading condition does not lead to immediate local damage, probability of local or global collapse is evaluated directly as $p_C = P[C|NLC]$.

The two local damage (LD) conditions herein considered are Internal Column Loss (ICL) and External column Loss (ECL). The evaluation of $P[CL|H]$ and $P[H]$ involves a risk analysis addressing the structural purpose and its environment, which involves subjective and epistemic uncertainties, and many non-structural factors [10]. Therefore, the column loss probability, given herein as $p_{CL} = \sum_H P[CL|H]P[H]$, is considered as an independent parameter in this manuscript, making the formulation threat-independent. By treating p_{CL} as an independent parameter, one can investigate the cost-benefit of designing for load bridging over lost columns, and find p_{CL} thresholds for which column loss analysis has positive cost-benefit

Column loss analysis is not required when the threat probability is smaller than the $h = 10^{-7}$ per year [11]. Thus, considering a design life of $t = 50$ years, this is equivalent to $p_{CL}^{min} = h \times t = 5 \times 10^{-6}$. Therefore, p_{CL} is considered, in this paper, as an independent parameter ranging from p_{CL}^{min} up to 1.

Practical design measures, such as providing binding, ductility, structural fuses, compressive arch and catenary actions, are not addressed herein. This manuscript specifically addresses the cost-effectiveness of designing for load bridging under discretionary column removal, the ideal safety margins of design check equations (eq. 2), and the probabilities p_{CL} for which alternate load path analysis has positive cost benefit. This is accomplished by an additional partial factor λ_{PC} in eq. (2):

$$\phi\, R_m \geq \lambda_{PC}(1.2\, D_n + 0.5\, L_n) \tag{3}$$

The initial construction cost is considered proportional to λ_{PC}, and is made non-dimensional by dividing by a reference construction cost ($R_m(\lambda_{PC} = 1)$):

$$C_{construction}(\lambda_{PC}) = \frac{R_m(\lambda_{PC})}{R_m(\lambda_{PC} = 1)} = \lambda_{PC} \tag{4}$$

Consequences of structural collapse involve the cost of shut-down, costs for removing debris and rebuilding, damage to building contents and surroundings, injury, death, and environmental damage. Since only the cost of reconstruction depends on design safety margins, consequences are considered herein by an independent cost parameter k. Thus. monetary consequences of structural collapse are given as a non-dimensional cost multiplier k times the reference cost. This cost term is also made non-dimensional by dividing by the reference cost:

$$C_{collapse} = k\, R_m(\lambda_{PC} = 1)\frac{1}{R_m(\lambda_{PC} = 1)} = k \tag{5}$$

The expected cost of collapse is given by the product of collapse cost and collapse probability. The total expected cost C_{TE} is obtained by multiplying each term of eq. (1) by the respective failure cost term [10]:

$$C_{TE}(\lambda_{PC}) = \lambda_{PC} + k_{NLC}\,\Phi[-\beta_{\mathrm{NLC}}(\lambda_{PC})] + k_{ICL}p_{ICL}\,\Phi[-\beta_{\mathrm{ICL}}(\lambda_{PC})] + k_{ECL}p_{ECL}\,\Phi[-\beta_{\mathrm{ECL}}(\lambda_{PC})] \tag{6}$$

where $\Phi[\ \ ]$ is the standard Gaussian cumulative distribution function, β is the reliability index, and p_{ICL} and p_{ECL} are the column loss probabilities. In view of that, the risk optimization employed herein is given by:

$$\begin{gathered}\text{Find } \lambda_{PL}^{*}\\ \text{which minimizes } C_{TE}(\lambda_{PC})\\ \text{subjected to } \lambda_{PC} > 0\end{gathered} \tag{7}$$

The cost-benefit analysis herein addressed focuses on the compromise between expected costs of collapse and costs of construction. Non-structural aspects are considered by the independent parameters p_{ICL} and p_{ECL} and cost multiplier k. Following the Joint Committee on Structural Safety (JCSS) [12], a full cost-benefit analysis is recommended for $k \geq 10$. In this manuscript, the values $k = 10$ and $k = 20$ (mainly) are considered.

Solution of the optimization problem in eq. (7) leads to the optimum design value λ_{PC}^{*}. This value represents the best cost-benefit that can be achieved in progressive failure analysis under column removal scenarios. Optimal cost-benefit cannot always be achieved, due to other structural and non-structural factors. In this regard, it is useful to define the terms “positive cost-benefit” and “negative cost-benefit”.

$$\begin{gathered}\text{if } C_{TE}(\lambda_{PC}^{new}) < C_{TE}(\lambda_{PC}),\text{ changing } \lambda_{PC} \text{ for } \lambda_{PC}^{new} \text{ has positive cost-benefit}\\ \text{if } C_{TE}(\lambda_{PC}^{new}) \approx C_{TE}(\lambda_{PC}),\text{ changing } \lambda_{PC} \text{ for } \lambda_{PC}^{new} \text{ has neutral cost-benefit}\\ \text{if } C_{TE}(\lambda_{PC}^{new}) > C_{TE}(\lambda_{PC}),\text{ changing } \lambda_{PC} \text{ for } \lambda_{PC}^{new} \text{ has negative cost-benefit}\end{gathered} \tag{8}$$

3 Implementation

3.1. Statistics for design of conventional buildings

This manuscript addresses cost-benefit analysis of a continuous beam, for which conventional load and strength statistics are widely available. Table 1 shows the load and resistance statistics employed herein, most of which have been used in past code calibration work [13,14].

Table 1. Statistics used

Variable	Mean (μ)	C.O.V. (σ/μ)	Distribution	Reference
Yield strength of steel (f_s)	1.28 f_{sk}	0.09	Normal	[13]
Model error - Resistance in bending of steel beams (M_B)	1.02	0.10	Normal	[14]
Plastic moment strength of steel beams (Z) - nondimensional	1.30	0.12	Normal	This paper.
Dead load (D)	1.05 L_n	0.10	Normal	[14]
Live load, arbitrary point in time value, a.p.t. (L_{apt})	0.25 L_n	0.55	Gamma	[14]
Live load, 50 year (L_{50})	1.0 L_n	0.25	Gumbel	[14]

3.2. Usual design

Usual design of steel and RC buildings under gravity loads is given by:

$$\phi R_m \geq 1.2\, D_n + 1.6\, L_n \tag{9}$$

while the limit state function used for normal loading conditions, is:

$$g_{NLC}(\lambda_{PC}, \boldsymbol{X}) = M_I R_I(\lambda_{PC}, f_c, f_y, \dots) - D - L_{50} \tag{10}$$

where $R_I(\quad)$ is the strength function for the intact structure, M_I is a non-dimensional strength model error variable, L_{50} is the fifty-year extreme live load, and $\boldsymbol{X}$ is the vector of random variables.

3.3. Design for progressive collapse

Under exceptional loading conditions, such as in column loss scenarios, the structural elements are designed according to eq. (2) and using mean resistance properties. Also, it is convenient to compare different solutions in terms of central safety factors, where the central safety for NLC is:

$$\lambda_{NCL} = \frac{R_I(\lambda_{PC}, f_{cm}, f_{ym}, \dots)}{\mu_D + \mu_{L50}} \tag{11}$$

The combination factor for live load 0.5 in eq. (2) is smaller than the 1.6 of eq. (9) since the damaged structure is not expected to withstand the 50-year extreme live load [7,8]. Therefore, the arbitrary-point-in-time live load (L_{apt}) is considered in column loss analysis, leading to the conditional limit state given as:

$$g_{CL}(\lambda_{PC}, \boldsymbol{X}) = M_{CL} R_{CL}(\lambda_{PC}, f_c, f_y, \dots) - D - L_{apt} \tag{12}$$

where M_{CL} is a non-dimensional strength model error variable, and $R_{CL}(\quad)$ is the strength function under column loss condition. In addition, the central safety factor under column loss condition is evaluated as:

$$\lambda_{CL} = \frac{R_{CL}(\lambda_{PC}, f_{cm}, f_{ym}, \dots)}{\mu_D + \mu_{L_{apt}}} \tag{13}$$

3.4. FOSM and FORM

Collapse probabilities p_C under normal loading or column loss conditions are evaluated as:

$$p_C = \int_{g(\lambda_{PC}, \boldsymbol{X}) \leq 0} f_{\boldsymbol{X}}(\boldsymbol{x}) d\boldsymbol{x} \tag{14}$$

where $f_{\mathbf{X}}(\mathbf{x})$ is the joint density function of the random variable vector $\mathbf{X}$. Under normal loading condition, $g(\lambda_{PC}, \mathbf{X})$ is given in eq. (10), while under column removal scenarios the limit state equation is given by eq. (12).

In this manuscript, collapse probabilities are evaluated by the Cornell reliability index, which is a First Order Second Moment (FOSM) analysis, and by the First Order Reliability Method (FORM). FORM involves a transformation of the random variable vector $\mathbf{X}$ and of the limit state functions $g(\lambda_{PC}, \mathbf{X})$ to standard Gaussian space [15]. The design point is found by solving a constrained optimization problem, so the limit state function is linearized at this point. This leads to the failure probabilities, which are evaluated as:

$$p_C = \Phi[-\beta] \tag{15}$$

where β is the Cornell (FOSM) or Hasofer-Lind (FORM) reliability index, corresponding to the distance from the design point to the origin of standard Gaussian space.

4 Plastic design of continuous steel beam

This manuscripit deals with a six-span continuous steel beam with the same rectangular cross section for all spans, as illustrated in Figure 1. Since the reliability problem has approximate analytical solution, this example is convenient in order to capture the essential aspects of the cost-benefit optimization problem for load bridging under column loss scenarios. The problem is solved for nominal live-to-dead load ratios of $L_n/D_n = 1$ and $L_n/D_n = 3$. In addition, solutions are non-dimensional in terms of nominal material strength.

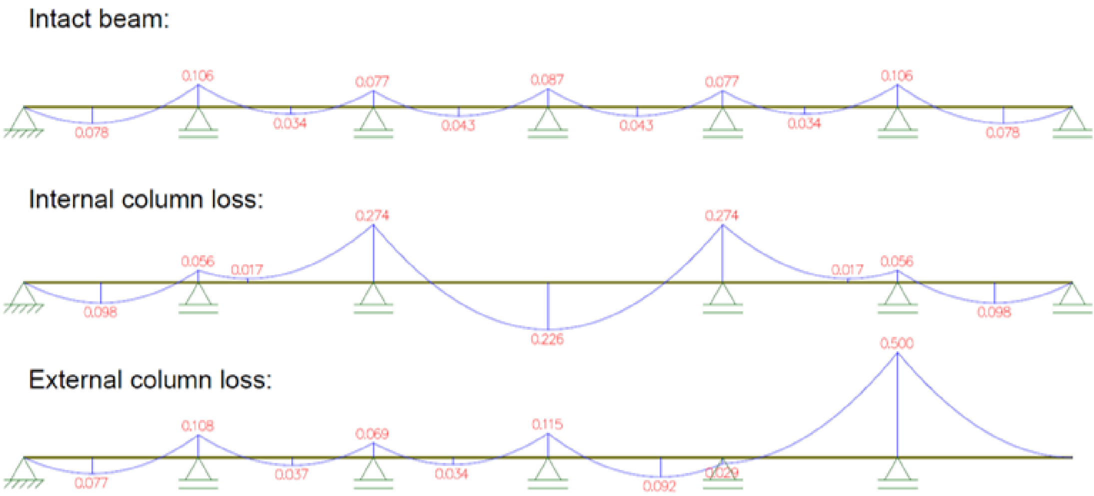

Figure 1. Bending moments on the beam subjected to a unitary distributed load for every loading condition.

Usual static analysis yields the maximum bending moments factors for each of the loading conditions considered herein, which are $m_{NLC} = 0.106$, $m_{ICL} = 0.274$, $m_{ECL} = 0.5$. A closed form solution for reliability index is obtained by considering resistance as the non-dimensional plastic modulus multiplied by plastic moment z_P, and by approximating load distributions as Gaussian. This makes the limit state linear in Gaussian random variables. The Cornell reliability index (FOSM) is written in terms of bending moment factors (m) as:

$$\beta(\lambda_{PC}, m) = \frac{E[g(\lambda_{PC}, X)]}{\sqrt{Var[g(\lambda_{PC}, X)]}} = \frac{z_p \mu_Z \lambda_{PC} - m(\mu_D + \mu_L)}{\sqrt{z_p^2 \sigma_Z^2 \lambda_{PC}^2 + m^2(\sigma_D^2 + \sigma_C^2)}} \tag{16}$$

For normal loading condition (β_{NLC}), the fifty year extreme load (L_{50}) is considered in eq. (16). For column loss conditions, β_{CL} is obtained using L_{apt}. Regarding the FORM solution, it is obtained considering the actual probability distributions alongside the following limit states equations:

$$\begin{aligned} g_{NLC}(\lambda_{PC}, X) &= z_p f_s M_B \lambda_{PC} - m_{NLC}(D + L_{50}) \\ g_{ICL}(\lambda_{PC}, X) &= z_p f_s M_B \lambda_{PC} - m_{ICL}(D + L_{apt}) \\ g_{ECL}(\lambda_{PC}, X) &= z_p f_s M_B \lambda_{PC} - m_{ECL}(D + L_{apt}) \end{aligned} \tag{17}$$

4.1. Results for $L_n = D_n = 1$ using Cornell index

Usual elastic design of the continuous beam, using eq. (9) with $\phi = 0.9$, $L_n = D_n = 1$ and unitary nominal strength leads to a required plastic section modulus $z_P^0 = 1.5\, z_E^0 = 0.49$ (and $\lambda_{NLC}^0 = 2.96$), which does not comply with eq. (2) for column loss scenarios.

The same probability is assumed for internal and external column loss events: $p_{ICL} = p_{ECL} = p_{CL}$, and results are computed for failure cost multiplier $k = 20$. Figure 2 shows the total expected cost objective functions for the continuous beam design for different column loss probabilities. Total expected cost functions grows for large λ_{PC} as a consequence of over-conservative design, and also grows very fast for small λ_{PC} due to cost of collapse failure.

It can be observed in Fig. 2 that column loss probabilities p_{CL} have a great impact in objective functions. For $p_{CL} = 1$ and $p_{CL} = 0.1$, the objective functions are dominated by expected cost of failure in column loss scenarios, and optimal values of λ_{PC} are around or greater than one. For $p_{CL} = p_{CL}^{min}$, column loss has insignificant impact on total expected costs, and the optimum design (for $\lambda_{PC} = 0.46$) is close to the usual design, under normal loading condition, since $\lambda_{NLC}^0 = 2.96$ and $\mu_Z \times \lambda_{NLC} \times \lambda_{PC}/\phi = 1.3 \times 3.91 \times 0.46/0.9 = 2.60$.

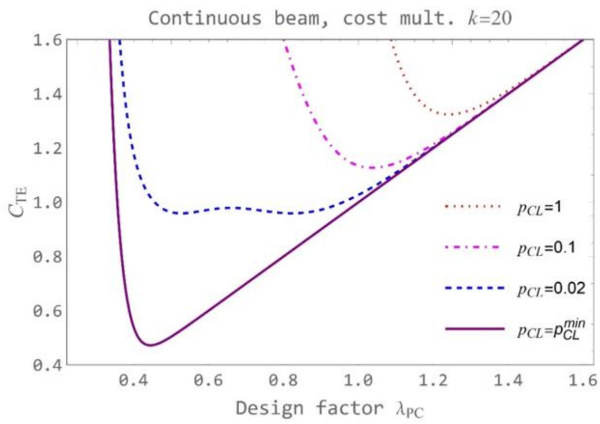

Figure 2. Total expected costs for $k = 20$ and different column loss probabilities

Figure 2 also shows that for $p_{CL} = 0.02$ the objective function becomes flat over a significant range of λ_{PC} values, going from 0.5 to almost 1.0. In this region, the optimal design is indifferent to λ_{PC} due to a trade-off between designing for column loss and for normal loading conditions. Therefore, designing for load bridging under discretionary column removal has positive cost-benefit only when column loss probability is above a threshold value p_{CL}^{th}. This threshold p_{CL}^{th} is herein defined as the value for which design for load bridging under discretionary column removal has neutral cost-benefit, in comparison to usual design.

When column loss probability is larger that this threshold, design for discretionary column removal has positive cost-benefit. For $p_{CL} < p_{CL}^{th}$, column loss design has negative cost-benefit, meaning that the higher constructions costs are not expected to be recovered during the structure lifespan. In addition, for $k = 20$, the column loss probability threshold is $p_{CL}^{th} = 0.02$, but if $k = 10$ is considered, $p_{CL}^{th} = 0.04$. This means the threshold value varies for different structures, different column removal scenarios, and costs multipliers.

4.2. Results for $L_n = 3D_n = 3/2$ using Cornell index and FORM

Since the standard deviation of live loads (L) is much greater than the standard deviation of D, the design for progressive collapse changes significantly by considering $L_n/D_n = 3$. Usual elastic design of the continuous beam, using eq. (9) with $\phi = 0.9$ and $L_n = 3D_n = 3/2$, leads to a required plastic section modulus of $z_P^0 = 1.5\, z_E^0 = 0.53$ (and $\lambda_{NLC}^0 = 3.21$).

Figure 3 shows the total expected cost objective functions for $k = 20$ and for different column loss probabilities (p_{CL}). Results are computed using the Cornell (FOSM) and Hasofer-Lind (FORM) reliability indexes. For this problem, live load uncertainty has greater importance in computed reliabilities. Nevertheless, results obtained using the Cornell reliability index and FORM are similar.

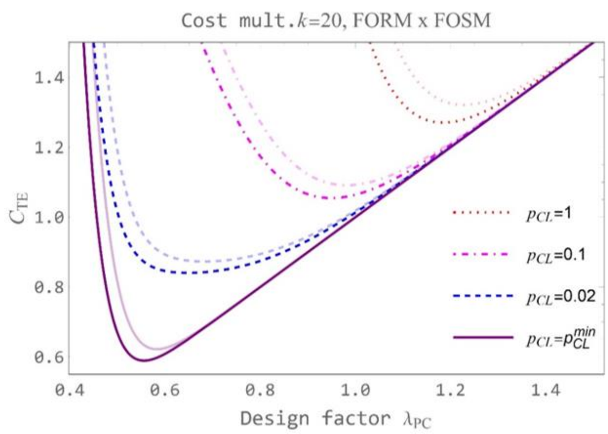

Figure 3. Total expected costs for $L_n = 3D_n = 3/2$ and $k = 20$, FOSM (shaded lines) x FORM (darker lines).

Similarly to $L_n/D_n = 1$, the threshold probability is also around $p_{th} \approx 0.02$. However, for $p_{CL} = p_{th} \approx 0.02$ and $L_n = 3D_n$ the objective function does not show the flat *plateau* observed in Fig. 2. Still, it is noticed that the optimal design is almost indifferent to λ_C in the range $0.55 \lesssim \lambda_C \lesssim 0.8$. Nevertheless, the flat *plateau* of Fig. 3 is not observed since the uncertainty in live load plays a more important role, which "competes" with p_{CL}.

5 Conclusions

In this paper, a proposed formulation for the risk-based cost-benefit analysis of progressive collapse of frame structures under column removal scenarios is applied in a continuous beam while considering an objective function that combines construction costs with expected costs of failure. Not only column loss probability shows significant impact on the total expected cost functions, a threshold value of column loss probability p_{CL}^{th} is noticed, for which optimal design for load bridging under discretionary column removal has neutral cost-benefit. Therefore, design for load bridging under discretionary column removal has positive cost-benefit only when column loss probability is above the threshold p_{CL}^{th}. The column loss probability threshold p_{CL}^{th} varies for different structures, for different column removal scenarios, for different reinforcement actions and as a function of failure cost multiplier. For a reference value $p_{CL}^{th} \approx 0.01$ and a lifetime of 50 year, column loss probability corresponds to an annual rate of column loss $h_{CL}^{th} = 2 \times 10^{-4}$ for $P[CL|H] = 1$. Such value is one to two orders of magnitude larger than the hazard rates for common threats (10^{-6} to 10^{-5} occurrences per year). Therefore, it may not be cost-effective to design frames for load bridging over lost columns for hazard rates smaller than $h_{CL}^{th} = 2 \times 10^{-4}$ per year. Based on results presented in this manuscript, it is also suggested that design factors for exceptional loading could be differentiated according to column loss probability.

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